MATH-013101

Only approved basic scientific calculators may be used.

This question paper consists of 3 printed pages, each of which is identified by the reference MATH-0131, together with 2 pages of formula sheet.

Examination for the Module MATH-0131

(January 2002)

Elementary Differential Calculus: Version 3

Time allowed : 2 hours

Attempt all the questions in Section A and **three** questions from Section B.

Each question in Section A carries 2 marks, and each question in Section B carries 20 marks. You must show your working in answers to all questions. A formula sheet is supplied with this paper.

SECTION A

Attempt all the questions in Section A.

A1. Expand $(x-4)(2x+1)$.

A2. Evaluate $8^{-1/3}$.

A3. Evaluate $\dfrac{3x^2y}{x^{1/2}y^3}$ when $x=4$ and $y=3$.

A4. What is the value of $\log_3 27$?

A5. Factorize $16x^2-1$.

A6. Solve the equation $x^2+2x=8$.

A7. Find the equation of the straight line with gradient 3 passing through the point $(3,2)$.

A8. Find the gradient of the line with equation $2y-6x+5=0$, and the coordinates of the point where it crosses the y-axis.

A9. The angle θ is acute and $\sin\theta=\dfrac{2}{3}$. Find the values of $\cos\theta$ and $\tan\theta$, leaving your answers as exact expressions involving square roots.

continued ...

A10. Find the equation of the circle with centre $(2, 1)$ and radius 2.

A11. Find $\frac{dy}{dx}$ when $y = 2x^7$.

A12. Find $\frac{dy}{dx}$ when $y = x^4 + \frac{1}{x^4}$.

A13. Find $\frac{dy}{dx}$ when $y = 3x^2 - 6x + 7$.

A14. Find $\frac{dy}{dx}$ when $y = (2x + 3)^{10}$.

A15. Find $\frac{dy}{dx}$ when $y = 3\cos x + 2\sin x$.

A16. Find $\frac{dy}{dx}$ when $y = e^{x^2}$.

A17. Find $\frac{dy}{dx}$ when $y = \ln(x^2 + 8)$.

A18. Find $\frac{d^2y}{dx^2}$ when $y = 10x^3 + 11x$.

A19. Without using a calculator, find exact expressions for $\sin\frac{\pi}{4}$ and $\cos\frac{\pi}{4}$.

A20. Find all solutions to the equation $\cos\theta = \frac{1}{\sqrt{2}}$ with $0 \leq \theta \leq 2\pi$.

SECTION B

Attempt **three** questions from Section B.

B1. **(a)** Find the turning points of the curve $y = x^3 - 3x^2 + 4$, and classify them as local maxima or minima. Give a rough sketch of the curve. *You must show all your working.*

Question B1 continues ...

(b) Find the gradient of the curve $y = 4x + x^{-2}$ at the point where $x = 1$.

Find the equation of the tangent to the curve at this point.

B2. **(a)** The points P, Q and R have coordinates $(1, 3)$, $(2, 4)$ and $(-1, -1)$, respectively. Find

(i) the equation of the straight line PQ;

(ii) the coordinates of the midpoint of the line PR;

(iii) the gradient of the line PR;

(iv) the equation of the perpendicular bisector of the line PR;

(v) the coordinates of the point where the perpendicular bisector of PR meets PQ.

(b) A circle has centre C with coordinates $(4, 4)$, and passes through the point A with coordinates $(0, 2)$. Find

(i) the radius of the circle;

(ii) the equation of the circle;

(iii) the gradient of the line AC;

(iv) the equation of the tangent to the circle at A.

B3. **(a)** Sketch the graph of the function $\sin 2x$ in the range $0 \leq x \leq 2\pi$ (radians), labelling the x-axis in multiples of $\pi/4$.

Find all solutions to the equation $\sin 2x = \dfrac{\sqrt{3}}{2}$ in the range $0 \leq x \leq 2\pi$.

(b) Use the identity $\sin^2 x + \cos^2 x = 1$ to find all solutions of the equation
$3\sin^2 x + \cos^2 x = 1 + \sin x$ in the range $0 \leq x \leq 2\pi$.

(c) In a triangle ABC, the angle B is a right angle, the hypoteneuse AC is 10cm, and the side BC is 8cm. The angle A is equal to θ. Find (without a calculator) exact values of $\sin\theta$, $\cos\theta$, $\sin 2\theta$ and $\cos 2\theta$.

B4. Differentiate each of the following functions with respect to x.

(a) $y = (x^4 + 2)^3 + (x^4 + 2)^{-1}$;

(b) $y = (2x^2 - 7x)\sin x$;

(c) $y = e^{3x}\cos 4x$;

(d) $y = \dfrac{\ln x}{x}$;

(e) $y = \arcsin(\sqrt{x})$.

END