MATH021201

This question paper consists of 5 printed pages, each of which is identified by the reference MATH–0212/0222

Examination for the Modules MATH–0212/0222

(May 1999)

ELEMENTARY INTEGRAL CALCULUS
Versions 1, 2

Time allowed : 2 hours

Attempt **all** the questions in Section A and **three** questions from Section B.
All questions in the same Section carry equal marks.
Only approved basic scientific calculators may be used.
A list of formulas which may be quoted is given at the end of the paper.

SECTION A

A1. Find $\displaystyle\int 4x^4\,dx$.

A2. Find $\displaystyle\int 5x^{-7}\,dx$.

A3. Find $\displaystyle\int x^{4/3}\,dx$.

A4. Find $\displaystyle\int (3\sin\theta - 4(\sec\theta)^2)\,d\theta$.

A5. Find $\displaystyle\int e^{3t}\,dt$.

A6. Evaluate $\displaystyle\int_0^{\pi/6} \cos 3u\,du$.

A7. Evaluate $\displaystyle\int_{-5}^{-2} \frac{1}{x}\,dx$ (to three decimal places).

A8. Find $\displaystyle\int \frac{1}{x-2}\,dx$.

A9. Find $\displaystyle\int \frac{1}{(u-4)^2}\,du$.

A10. Find $\displaystyle\int \frac{2}{z^2+1}\,dz$.

continued ...

A11. Find the area between the graph of $y = \cos x$ and the x-axis for x between 0 and $\pi/2$.

A12. Factorise $x^2 - 7x - 18$.

A13. Find the solutions of the equation $8x^2 + 4x = 0$.

A14. Find numbers a and b such that $x^2 + 10x + 16 = (x + a)^2 - b^2$.

A15. Find the centre and radius of the circle whose equation is $x^2 + y^2 + 6x - 4y - 3 = 0$.

A16. Find the solutions (to three decimal places) of the equation $2x^2 + 5x - 1 = 0$.

A17. What is the coefficient of x^2y^2 in $(x + y)^4$?

A18. Find the quotient and the remainder when $x^3 + 3$ is divided by $x^2 - 2$.

A19. Find numbers A and B such that $\dfrac{x-4}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2}$.

A20. Find the area (to one decimal place) of the triangle ABC in which $AB = 7$cm, $BC = 6$cm and $\angle ABC = 60°$.

SECTION B

B1. **(a)** Find $\displaystyle\int \frac{4}{4x-3}\,dx$.

(b) Find numbers A and B such that $\dfrac{14x-5}{(x+2)(4x-3)} = \dfrac{A}{x+2} + \dfrac{B}{4x-3}$. Hence find $\displaystyle\int \frac{14x-5}{(x+2)(4x-3)}\,dx$.

(c) Use the substitution $u = x^4 + 2$ to evaluate $\displaystyle\int_0^1 \frac{x^3}{(x^4+2)^2}\,dx$.

(d) Use the method of integration by parts to find $\displaystyle\int x \sin 2x\,dx$.

B2. **(a)** In the triangle XYZ, $YZ = 14$cm, $\angle XZY = 53°$ and $\angle YXZ = 26°$. Find the length of XZ and the area of the triangle (to two decimal places).

(b) In the quadrilateral $PQRS$, $PQ = 3$cm, $QR = 6$cm, $RS = 5$cm, $PS = 4$cm and $\angle PSR$ is a right angle. Find the length of PR, the angle PQR and the area of the quadrilateral (to two decimal places).

B3. **(a)** Find the coefficient of $x^{12}y^4$ in $(x+y)^{16}$. Hence find the coefficient of x^{12} in $(x+3)^{16}$, and the coefficient of x^{12} in $(3x-1)^{16}$.

(b) Find the quotient and the remainder when $x^5 + 3x^4 + 4x^3 + 2x - 1$ is divided by $x^3 + 2x^2 - 1$.

(c) Find numbers A, B, C such that

$$\frac{4x^2 - 22x + 22}{(x+1)(x-3)^2} = \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}.$$

B4. **(a)** Find the area between the graph of $y = x^2 + 1$ and the x-axis for x between -2 and 2.

(b) By rotating the line $y = 4x$ about the x-axis, find the volume of a cone which has height 2cm and circular base with radius 8cm.

(c) Find the area of the region between the graph of $y = \dfrac{1}{\sqrt{x}}$ and the x-axis for x between 1 and 9. Find the volume obtained by rotating this region about the x-axis. (Give the volume to one decimal place.)

END

Elementary Differential and Integral Calculus
FORMULA SHEET

Exponents

$x^a \cdot x^b = x^{a+b}$, $a^x \cdot b^x = (ab)^x$, $(x^a)^b = x^{ab}$, $x^0 = 1$.

Logarithms

$\log xy = \log x + \log y$, $\log x^a = a \log x$, $\log 1 = 0$, $e^{\log x} = x$, $\log e^y = y$, $a^x = e^{x \log a}$.

Trigonometry

$\cos 0 = \sin \frac{1}{2}\pi = 1$, $\sin 0 = \cos \frac{1}{2}\pi = 0$,
$\cos^2\theta + \sin^2\theta = 1$, $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$,
$\cos(A+B) = \cos A \cos B - \sin A \sin B$, $\cos 2\theta = \cos^2\theta - \sin^2\theta$,
$\sin(A+B) = \sin A \cos B + \cos A \sin B$, $\sin 2\theta = 2\sin\theta\cos\theta$,
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $1 + \tan^2\theta = \sec^2\theta$.

Inverse Functions

$y = \sin^{-1} x$ means $x = \sin y$ and $-\frac{1}{2}\pi \leqslant y \leqslant \frac{1}{2}\pi$.
$y = \cos^{-1} x$ means $x = \cos y$ and $0 \leqslant y \leqslant \pi$.
$y = \tan^{-1} x$ means $x = \tan y$ and $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$.
$y = x^{1/n}$ means $x = y^n$. $y = \log x$ means $x = e^y$.

Alternative Notation

$\arcsin x = \sin^{-1} x$, $\arccos x = \cos^{-1} x$, $\arctan x = \tan^{-1} x$, $\log_e x = \log x$.
Note: $\sin^{-1} x \neq (\sin x)^{-1}$, $\cos^{-1} x \neq (\cos x)^{-1}$, $\tan^{-1} x \neq (\tan x)^{-1}$.
However: $\sin^2 x = (\sin x)^2$, $\cos^2 x = (\cos x)^2$, $\tan^2 x = (\tan x)^2$.

Lines

The line $y = mx + c$ has slope m.

The line through (x_1, y_1) with slope m has equation $y - y_1 = m(x - x_1)$.

The line through (x_1, y_1) and (x_2, y_2) has slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and equation $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$.

The line $y = mx + c$ is perpendicular to the line $y = m'x + c'$ if $mm' = -1$.

Circles

The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

The circle with centre (a, b) and radius r is given by $(x - a)^2 + (y - b)^2 = r^2$.

Triangles

In a triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}; \qquad a^2 = b^2 + c^2 - 2bc\cos A.$$

Pascal's Triangle

$(x+y)^2 = x^2 + 2xy + y^2$, $\quad (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ and so on.

The coefficients in $(x+y)^n$ form the nth row of Pascal's triangle:

$$\begin{array}{ccccccccc} & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

and so on.

Quadratics

If $ax^2 + bx + c = 0$, with $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Calculus

If $y = u + v$ then $\dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$. $\qquad$ If $y = uv$ then $\dfrac{dy}{dx} = \dfrac{du}{dx}v + u\dfrac{dv}{dx}$.

If $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \left\{\dfrac{du}{dx}v - u\dfrac{dv}{dx}\right\} \Big/ v^2$.

$\displaystyle\int (u+v)\,dx = \int u\,dx + \int v\,dx.$ $\qquad$ $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int \frac{du}{dx}v\,dx.$

If y is a function of u where u is a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad \text{and} \quad \int y\frac{du}{dx}\,dx = \int y\,du.$$

Standard Derivatives and Integrals

If $y = x^a$ then $\dfrac{dy}{dx} = a\,x^{a-1}$; and $\displaystyle\int x^a\,dx = \frac{x^{a+1}}{a+1} +$ constant $\quad (a \neq -1)$.

If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$; and $\displaystyle\int \sin x\,dx = -\cos x +$ constant.

If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$; and $\displaystyle\int \cos x\,dx = \sin x +$ constant.

If $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$; and $\displaystyle\int \tan x\,dx = \log|\sec x| +$ constant.

If $y = e^x$ then $\dfrac{dy}{dx} = e^x$; and $\displaystyle\int e^x\,dx = e^x +$ constant.

If $y = \log x$ then $\dfrac{dy}{dx} = \dfrac{1}{x}$; and $\displaystyle\int \frac{1}{x}\,dx = \log|x| +$ constant.

If $y = \sin^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$; and $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x +$ constant.

If $y = \cos^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{1-x^2}}$.

If $y = \tan^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$; and $\displaystyle\int \frac{1}{1+x^2}\,dx = \tan^{-1} x +$ constant.