King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP2621 Astrophysics

Summer 2005

Time allowed: THREE Hours

**Candidates should answer all SIX parts of SECTION A,
and no more than TWO questions from SECTION B.
No credit will be given for answering further questions.**

The approximate mark for each part of a question is indicated in square brackets.

**You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.**

TURN OVER WHEN INSTRUCTED

Physical Constants

Permittivity of free space	ϵ_0	=	8.854×10^{-12}	$\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Speed of light in free space	c	=	2.998×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	=	6.673×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Elementary charge	e	=	1.602×10^{-19}	C
Electron rest mass	m_{e}	=	9.109×10^{-31}	kg
Unified atomic mass unit	m_{u}	=	1.661×10^{-27}	kg
Proton rest mass	m_{p}	=	1.673×10^{-27}	kg
Neutron rest mass	m_{n}	=	1.675×10^{-27}	kg
Planck constant	h	=	6.626×10^{-34}	$\mathrm{J\,s}$
Boltzmann constant	k_{B}	=	1.381×10^{-23}	$\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.670×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Gas constant	R	=	8.314	$\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_{A}	=	6.022×10^{23}	$\mathrm{mol^{-1}}$
Molar volume of ideal gas at STP		=	2.241×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.013×10^{5}	$\mathrm{N\,m^{-2}}$
Electron volt	$1\,\mathrm{eV}$	=	1.602×10^{-19}	J
Mass of the Sun	$M_{\odot}$	=	1.989×10^{30}	kg
Radius of the Sun	$R_{\odot}$	=	6.960×10^{8}	m

SECTION A – Answer all SIX parts of this section

1.1) A star is observed to have a parallax of $p = 0.02$ arcseconds and an apparent magnitude of $m = 2.0$. What is the star's distance (in parsecs) and its absolute magnitude?

[7 marks]

1.2) The Saha equation gives the ratio of the number, N, of atoms in ionisation stage $(i+1)$ to the number in adjacent ionisation stage i. It can be written as

$$\frac{N_{i+1}}{N_i} = \frac{2k_{\mathrm{B}}T}{p_{\mathrm{e}}}\frac{Z_{i+1}}{Z_i}\left(\frac{2\pi m_{\mathrm{e}}k_{\mathrm{B}}T}{h^2}\right)^{3/2}\exp\left(-\frac{\chi_i}{k_{\mathrm{B}}T}\right),$$

where all the symbols have their usual meanings. Consider a stellar atmosphere composed purely of hydrogen, at a temperature of 12 500 K and an electron pressure of $15\,\mathrm{N\,m^{-2}}$. Given that the partition functions for neutral and for ionised hydrogen are 2 and 1, respectively, and that the ionisation energy of hydrogen is 13.6 eV, use the Saha equation to calculate the ratio of ionised to neutral hydrogen in this stellar atmosphere.

[7 marks]

1.3) One of the equations of stellar structure is

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\kappa(r)\rho(r)L(r)}{64\pi\sigma r^2 T^3}.$$

Define all the terms in this equation and state what physical process the equation represents. Describe the conditions under which the equation becomes invalid.

[7 marks]

1.4) The main-sequence timescale, τ_{ms}, of a star may be roughly estimated as the ratio of the stellar mass, M, to its luminosity, L. In the case of the Sun, $\tau_{\mathrm{ms}} = 1.5 \times 10^{10}$ yrs. By deriving a relationship between τ_{ms} and $M[M_\odot]$, the stellar mass in solar mass units, calculate the main-sequence timescale of a $4\,M_\odot$ star.

[From stellar evolution theories, $L \propto M^{3.5}$.]

[7 marks]

1.5) Explain what is meant by *degenerate electron pressure* and by *helium flash*. State, with reasons, if a helium flash can occur in a non-degenerate gas. Stars of what mass experience a helium flash event, and at what stage of their evolution?

[7 marks]

1.6) Define the *Schwarzschild radius*, $R_{\rm sc}$, and, using semi-classical arguments, show that for a black hole of mass, M, $R_{\rm sc} = 2GM/c^2$. Hence calculate $R_{\rm sc}$ for a $3\,M_\odot$ black hole.

[7 marks]

SECTION B – Answer TWO questions

2a) i) The Boltzmann equation gives the ratio of the number, N, of atoms with energy E_b to the number with energy E_a. It can be written as

$$\frac{N(E_b)}{N(E_a)} = \frac{g_b}{g_a} \exp\left(-\frac{E_b - E_a}{k_{\mathrm{B}}T}\right),$$

where all the symbols have their usual meanings. Consider a gas of *neutral* hydrogen atoms. Given that the ionisation energy of hydrogen is 13.6 eV, use the Boltzmann equation to find the temperature at which there are 10^5 as many atoms with electrons in the ground state as there are with electrons in the first excited state.

[7 marks]

ii) Explain what is meant by Local Thermodynamic Equilibrium (LTE), naming the expressions that describe the source function, the excitation equilibrium and the ionisation equilibrium. Give an example of a region in a star where LTE is a good assumption, stating why this is the case.

[7 marks]

b) i) By considering the pressure and gravitational forces acting on a mass element, $\mathrm{d}M(r)$, of area, $\mathrm{d}A$, and thickness, $\mathrm{d}r$, at a radius, r, from the centre of a sphere of gas of density, $\rho(r)$, derive the equation of hydrostatic equilibrium

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)\rho(r)}{r^2},$$

where $M(r)$ is the mass enclosed at radius, r, and P is the pressure.

[7 marks]

ii) Consider a star to be a sphere of gas of constant density, ρ, in hydrostatic equilibrium. Use the equation of hydrostatic equilibrium and the equation of mass conservation

$$\frac{\mathrm{d}M(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

to show that the central pressure, P_c, of the star is

$$P_c = \frac{3}{8\pi}\frac{GM^2}{R^4}.$$

where M and R are the mass and radius of the star, respectively. Calculate a numerical value for the Sun's central pressure assuming this model. Why is this an underestimate of the true central pressure of the Sun?

[9 marks]

3a) Sketch and label a Hertzsprung-Russell (H-R) diagram. Indicate the main sequence and the location of the Sun, and the approximate positions of the red giant, red supergiant and white dwarf stars.

[7 marks]

b) Describe the evolution of a $1\,M_\odot$ star from the time when hydrogen is exhausted in the core to the onset of the thermal pulses, clearly labelling all stages on the H-R diagram.

[15 marks]

c) Explain how an H-R diagram can be used to estimate the age of a star cluster.

[4 marks]

d) State the Vogt-Russell theorem for stellar evolution. For what type of star does the theorem hold?

[4 marks]

4a) i) List and briefly describe the objects that may be formed in the last stage of evolution of low-, intermediate- and high-mass stars.

[5 marks]

ii) Briefly describe the main stages of core collapse in a massive star, from the onset of photodisintegration of the heavy nuclei to the formation of a compact object.

[10 marks]

b) A white dwarf is supported against gravitational collapse by the degenerate gas pressure, P. This is proportional to a power of the gas density, ρ, according to the relationship

$$P \propto \rho^{5/3}.$$

In hydrostatic equilibrium, an approximate relationship between P and the mass, M, and radius, R, of a white dwarf is

$$P \propto \frac{M^2}{R^4}.$$

Derive a relationship between the radius of a white dwarf and its mass. Comment on the significance of this relationship.

[8 marks]

c) i) Define the *Chandrasekhar limit*, $M_{\rm ch}$.

[2 marks]

ii) A simple treatment for a relativistic white dwarf gives

$$M_{\rm ch} = \frac{5.87}{\mu^2} M_{\odot}.$$

Define μ in this expression. Given that for iron, the atomic weight is 56 and the atomic number is 26, calculate $M_{\rm ch}$.

[5 marks]