King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/3240 Theoretical nuclear and particle physics

Summer 2000

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A, and TWO questions from SECTION B.

Separate answer books must be used for each Section of the paper.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College Calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Speed of light $c = 2.998\times10^{8}\ \mathrm{m\,s^{-1}}$

Rest mass of electron $m_e = 0.511\ \mathrm{MeV\,c^{-2}}$

Rest mass of proton $m_p = 1.673\times10^{-27}\ \mathrm{kg} = 938\ \mathrm{MeV\,c^{-2}}$

Rest mass of Z^0 $m_Z \approx 93\ \mathrm{GeV\,c^{-2}}$

Reduced Planck constant $\hbar = 1.055\times10^{-34}\ \mathrm{J\,s}$

SECTION A – Answer SIX parts of this section

1.1 Which property defines whether a particle is a *fermion* or a *boson*? State whether the following particles are fermions or bosons: electron, pion, neutron, strange quark, photon.

[7 marks]

1.2 Three particles, *A*, *B* and *C* decay via the strong, weak and electromagnetic force respectively. Starting with the shortest, put the particles in order of their lifetimes. Given that the range of the strong force is $\sim10^{-15}\ \mathrm{m}$, *estimate* a likely lifetime for particle *A*.

[7 marks]

1.3 By considering the relevant conservation laws determine which of the following reactions and decays are allowed by first-order processes.

(i) $\Sigma^0 \to \Lambda + \gamma$

(ii) $K^0 \to \pi^0 + \gamma$

(iii) $K^+ + n \to \Lambda + \pi^+$

(iv) $K^- + n \to \Lambda + \pi^-$

For those which are allowed, state which force is involved. For those which are not allowed, state a conservation law which is violated.

[7 marks]

1.4 The interaction $e^+ + e^- \to e^+ + e^-$ can proceed in several ways. Draw three possible Feynman diagrams describing the process, and state which force is responsible in each case. At least one of your Feynman diagrams must be geometrically dissimilar to the others.

[7 marks]

1.5 A paper entitled "Observation of a meson with strangeness –2" is submitted for publication in *Physical Review Letters*. Explain why, if true, this observation would be incompatible with the simple quark model. What values of strangeness are allowed in the quark model for mesons and baryons with charges of –1?

[7 marks]

1.6 Briefly describe the processes by which electromagnetic and hadron calorimeters measure the energies of particles. Explain why electromagnetic calorimeters are more accurate than hadron calorimeters.

[7 marks]

SEE NEXT PAGE

1.7 Write down the main processes by which neutrinos are produced in the upper atmosphere following the interactions of primary cosmic rays. Explain why you would expect approximately twice as many muon neutrinos (ν_μ) as electron neutrinos (ν_e) to be produced. State a possible reason why the relative detection rate ν_μ/ν_e in a ground based detector is not equal to two.

[7 marks]

1.8 The strong interaction is considered to take place through the exchange of gluons between quarks. Which discrete quantum number is exchanged? Describe qualitatively an important difference between gluon exchange and the exchange of photons in the electromagnetic interaction.

[7 marks]

SECTION B – Answer TWO questions

2. a) Describe qualitatively how Gell-Mann predicted the charge and strangeness of the Ω^-.

[6 marks]

b) Given that the approximate masses of the particles Δ, Σ^* and Ξ^* are $1230\,\text{MeV}\,c^{-2}$, $1385\,\text{MeV}\,c^{-2}$ and $1530\,\text{MeV}\,c^{-2}$, respectively, *estimate* the mass of the Ω^-.

[4 marks]

c) Gell-Mann's prediction of the properties of the Ω^- helped to establish the credibility of the quark model in which there were three quarks, up, down and strange. Despite this, there were two main early objections to this model. What were they? How did the introduction of the quantum number *colour* help to overcome these objections?

[6 marks]

d) Write down the quark contents of the neutron (n), the negative pion (π^-) and the negative sigma (Σ^-). Draw Feynman diagrams, in which the quarks are explicitly represented, describing the decays

$$\text{(i)}\ \Sigma^- \to n + \pi^-$$

$$\text{(ii)}\ \Sigma^- \to n + e^- + \bar{\nu}_e$$

[6 marks]

e) From the mass data given in part b), *estimate* the mass of the strange quark and hence obtain a value for the mass of a meson containing a strange quark and a strange antiquark. Why is this estimate likely to be inaccurate?

[5 marks]

f) The lightest meson containing a charmed quark and a charmed antiquark has a mass of $3.05\,\text{GeV}\,c^{-2}$. *Estimate* the mass of the charmed quark and state why the value is reasonably accurate.

[3 marks]

3. a) Write down the complete set of spatial symmetry operations which can be applied to a square.

[6 marks]

b) Define the group properties of *closure*, *identity*, *inverse* and *associativity*.

[4 marks]

c) Show, by using selected symmetry operations from part a), that the set satisfies the properties defined in part b).

[6 marks]

The table below gives the Clebsch-Gordan coefficients for combining two states with angular momenta $j_1 = 1$ and $j_2 = 1$. You may use these coefficients to answer parts d), e) and f).

		J:	2	2	1	2	1	0	2	1	2
m_1	m_2	*M:*	+2	+1	+1	0	0	0	–1	–1	–2
+1	+1		1								
+1	0			$\sqrt{1/2}$	$\sqrt{1/2}$						
0	+1			$\sqrt{1/2}$	$-\sqrt{1/2}$						
+1	–1					$\sqrt{1/6}$	$\sqrt{1/2}$	$\sqrt{1/3}$			
0	0					$\sqrt{2/3}$	0	$-\sqrt{1/3}$			
–1	+1					$\sqrt{1/6}$	$-\sqrt{1/2}$	$\sqrt{1/3}$			
0	–1								$\sqrt{1/2}$	$\sqrt{1/2}$	
–1	0								$\sqrt{1/2}$	$-\sqrt{1/2}$	
–1	–1										1

d) Determine the relative amplitudes of the states $|J, M\rangle$ which result from the following combinations of states:

(i) $|j_1, m_1\rangle = |1,+1\rangle$, $|j_2, m_2\rangle = |1,0\rangle$

(ii) $|j_1, m_1\rangle = |1,+1\rangle$, $|j_2, m_2\rangle = |1,-1\rangle$

(iii) $|j_1, m_1\rangle = |1,0\rangle$, $|j_2, m_2\rangle = |1,0\rangle$

[8 marks]

e) Determine the relative amplitudes of the $|1, m_1\rangle$, $|1, m_2\rangle$ combinations that contribute to the state $|J, M\rangle = |0,0\rangle$.

[3 marks]

f) What are the relative intensities of the $|1, m_1\rangle$, $|1, m_2\rangle$ combinations that contribute to $|J, M\rangle = |2,0\rangle$?

[3 marks]

SEE NEXT PAGE

4. a) Define the system of natural units. Hence obtain values, in SI units, for the natural units of length, energy and time.

[6 marks]

b) Define what is meant by (i) the four-momentum of a particle, (ii) a conserved quantity and (iii) an invariant quantity. Calculate the square of the four-momentum of a proton of energy 1 GeV. Give one example of a conserved quantity and one of an invariant quantity.

[9 marks]

c) Show that when a particle of mass m collides with a stationary particle of mass M, the minimum energy required to produce a state of mass M* is, in natural units,

$$E_{\text{threshold}} = \frac{M^{*2} - M^2 - m^2}{2M}.$$

[6 marks]

d) Determine the threshold energy for producing a Z^0 boson when beams of equal energy electrons and positrons are made to collide head on.

[3 marks]

e) Estimate the beam energy required to achieve the same centre-of-mass energy as in part (d) when a beam of electrons strikes a stationary proton target. Comment on your answer.

[6 marks]

5. a) Draw the weight diagrams of the fundamental $SU(3)$ triplet **3** and its conjugate $\overline{\mathbf{3}}$.

[4 marks]

b) Draw the $SU(3)$ weight diagram (4,1), including the site multiplicities, and use it to illustrate the three fundamental properties of $SU(3)$ multiplets.

[10 marks]

c) Show that $\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$ and $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$. Relate these to the $SU(3)$ multiplets representing mesons and baryons.

[16 marks]

FINAL PAGE