MATH193201

This question paper consists of 2 printed pages, each of which is identified by the reference **MATH1932**.

Examination for the Module MATH1932
(January 2007)

Calculus, Ordinary Differential Equations and Several Variable Calculus

Time allowed: **2 hours**

Attempt not more than four questions
All questions carry equal marks

1. (a) Differentiate the following functions with respect to x

i) $\ln[1+\sin(x)]$, ii) $\tanh^2(x+x^3)$.

(b) The function f is defined by $f(x) = \dfrac{x^2+1}{x^2-4}$.

i) Determine the first and second derivatives of f with respect to x.
ii) Show that f has a single stationary point which is a maximum and find the corresponding stationary value.
iii) Investigate the behaviour of f as $x \to 2$ from above and below.
iv) Sketch the graph of f.

2. (a) Define the functions $\sinh(x)$ and $\cosh(x)$ in terms of the exponential function. Use your definitions to show that

i) $\cosh^2(x) - \sinh^2(x) = 1$, ii) $\sinh(x)\cosh(y) + \sinh(y)\cosh(x) = \sinh(x+y)$.

(b) Determine the following integrals:

i) $\displaystyle\int x\sinh(x)\,dx$, ii) $\displaystyle\int \frac{x-1}{x^2+1}\,dx$, iii) $\displaystyle\int_0^1 \sin^{-1}(x)\,dx$.

3. (a) Find the Taylor series expansion about $x = 0$ in powers of x up to and including the term in x^4 for the function $f(x) = e^{\sin(x)}$.

(b) Show that the Taylor series expansion about $x = 0$ of the function $\dfrac{1}{1-x^2}$ is $\dfrac{1}{1-x^2} = \displaystyle\sum_{n=0}^{\infty} x^{2n}$.

(c) By integrating the series in part (b), find the Taylor series for $\tanh^{-1}(x)$, using the condition $\tanh^{-1}(0) = 0$ to fix the constant of integration.

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4. (a) Find the general solution to the first order differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x} = x^2 \cos(x).$$

(b) Find the solution to the second order differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = 8\cos(x)$$

subject to the boundary conditions $y = 0$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = 0$.

5. (a) Find all of the first and second order partial derivatives of the function

$$f(x, y) = x^3 + 3x^2y - x^2 - y^2 - 1$$

(b) Locate and classify the stationary points of $f(x, y)$ and find the corresponding stationary values.

END