King’s College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the Authority of the Academic Board.

B.Sc. EXAMINATION

CP/2720 COMPUTATIONAL PHYSICS

SUMMER 1999

Time allowed: **THREE HOURS**

Candidates must answer any SIX questions from SECTION A, and BOTH questions from SECTION B.

Separate answer books must be used for each section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You can gain marks for later sections of a question even if you cannot do the earlier sections.

TURN OVER WHEN INSTRUCTED

SECTION A – Answer SIX parts of this section

1.1) Explain how rounding errors and truncation errors arise in numerical calculations. What can be done to minimise their effect?

[7 marks]

1.2) Show how the Monte Carlo method can be used to find the area of a circle, and hence to evaluate π.

[7 marks]

1.3) If the sum of a convergent series, after n terms, is S_n, show by comparing it to a geometric series, that a good estimate of its infinite sum is:

$$S'_n = S_{n+1} - \frac{(S_{n+1} - S_n)^2}{S_{n+1} - 2S_n + S_{n-1}}$$

Note that the sum of a geometric series is $\sum_{j=0}^{n} ar^j = a\frac{1-r^{n+1}}{1-r}$.

[7 marks]

1.4) The recurrence relation for Chebyshev polynomials, $T_n(x)$ is:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \quad n \geq 1$$

Show that this relation is only stable for $0<x<1$.

[7 marks]

1.5) Use a Taylor series expansion of $f(x+\Delta x)$ to show that the symmetric difference formula $f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$ gives a more accurate estimate of $f'(x)$ than the one-sided version: $f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$.

[7 marks]

1.6) Explain how to use the Golden Section search method to find the minimum of a function $f(x)$.

[7 marks]

1.7) Explain (in the form of a series of steps) how you would use the simulated annealing technique to estimate the minimum of a function of N variables, $f(x_1, x_2, \ldots, x_N)$. You need not discuss choice of the “temperature” variable or the step size.

[7 marks]

1.8) Describe the fourth-order Runge-Kutta method of solving ordinary differential equations whose boundary conditions are in the form of the initial values of the function and its derivatives.

[7 marks]

SECTION B – Answer BOTH questions

In EACH question, answer just ONE of the parts (a, b, or c).

Explain how you would try to solve the problem numerically, including an explanation of why you would choose the method you describe and how you would assess the accuracy and reliability of the solution. A detailed description of the method is required, not a computer program.

2) Select ONE of the problems (a, b or c) described below:

a) A circular drum skin of radius 30 cm is fixed around its circumference. When it is struck in the centre, the vertical displacement $z(r, \theta, t)$ of the drum skin satisfies the (circularly symmetric) wave equation.:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial z}{\partial r}\right)=\frac{1}{u^2}\frac{\partial^2 z}{\partial t^2}$$

If the initial displacement of the skin is a cone shape, with a maximum displacement of 1 cm at the centre, find the frequency of vibration at the centre given that $u = 120\ \mathrm{ms}^{-1}$.

[30 marks]

b) It takes 5 minutes to hard boil a hen's egg (diameter 4 cm) when it is put into boiling water straight from the fridge (at 0º C). How long does it take to hard boil an ostrich's egg (diameter 20 cm)? (An egg is hard boiled when the centre reaches 80º C). State all your assumptions. The heat conduction equation with spherical symmetry is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{k}\frac{\partial T}{\partial t}.$$

[30 marks]

c) The equation of motion for the angle, θ, of a simple pendulum, of fixed length l, is

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+\frac{g}{l}\sin\theta=0.$$

You usually approximate this by setting $\sin\theta=\theta$, for small θ.

A pendulum of length 1 m is displaced by θ_0 at $t = 0$. Explain how you would solve the full differential equation numerically in order to find the value of the angle θ_0 such that the period of the pendulum is exactly 2.25 s. You should outline two methods of solution, and explain why you chose one of them.

[30 marks]

3) Select ONE of the problems (a, b or c) described below:

a) Describe two methods to evaluate $\int_{-\pi}^{\pi} \frac{\sin x}{x} \mathrm{d}x$. Which of your methods is the more efficient?

[27 marks]

Give a rough estimate of the value of the answer.

[3 marks]

b) An thin isolated conducting wire of length 10 cm has a static charge on it of 1 μC. Consider the potential energy associated with the repulsion of charges at different points along the wire. Describe a numerical method to calculate the distribution of charge along the wire. Sketch the distribution that you would expect.

[30 marks]

c) Outline a numerical method to find all the roots of the equation

$$z^4 + 2z + 2 = 0$$

where z is complex.

[30 marks]

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