| PAPER CODE NO. COMP 108 | EXAMINER : Leszek Gasieniec |
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# MAY 2003 EXAMINATIONS

Bachelor of Arts : Year 1
Bachelor of Arts : Year 2
Bachelor of Science : Year 1
Bachelor of Science : Year 2
Master of Mathematics: Year 2
No qualification aimed for: Year 1

**ALGORITHMIC FOUNDATIONS**

TIME ALLOWED : Two Hours

INSTRUCTIONS TO CANDIDATES

Candidates will be assessed on their best four answers.

If you attempt to answer more than the required number of questions (in any section), the marks awarded for the excess questions will be discarded (starting with your lowest mark).

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## Question 1

**1.A** Prove or disprove using a truth table that:

$$\textbf{not}P \textbf{ or } (Q \;=>\; R)$$

is logically equivalent to:

$$(P \;=>\; \textbf{not}Q) \textbf{ or } R$$

[10 marks]

**1.B** Trace the values of $i$ and $j$ in the following algorithm when $m = 3$ and $n = 13$.

**begin**
    *Input* $m, n$;
    $i := 0; j := m$;
    **while** $j < n$ **do**
        **begin**
            $i := i + 1; j := j + m$;
        **end**
    *Output* $i$;
**end**

What is the output of the algorithm for general values of integers $m$ and $n$?

[10 marks]

**1.C** How many distinct rearrangements are there of the letters in the magic spell *ALOHAMORA*.

[5 marks]

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## Question 2

**2.A** Prove, using mathematical induction, that for any integer $n \geq 0$

$$1 + 2^1 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 1.$$

**[10 marks]**

**2.B** The function $f : A \to B$ is given by $f(x) = 1 + \frac{2}{x}$ where $A$ denotes the set of real numbers excluding 0 and $B$ denotes the set of real numbers excluding 1. Show that $f$ is *bijective* and determine the *inverse function*.

Justify your answers.

**[10 marks]**

**2.C** State the definition of two *De Morgan's laws* in algebra of sets. Illustrate the definition of one of De Morgan's laws using the Venn diagrams.

**[5 marks]**

**Question 3**

**3.A** Consider relation $R \subseteq \mathbf{Z} \times \mathbf{Z}$, s.t., for any $x, y \in \mathbf{Z}$, $xRy$ if and only if $x = y + 5n$, for some $n \in \mathbf{Z}$. Prove that $R$ is an *equivalence relation*. List all equivalence classes in relation $R$.

**[15 marks]**

**3.B** A gentleman living in Salt Lake City has 3 wives and 7 children. Prove that there must be two brothers or two sisters coming from the same mother in this lovely family.

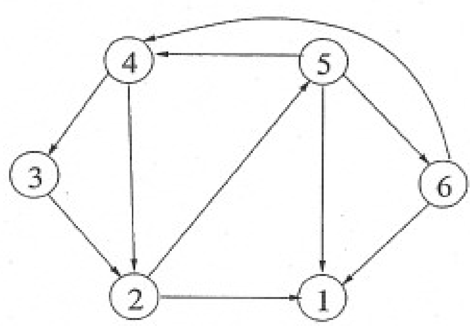
**[10 marks]**

## Question 4

**4.A** A *palindrome* is a sequence of characters that reads the same from left to right and from right to left. For example, strings $ABBA$ and $KAYAK$ are palindromes. Let $\mathcal{C}_n$ be a nonempty sequence of characters $c_1, c_2, c_3, ..., c_n$. Design and write pseudocode of a procedure establishing whether the sequence $\mathcal{C}_n$ represents a palindrome. What is the time complexity of your solution?

**[15 marks]**

**4.B** Consider the following directed graph $G_1$:




1. Explain why graph $G_1$ isn't strongly connected. [2 marks]
2. Propose one additional edge to make $G_1$ strongly connected. [2 marks]
3. What is the in-degree and what is the out-degree of node 2? [2 marks]
4. What is the shortest directed path from node 3 to 4? [2 marks]
5. Is there any cycle in $G_1$ of size 5? [2 marks]

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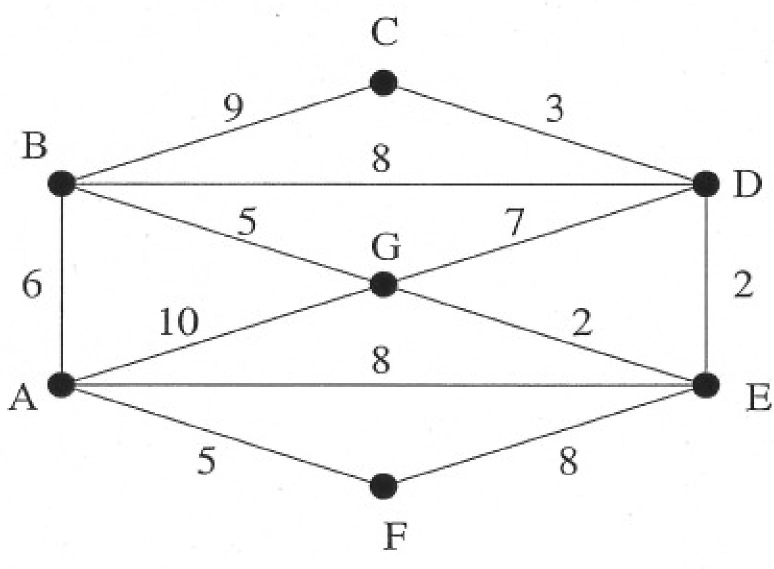
## Question 5

**5.A** The graph $G_2$ represents a road network connecting a set of seven towns: $A, B, C, D, E, F$ and $G$, where all distances are given in miles. Use Kruskal's algorithm to find a road network (a spanning tree in $G_2$) of minimal total length connecting all the towns. List all edges belonging to the final solution in the order they have been selected.




[15 marks]

**5.B** Give a proof by contradiction that for any integer $n$ and $m$

$$n^2 + m^2 \text{ is even} \Rightarrow (n, m \text{ are both } \textit{odd}) \text{ or } (n, m \text{ are both } \textit{even})$$

is true.

[10 marks]