MATH037001

This question paper consists of 3 printed pages, each of which is identified bythe reference MATH0370

UNIVERSITY OF LEEDS©
Examination for the Module MATH0370
(May-June 2004)
INTRODUCTION TO APPLIED MATHEMATICS 2

Time allowed: **2 hours**
Answer ALL questions. A formula sheet is provided
Always state clearly any formula that you use and your reasons for using it.
Take g = 10m/s^2 unless otherwise indicated.
THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION

1. An aircraft is at a location

$$\mathbf{r} = t^{-2}\,\mathbf{i} + t^{-1}\sin(t)\,\mathbf{j} + t\exp(-t)\,\mathbf{k} \quad \text{km}$$

relative to an airfield at time (hours) $t > 0$.
(a) Find the velocity **v** and the acceleration **f** at time t.
(b) Find the speed of the plane at time t.
(c) After a large time (i.e. $t \to \infty$) find the location of the aircraft.

[8 marks]

2. A particle of mass 5 kg is acted upon by a force **F** given by

$$\mathbf{F} = 240\,t^2\,\mathbf{i} + 30\,\mathbf{j} + 60\,t\,\mathbf{k}$$

If at time $t = 0$ the velocity $\mathbf{v} = -\mathbf{i} + 2\,\mathbf{j}$ then find the velocity and the momentum of the particle at time t. Further if the particle is at $\mathbf{r} = 2\,\mathbf{i} + 3\,\mathbf{j} - 4\,\mathbf{k}$ at time $t = 1$ then find the position vector of the particle at time t.

[12 marks]

3. A ship P sets out from a port at time $t = 0$, and has position vector $\mathbf{r} = 5\,\mathbf{i} + 3\,t\,\mathbf{j}$ and another ship Q sets out from another port also at time $t = 0$, and has position vector $\mathbf{r} = 4\,t\,\mathbf{i} + 3\,\mathbf{j}$.
(Distance is measured in km, time in h, and speed in km/h.) Find
(a) the position of Q relative to P.
(b) the velocity of Q relative to P.
(c) the time when Q is nearest to P.

[8 marks]

4. An object is thrown vertically downward. During the tenth second of travel it fell twice as far as during the fifth second. Show that the speed at which it was thrown is 5 m/s.

[10 marks]

5. Four children, each of mass 50 kg, have truck of mass 100 kg. If 2 sit inside the truck and 2 push, the acceleration is 1.0 m/s^2. If 1 sits inside the truck and 3 push, the acceleration is 3.0 m/s^2. Assuming that the resistance to motion is proportional to the sum of the mass of the truck plus the mass of the children inside the truck, find how hard each child pushes (assuming that they all push the same).

[8 marks]

CONTINUED.....

6. An engine of mass 200 tonnes pushes a coach of mass 40 tonnes with an acceleration of 0.2 m/s^2. Draw the diagram showing (i) the force and acceleration of the train, and (ii) the force and the acceleration of the coach. Neglecting resistances, find the force transmitted through the coupling and the driving force of the engine.

[6 marks]

7.A ball is thrown with an intial speed of 20 m/s at an angle of $\pi/6$ to the horizontal from a height of 15 m above a horizontal ground. Air resistance can be neglected and the ball travels in a vertical plane. Find

(a) the time of flight of the ball.
(b) the horizontal distance traveled by the ball before it stikes the ground.
(c) the speed and direction of the ball immeadiately before it hits the ground.

[12 marks]

8. A walker of mass 60 kg climbs a height 300 m in 2 hours. Find

(a) his increase in potential energy.
(b) the power he develops assuming that he is working at a constant rate all the time.

[4 marks]

9. A block A of mass 44 kg is connected to a load B of mass 22 kg by a light inextensible string. The block A is placed on a horizontal table with a coefficient of friction 0.2 and the string passes over a smooth frictionless pulley and the load B hangs freely over the edge of the table.

(a) Find the acceleration of the block A.
(b) A load C is now placed on the top of the block A. Find the minimum load C such that no motion takes place.

[12 marks]

10. A 20 kg mass is accelerated from rest under the action of a force of 40 N. After 4 seconds find the distance travelled by the mass, and its speed and kinetic energy. Further, find the work done by the force in the first 4 seconds.

[6 marks]

11. A billiard ball, A, of mass 0.1kg moves with a constant speed 8 m/s and directly impacts on an identical ball, B, moving towards A with a speed of 2 m/s. If the ball A is brought to rest, what is

(a) the coefficient of restitution between the balls?
(b) the speed at which the ball B starts to move after impact?
(c) the magnidue of the impact on each ball?

[6 marks]

12. The coefficient of friction between a car's tyres and the road is 0 2. At what speed is it safe for the driver to round a corner of radius 20 m if the road surface is horizontal?

[5 marks]

13. A toy car of mass 2 kg is made to move in a circle by means of a sting of length 8 m attached to a pole at the centre of the circle. The car rotates at a constant rate of 3 radians per second. Find the tension in the string.

[3 marks]

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Introduction to Applied Mathematics 2

Formula Sheet

Exponents:

$$x^a x^b = x^{a+b}, \quad a^x b^x = (ab)^x, \quad (x^a)^b = x^{ab}, \quad x^0 = 1, \quad x = e^{\ln x}, \quad x^a = e^{a \ln x}.$$

Logarithms:

$$\ln xy = \ln x + \ln y, \quad \ln x^a = a \ln x, \quad \ln 1 = 0.$$

Trigonometry:

$$\cos 0 = \sin(\pi/2) = 1, \quad \sin 0 = \cos(\pi/2) = 0, \quad \sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2},$$

$$\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2, \quad \cos(\pi/3) = \sin(\pi/6) = 1/2.$$

$$\cos^2\theta + \sin^2\theta = 1,$$

$$\cos(-\theta) = \cos\theta, \quad \sin(-\theta) = -\sin\theta,$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta,$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B, \quad \sin 2\theta = 2 \sin\theta \cos\theta,$$

$$\tan\theta = \sin\theta / \cos\theta, \quad \sec\theta = 1/\cos\theta, \quad 1 + \tan^2\theta = \sec^2\theta.$$

Radians:

$$\pi \text{ rad} = 180^\circ, \quad \pi/2 \text{ rad} = 90^\circ, \quad \pi/4 \text{ rad} = 45^\circ, \quad \pi/3 \text{ rad} = 60^\circ, \quad \pi/6 \text{ rad} = 30^\circ.$$

Quadratic equations:

$$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

END.