

MATH171501

This question paper consists of 5 printed pages, each of which is identified by the reference **MATH171501**.

Statistical tables are attached at the end of the question paper. Only approved basic scientific calculators may be used.

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Examination for the Module MATH1715
(January 2007)

INTRODUCTION TO PROBABILITY

Time allowed: **2 hours**

Attempt ALL questions in Section A and THREE questions from Section B.

For Section A, only write down a single letter answer for each question.

Section A is worth 40% of the examinations marks.

All questions within each section carry equal marks.

Section A

Attempt ALL questions in Section A.

For each question, write down a single letter answer.

- A1.** In a survey of water supplies to various cities, 35% had type A impurity, 55% had type B and 30% had neither impurity. If a city is chosen at random, what is the probability that it has an impurity in its water supply?

A: 0.45 B: 0.6 C: 0.65 D: 0.7 E: 0.9

- A2.** A retailer offers a replacement policy for a new dishwasher that will pay the full product cost of £300 if it breaks down within a year. The retailer's statistics show that about 4% of dishwashers of that brand will break down within a year. The administrative cost per policy is £3 at the outset plus an extra £10 if a claim has been made. What should be the policy premium (i.e., the amount the customer is charged) in order that the retailer's expected net profit per policy be £5?

A: £19.60 B: £20.40 C: £19.40 D: £21.40 E: £20.34

- A3.** Suppose that 3% of the families in a large city have an annual income of over £60,000. Using a suitable approximation, evaluate the probability that, of 70 random families, at most two have an annual income of over £60,000.

A: 0.62 B: 0.35 C: 0.27 D: 0.38 E: 0.65

- A4.** A doctor has five patients with migraine headaches. He prescribes for all five a drug that relieves the headaches of 82% of such patients. What is the probability that the drug will not relieve the headaches of two of these patients?

A: 0.032 B: 0.551 C: 0.179 D: 0.222 E: 0.956

- A5.** You roll two fair dice until the sum of the two readings on the dice equals 8. How many rolls would you expect to make?

A: 6 B: 7.2 C: 8 D: 11 E: 5.14

- A6.** Every day the average number of wrong phone calls received by a certain mail-order company is four. Assuming that the number of wrong phone calls has a Poisson distribution, what is the probability that the company will receive exactly four wrong phone calls tomorrow?

A: 0.805 B: 0.629 C: 0.567 D: 0.371 E: 0.195

- A7.** The length of an aluminium-coated steel sheet manufactured by a certain factory is approximately normally distributed with mean 75 cm and standard deviation 1 cm. What is the probability that a randomly selected sheet manufactured by this factory is between 74.5 and 75.8 cm?

A: 0.7445 B: 0.4796 C: 0.0966 D: 0.5966 E: 0.5304

- A8.** Suppose that the sex of a child can be modelled as independent rolls of a 35-sided die, with 18 faces marked ‘boy’ and the other 17 marked ‘girl’. Under a stopping rule: “stop after the first son”, what proportion of completed families would comprise at least four children?

A: 0.115 B: 0.056 C: 0.070 D: 0.486 E: 0.136

- A9.** On a multiple-choice exam with five choices for each question, a student either knows the answer to a question or chooses an answer at random. If the probability that the student knows a correct answer is $\frac{2}{3}$, what is the probability that an answer that was marked as correct, was not chosen randomly?

A: $\frac{1}{11}$ B: $\frac{1}{3}$ C: $\frac{10}{11}$ D: $\frac{9}{10}$ E: $\frac{11}{15}$

- A10.** Let X be a random variable with mean 1 and variance 2. Let $Y = -2X + 3$. What is the correlation coefficient between X and Y ?

A: -1 B: -0.5 C: 0 D: 0.5 E: 1

Section B

Attempt THREE questions from Section B.

- B1.** (a) Let A , B , and C be arbitrary events. Using Venn diagrams, determine if the following equality is correct:

$$(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C).$$

- (b) A die is loaded, so that the probability a face turns up is proportional to the number on that face (that is, $1, 2, \dots, 6$). If the die is thrown, what is the probability that an even number occurs?
- (c) The demand for a certain weekly magazine at a newsstand is a random variable X with probability mass function $p_X(x)$ given by

x	3	4	5	6
$p_X(x)$	0.20	0.44	0.32	0.04

The magazine sells for £1.50, and the cost to the owner is 50 pence per copy. Unsold copies cannot be returned to the publisher.

- (i) What is the expected weekly demand for this magazine?
- (ii) If the owner orders three copies of the magazine, his net profit will be £3.00. What will be his expected profit if he orders four copies?
- (iii) How many copies should be ordered per week in order to maximise the expected profit?

- B2.** (a) An event occurs at least once in four independent trials with probability 0.76. What is the probability (to 3 decimal places) of its occurrence in one trial?
- (b) For a continuous random variable X with probability density function $f_X(x)$, state the rule to compute the expected value of $Y = g(X)$, where $y = g(x)$ is a given function.
- (c) Suppose that a random variable X has an exponential distribution with parameter $\lambda = 2$, and let $Y = e^{-X}$.
- Determine the set of possible values of the random variable Y , and obtain its cumulative distribution function $F_Y(y) = P\{Y \leq y\}$.
 - Using the result of part (i) in order to derive the probability density function $f_Y(y)$, or otherwise, evaluate the mean value $E(Y)$.
 - Let Y_1 be the first digit in a decimal expansion of Y . Obtain the probability that $Y_1 = 6$.
- B3.** (a) Four people are chosen at random from six couples. What is the probability that two men and two women are selected?
- (b) An elevator in a building starts with 4 people and stops at 7 floors. If each passenger is equally likely to get off at any floor, independently of the others, what is the probability (to 3 significant figures) that at least two passengers get off at the same floor?
- (c) An insurance company classifies people into one of three classes — good risk (20%), average risk (50%) and bad risk (30%). It also believes that the probability of an accident claim arising in any one-year period is 0.05 for good risk clients, 0.15 for average risk clients and 0.30 for bad risk clients.
- A new client takes out insurance with them. What is the probability that he will make an accident claim in the first year?
 - Given that a policy holder had no accidents in 2006, find the probability that he is of average risk.
- B4.** (a) The time it takes for a student to finish an aptitude test (in hours) has a probability density function of the form
- $$f(x) = \begin{cases} c(1-x)(2-x) & \text{if } 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$
- Determine the constant c .
 - How long will it take on average for a student to complete the test?
 - What is the probability that a randomly selected student will finish the test in less than 78 minutes?
- (b) Let X_1 , X_2 and X_3 be independent random variables with zero mean and variance 1. Determine the covariance between $Y_1 = 2X_1 - X_2$ and $Y_2 = X_2 + 2X_3$.
- (c) Suppose that a cloud seeded with silver iodide will show splendid growth with probability 0.6. Let 54 clouds be seeded with silver iodide. Using a normal approximation with the continuity correction, obtain the probability that at least 32 clouds will show splendid growth.

Normal Distribution Function Tables

The first table gives

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}y^2} dy$$

and this corresponds to the shaded area in the figure to the right. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x . When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with mean zero is symmetric about zero. For interpolation use the formula

$$\Phi(x) \approx \Phi(x_1) + \frac{x - x_1}{x_2 - x_1} (\Phi(x_2) - \Phi(x_1))$$

$$(x_1 < x < x_2)$$

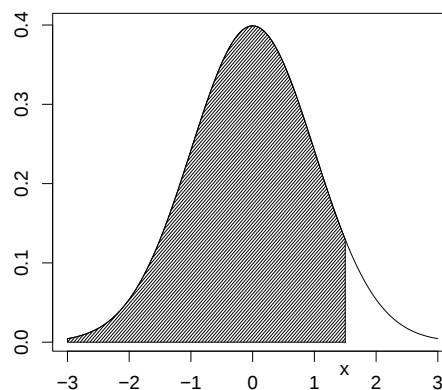


Table 1

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.9798	2.55	0.9946
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.9821	2.60	0.9953
0.15	0.5596	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.9842	2.65	0.9960
0.20	0.5793	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.9861	2.70	0.9965
0.25	0.5987	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.9878	2.75	0.9970
0.30	0.6179	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.9893	2.80	0.9974
0.35	0.6368	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.9906	2.85	0.9978
0.40	0.6554	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.9918	2.90	0.9981
0.45	0.6736	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.9929	2.95	0.9984
0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772	2.50	0.9938	3.00	0.9987

The inverse function $\Phi^{-1}(p)$ is tabulated below for various values of p .

Table 2

p	0.900	0.950	0.975	0.990	0.995	0.999	0.9995
$\Phi^{-1}(p)$	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905

END