King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP3380 Optics

Summer 2003

Time allowed: THREE Hours

Candidates should answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light in free space	c	$= 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck constant	h	$= 6.626 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	k	$= 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

SECTION A – Answer SIX parts of this section

1.1) A small object is located at the bottom of a jar containing a transparent oil of depth d that has a refractive index n. Explain, with the aid of a diagram, why an observer looking vertically down into the jar will see the object at an apparent depth d' that differs from the real depth d. Derive an expression for the apparent depth d'.

What is the difference between the shortest optical path length from the object to the surface of the oil, and the apparent depth d'?

[7 marks]

1.2) With the aid of a diagram, explain briefly how a Young's double-slit interference experiment could be used to measure the spatial coherence of a beam of light.

[7 marks]

1.3) Briefly distinguish between the conditions needed to observe Fraunhofer and Fresnel diffraction.

The diffraction pattern from a single narrow slit of adjustable width is observed. Explain how it is possible to tell whether it is a Fresnel or a Fraunhofer pattern simply by changing the width of the slit.

[7 marks]

1.4) A regular linear array of identical apertures is illuminated by a plane wave of monochromatic light. The diffracted intensity in the far-field can be written as the product of a constant and two other factors, commonly referred to as the 'diffraction envelope' and the 'interference term'. Explain briefly the physical significance of these two factors.

[7 marks]

1.5) Explain briefly whether the following statements are true or false.

The resolution of a lens-based imaging system can be improved by:

a) Using lenses that have smaller numerical apertures.

b) Using illumination that has a shorter wavelength.

c) Adding an additional lens to the final part of the optical system to produce an image with larger magnification.

[7 marks]

1.6) Draw a clearly-labelled diagram that illustrates the Zernike method of phase contrast imaging.

Under what condition can the Zernike method be described as a *linear* method of phase contrast imaging? Explain briefly in what way the Zernike method is *linear*.

[7 marks]

1.7) Explain, with the aid of a diagram, why it is necessary for the light source used to record an off-axis Fresnel hologram to be both spatially and temporally coherent.

[7 marks]

1.8) With the aid of a diagram, explain how it is possible to use white light to view the holographic image produced by a reflection hologram, even though monochromatic illumination is needed to record the hologram.

[7 marks]

SECTION B – Answer TWO questions

2(a) State the boundary conditions that are used when deriving the Fresnel equations for the amplitude reflection and transmission coefficients at the interface between two uniform transparent dielectric media.

[4 marks]

At normal incidence, the amplitude reflection coefficient R for light travelling in a uniform transparent medium with refractive index n_1, that is incident on the boundary with another transparent medium of refractive index n_2, is given by

$$R = \frac{n_1 - n_2}{n_1 + n_2}$$

Derive an expression for the intensity transmittance across the same boundary at normal incidence.

[4 marks]

In a semiconductor laser, the lasing medium is a crystal of gallium arsenide, with a refractive index of 3.6 for the light produced by the lasing transition. Calculate the fraction of the light intensity generated within the crystal that can emerge into free space when it is normally incident on a face of the crystal. Hence explain why simply polishing two opposite faces of the crystal is unlikely to allow the gallium arsenide crystal to act effectively as a resonant cavity.

[4 marks]

Describe an alternative treatment that could be applied to opposite faces of the crystal that is likely to be more effective in producing a resonant cavity.

[4 marks]

(b) Explain briefly why a laser is commonly used as the source of illumination when recording a hologram with visible light.

[3 marks]

Identify three problems that adversely affect the use of a Gabor in-line hologram in forming a 3-dimensional image of an object.

[7 marks]

A Gabor hologram is to be recorded with coherent monochromatic light of wavelength 600 nm using a sheet of photographic emulsion that is capable of resolving 400 lines per mm and lies perpendicular to the direction of the reference wave. Calculate the maximum possible angle between the reference wave and any scattered waves that can contribute to the formation of the hologram.

[4 marks]

3(a) Write down an expression for the amplitude in the far-field diffraction pattern of an aperture with transmission function $f(x,y)$ when it is illuminated by a monochromatic plane wave propagating along a normal to the aperture plane. *Without detailed derivation* outline how this leads to an expression for the diffracted intensity from a regular linear array of N identical apertures, each separated by a distance d, that is of the form

$$I(u,v) = I_0 \left|F(u,v)\right|^2 \left(\frac{\sin N\pi ud}{\sin \pi ud}\right)^2$$

where $F(u,v)$ denotes the Fourier transform of $f(x,y)$. Hence obtain an expression for the values of u at which the principal maxima will occur.

[10 marks]

(b) Write down an expression for the chromatic resolving power of a linear grating with N slits, defining all the symbols used.

[2 marks]

A linear grating with 20 slits is illuminated by green light from a cadmium lamp. This light consists of two closely spaced wavelengths with a mean wavelength of 535.8 nm. A series of bright diffraction spots are observed in the diffraction pattern 1.5 m from the grating. The diffraction spots near the centre of the pattern are evenly spaced with a mean separation of 6 mm in the diffraction plane, except that on moving away from the centre of the pattern every 9th spot is absent. When the 7th order and higher order spots (taking zero at the centre of the pattern) are studied carefully it is seen that there are in fact two spots very close together.

Determine the values of a and d for this grating, and calculate the difference in the wavelengths of the two green cadmium lines.

[10 marks]

(c) Explain briefly how a Fraunhofer diffraction experiment could be used to measure the diameter of a fine cylindrical wire. In the experiment, light of wavelength 550 nm is used. What range of wire diameters could be measured in the experiment you describe?

[8 marks]

4) State briefly the two principal ideas that form the basis of the Abbe theory of image formation, and explain why the concept of a *pupil function* is useful when analysing the behaviour of a lens-based imaging system.

[6 marks]

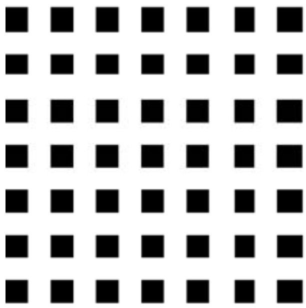
Figure 4.1 shows an object that consists of a regular array of square apertures. Figure 4.2 shows the intensity distribution in the back focal plane of a lens when the object is imaged by the lens, and is illuminated by a coherent monochromatic plane wave. The shaded boxes in Figure 4.2 show the edges of a variable-width slit that is located in the back focal plane.

a) Describe how the appearance of the image will change as the slit width is slowly reduced until it allows only the central horizontal line of spots in the diffraction pattern to pass through.

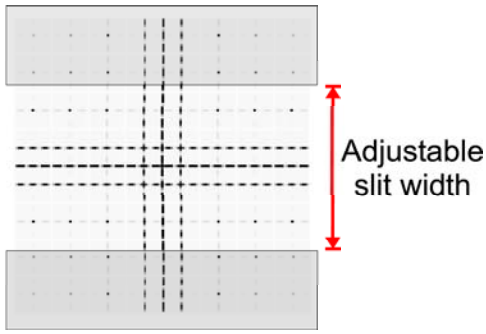
[6 marks]

b) Describe the appearance of the image after the narrow slit in the back focal plane has been rotated through an angle of 45°, so that it allows only one diagonal line of spots in the pattern to pass.

[4 marks]



4.1 Object



4.2 Diffraction pattern

State the mathematical relationship between the *Pupil Function* of an optical system and the *Point Spread Function* for the same system.

[4 marks]

The pupil function for a telescope is defined by a circular aperture of diameter a that is centred on the optical axis, in the back focal plane of the first lens. Compare qualitatively the point spread function for this telescope with the point spread function for another telescope that is identical in all respects *except* that its pupil function is defined by a square aperture of side a.

Comment on the relative merits of the two telescopes. If their imaging resolution is determined using the Rayleigh criterion, explain briefly which of the two optical systems might be more useful when trying to resolve two closely-spaced point-like objects, such as a binary star system.

[10 marks]

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5) Write down an expression for the ratio of the rate of stimulated emission to the rate of spontaneous emission for an equilibrium system with only two energy levels. Use this expression to estimate the wavelength for which this ratio is unity at a temperature of 5000 K.

[7 marks]

Identify four properties of a photon generated by *stimulated emission* that are particularly useful for the successful operation of a laser.

[4 marks]

Explain briefly why a resonant cavity often forms part of a laser light source, and suggest three design features of such a cavity that will enhance the performance of the laser.

[10 marks]

A gas laser with a resonant cavity, 600 mm long, has laser emission at a wavelength of 632.8 nm, with a Doppler-broadened transition width $\Delta\nu \approx 1.3$ GHz. Assuming that the gas refractive index is $n = 1$, determine the number of resonant longitudinal modes that can be sustained by this laser.

[7 marks]

What is the maximum frequency difference possible between the centre of the emission line and the nearest longitudinal mode of the laser?

[2 marks]