MATH037001

This question paper consists of 2 printed pages, each of which is identified by the reference **MATH037001**.

Only approved basic scientific calculators may be used

Examination for the Module MATH0370

(June 2005)

Introduction to Applied Mathematics 2

Time allowed: **2 hours**

Attempt all questions. Marks for each question are given in the right hand margin.

Take $g = 10\text{ms}^{-2}$ unless stated otherwise.

1. The position, at time t, of a toy car is given by $\mathbf{r}(t) = \frac{1}{2}\cos 2t\mathbf{i} - \frac{1}{2}\sin 2t\mathbf{j} + te^{-t}\mathbf{k}$ m. Find

(a) the velocity and acceleration (in vector form) of the car at time t and

(b) the speed of the car at time $t = \frac{\pi}{2}$s.

Show that, at large t, the magnitude of the acceleration tends towards a constant value. (6)

2. A particle of mass 3kg is acted on by a variable force given by $\mathbf{F}(t) = 36\mathbf{i} + 12(t+1)^{-2}\mathbf{j} - 18t\mathbf{k}$. If at time $t = 0$ the velocity of the particle is $\mathbf{v} = -4\mathbf{j} + \mathbf{k}$ find the velocity and momentum at time t. Further, if the particle is at $\mathbf{r} = 0$ initially then find the position of the particle at time $t = 2$. (9)

3. A swimmer who can swim at a speed of 5ms^{-1} in the still water of a swimming pool needs to cross a river whose width is 20m. The river flows at 3ms^{-1} and she sets off directly across the river.

(a) Find the time it takes her to cross the river and the distance she drifts down the river while crossing.

(b) What direction would she need to set off in if she was to cross the river directly?

(c) Why is it not possible for her to cross the river directly if it flows at a speed greater than 5ms^{-1}? (6)

4. A ship is towed at a constant speed by the cables from two tug boats. If the tension in the cables are both 8×10^6N and the water supplies a resistance force of magnitude 11×10^6N find the directions that the towing cables make with the direction of motion. (4)

5. A 120 tonne railway engine pulls two coaches, each of mass 40 tonnes, along a horizontal track with an acceleration of 0.5ms^{-2}. Assuming that the engine and each coach experience a resistive force equal to 0.2 times its mass

(a) draw a diagram showing all the forces acting on the engine and coaches,

(b) find the force transmitted through each coupling, and

(c) find the driving force of the engine. (5)

6. (a) A block of mass 4kg is on the point of slipping down a rough plane inclined at an angle of 30^o to the horizontal. Find the coefficient of friction between the block and the plane.

(b) A horizontal force of $8g$N is able to push the block up the plane. Find the acceleration of the block. (10)

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7. A block of mass $2m$ kg rests on a rough table and is connected to a freely hanging block of mass $5m$ kg by a light, inextensible string that passes over a smooth pulley on the edge of the table. If the freely hanging mass accelerates downwards at $\frac{3}{5}g\text{ms}^{-2}$ find the coefficient of friction between the $2m$ mass and the table and the force on the pulley. (9)

8. A stone is dropped from the top of a vertical cliff. 1 second later another stone is thrown down from the same height with a speed of 16ms^{-1}. If the two stones land at the same time find the height of the cliff. (7)

9. When projected at an angle $\tan^{-1}\left(\frac{3}{4}\right)$, a projectile falls 40m short of a target in a horizontal plane through the point of projection. When the angle of elevation is 45^{o}, the projectile overshoots the target by 50m. Show that the target is at a horizontal distance of 2200m from the point of projection. (8)

10. A ball of mass $4m$ kg is moving with speed $2u\text{ ms}^{-1}$ and collides with a second ball of mass $5m$ kg which is moving with speed $u\text{ ms}^{-1}$ in the opposite direction. If the coefficient of restitution is $\frac{1}{2}$, find the velocities of the balls immediately after the impact and the loss in energy due to the collision. (6)

11. A food parcel of mass 10kg is dropped from a helicopter from a height of 200m. Calculate the loss in energy if the resistance is such that it restricts the speed of the food parcel to 20ms^{-1} at the ground. (2)

12. On a snowy day the coefficient of friction between the road and the tyres of a car is 0.15. At what speed is it safe to round a corner of radius 100m if the road is horizontal? (4)

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