King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/1400 Classical Mechanics and Special Relativity

Summer 1998

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

Separate answer books must be used for each Section of the paper.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Acceleration due to gravity at the Earth's surface, $g = 9.8\ \mathrm{m\ s^{-2}}$

Section A - Answer **SIX** parts of this section

1.1) An elastic cord obeys Hooke's law with a constant of proportionality k. Show that the work done in stretching the cord by a distance x beyond its natural length is $0.5kx^2$.

[7 marks]

1.2) State Kepler's three laws of planetary motion.

[7 marks]

1.3) Define *Simple Harmonic Motion.* Write down a differential equation which relates the displacement of a simple harmonic oscillator to the frequency of oscillation, taking care to explain the meaning of each symbol used.

[7 marks]

1.4) What is meant by the *moment of inertia* (I) of a body? Show that the kinetic energy associated with the rotation of a body is $0.5I\omega^2$, where ω is the angular frequency of rotation of the body.

[7 marks]

1.5) The carbon and oxygen atoms in the diatomic molecule carbon monoxide have masses m_C and m_O respectively, and are separated by a distance r. Show that the distance of the centre of mass of the molecule from the oxygen atom is $m_C r/(m_C + m_O)$.

[7 marks]

1.6) State the principles of conservation of *mechanical energy* and of *linear momentum*, indicating any limitations on their applicability.

[7 marks]

1.7) State the physical principle upon which the Special Theory of Relativity is based. What particular consequence does this principle have for the velocity of light?

[7 marks]

1.8) What is meant by the *coefficient of restitution* used in connection with a collision process? What is the value of this coefficient for (a) a perfectly elastic collision and (b) a totally inelastic collision?

[7 marks]

Section B - Answer **TWO** questions from this section

2) A railway truck of mass M moves with a velocity v along a straight track and collides with a stationary truck of mass m. If the impact is perfectly elastic, show that, after the collision, the velocity of the truck of mass M is $\frac{M-m}{M+m}v$ while that of the truck of mass m is $\frac{2M}{M+m}v$. (Derive all equations used. Assume that the trucks remain on the track and neglect the effects of friction and air resistance.)

[15 marks]

Use your result to determine the result of the collision if

(a) the trucks are of equal mass;

(b) the mass M is much greater than m;

(c) the mass m is much greater than M.

[9 marks]

In the general case, determine the result of the collision if the impact is totally inelastic.

[6 marks]

3) Define the terms *angular acceleration* and *torque*. How are the two related?

[6 marks]

Show, from first principles, that the moment of inertia of a uniform solid cylinder about its axis is $0.5mR^2$, where m and R are, respectively, the mass and radius of the cylinder.

[12 marks]

The uniform solid cylinder has a mass of 60 kg and a radius of 250 mm, and is supported with its axis horizontal in such a way that it can rotate freely. A light rope passes over the cylinder and hangs vertically on either side. A crate of mass M is attached to one end of the rope, while the other end is attached to a counterweight of mass $5M$. When the crate is released from rest, it is found to move upwards with a constant acceleration equal in magnitude to half that of free fall. Assuming that the rope does not slip on the cylinder, determine the mass of the crate.

[12 marks]

4) Explain the meaning of the terms (a) *forced vibration* and (b) *resonance* in an oscillatory system.

[6 marks]

A particle of mass m moving on a smooth, horizontal surface is attached to one end of a horizontal spring of force constant k, the other end of which is firmly anchored. The natural frequency of oscillation of this system is given by $\omega_0 = k/m$. The mass experiences an oscillatory horizontal force $F = F_O \sin \omega t$, where the symbols have their usual meaning, directed along the axis of the spring. Obtain an expression for the amplitude of oscillation of the mass when the steady state has been reached, and determine the resonant frequency. (Neglect the effects of friction and viscosity.)

[16 marks]

What is the theoretical maximum amplitude of the oscillation? Why is this not achieved in practice?

[8 marks]

5) Show with the aid of diagrams, but giving no detailed analysis, how the behaviour of light in the Special Theory of Relativity leads to the concept of time dilation. Explain why the Lorentz-Fitzgerald contraction must follow from this concept.

[10 marks]

In experiments to determine the half life of μ-mesons, the half life in the meson's frame of reference is found to be 2.2 μs, while that in the laboratory frame is 4.0 μs. Deduce the speed of the mesons with respect to the laboratory.

[7 marks]

A pole is 2.5 m long in its rest frame. At what speed must it be moving relative to an observer whose measurements indicate that it will fit into a 2.0 m enclosure? What is the length of the enclosure as measured by an observer travelling with the pole?

[7 marks]

Show diagrammatically why clocks that are synchronised in one frame of reference may not appear synchronised to an observer in motion relative to the clocks.

[6 marks]