MATH037001

This question paper consists of 3 printed pages, each of which is identified by the reference **MATH0370**

UNIVERSITY OF LEEDS©

Examination for the Module MATH0370

(May-June 2003)

INTRODUCTION TO APPLIED MATHEMATICS 2

Time allowed: **2 hours**

Answer ALL questions. A formula sheet is provided.
Always state clearly any formula that you use and your reasons for using it.
Take $\mathbf{g = 10m/s^2}$ unless otherwise indicated.
THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

1. A toy plane is at the location

$$\mathbf{r} = [(1+t)^{-1}\mathbf{i} - 2\exp(-3t)\mathbf{j} + t\exp(-t)\mathbf{k}] \quad \text{metres}$$

relative to a stationary child at time $t \geq 0$.

(a) Find the velocity $\mathbf{v}$ and the acceleration $\mathbf{f}$ at time t.

(b) Find the speed of the plane at time t.

(c) After a large time (i.e. $t \to \infty$) find the location of the plane and its speed.

[9 marks]

2. Find the position $\mathbf{r}(t)$ of a projectile given that its acceleration is given by

$$\mathbf{f} = 48t^2\mathbf{i} + 6\mathbf{j} + 12t\mathbf{k}$$

and that at time $t = 0$, $\mathbf{r}(0) = 3\mathbf{i} + 5\mathbf{k}$, and $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$. [7 marks]

3. The driver of a car travelling due west on a straight road at 40km/hr is watching a train moving due south at 30km/hr. What is the apparent velocity, speed and direction of the train? [7 marks]

4. A stone is **dropped** from the top of a cliff. One second later another stone is thrown downwards from the same height at 11 m/s. The two stones land at the same time. Find the height of the cliff. [9 marks]

5. A tug tows 3 barges in line and has an acceleration of 1 m/s^2. Each barge weighs 80 tonnes and the water offers a resistance of 625 kN to each barge. If the towing ropes are horizontal and along the line of the barges, find the tension in each coupling. [9 marks]

6. A block mass 40kg slides on a rough slope with a constant acceleration of 2 m/s^2. The slope is inclined at 30° to the horizontal. Find the frictional force and the coefficient of friction. [8 marks]

CONTINUED....

7. A block of unknown mass M is connected to a 20kg load hanging over the edge of a table by a light inextensible string which passes over a smooth frictionless pulley. The block slides across the horizontal table with an acceleration of 5 m/s^2 and the coefficient of friction between the table and the mass M is 0.5. Find M. [7 marks]

8. A shot is thrown with a initial speed of 20 $\sqrt{2}$ m/s at an angle of $\pi/4$ to the horizontal from a height of 2.05m above a horizontal ground. Air resistance can be neglected and the shot travels in a vertical plane. Find

   (i) the maximum height reached by the shot and the horizontal distance from where it was thrown when it reaches that height.

   (ii) the time of the flight of the shot.

   (iii) the distance from where the shot was thrown to where it hits the ground.

   (iv) the speed and direction of the shot immediately before it hits the ground.

   [19 marks]

9. A mountaineer of mass 120 kg (including his back pack) climbs 240m in 1 hour. What is the power he develops assuming that he is working at a constant rate all the time. [5 marks]

10. A billiard ball, A, is moving with a speed of 12 m/s collides with an identical billiard ball, B, which is at rest. The collision is along the line joining their centres. If the ball B starts to move with a speed of 9 m/s find the coefficient of restitution and the speed of the ball A after impact. [6 marks]

11. A ball of mass 4 kg is dropped from rest at a point 5m above the ground. Find its velocity immediately before it hits the ground. It bounces and immediately after hitting the ground it has a velocity of 2 m/s upwards. What impulse has the ground exerted on the ball? How much kinetic energy was lost during the impact and what has happened to it? [8 marks]

12. On a wet day the coefficient of friction between a car's tyres and the gound is 0.1. At what speed is it safe for the driver to round a corner of radius 25m if the road surface is horizontal? [6 marks]

CONTINUED.....

MATH037001

Introduction to Applied Mathematics 2

Formula Sheet

Exponents:

$$x^a x^b = x^{a+b}, \quad a^x b^x = (ab)^x, \quad (x^a)^b = x^{ab}, \quad x^0 = 1, \quad x = e^{\ln x}, \quad x^a = e^{a \ln x}.$$

Logarithms:

$$\ln xy = \ln x + \ln y, \quad \ln x^a = a \ln x, \quad \ln 1 = 0.$$

Trigonometry:

$$\cos 0 = \sin(\pi/2) = 1, \quad \sin 0 = \cos(\pi/2) = 0, \quad \sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2},$$

$$\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2, \quad \cos(\pi/3) = \sin(\pi/6) = 1/2.$$

$$\cos^2\theta + \sin^2\theta = 1,$$

$$\cos(-\theta) = \cos\theta, \quad \sin(-\theta) = -\sin\theta,$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta,$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B, \quad \sin 2\theta = 2 \sin\theta \cos\theta,$$

$$\tan\theta = \sin\theta / \cos\theta, \quad \sec\theta = 1/\cos\theta, \quad 1 + \tan^2\theta = \sec^2\theta.$$

Radians:

$$\pi \,\text{rad} = 180^\circ, \quad \pi/2 \,\text{rad} = 90^\circ, \quad \pi/4 \,\text{rad} = 45^\circ, \quad \pi/3 \,\text{rad} = 60^\circ, \quad \pi/6 \,\text{rad} = 30^\circ.$$

Quadratic equations:

$$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

END.