King's College London

UNIVERSITY OF LONDON

B.Sc. EXAMINATION

CP/1490 Structure of Matter

Summer 2003

Time allowed: THREE Hours

Candidates should answer SIX parts of SECTION A, and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Planck constant	h	$= 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Electronic charge	e	$= 1.60 \times 10^{-19}\,\mathrm{C}$
Speed of light in free space	c	$= 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Electron mass	m_e	$= 9.11 \times 10^{-31}\,\mathrm{kg}$
Atomic mass unit	u	$= 1.66 \times 10^{-27}\,\mathrm{kg}$
Neutron mass	m_n	$= 1.0087\,u$
Proton mass	m_p	$= 1.0077\,u$
Boltzmann constant	k_B	$= 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Acceleration due to gravity	g	$= 9.81\,\mathrm{m\,s^{-2}}$

SECTION A – Answer SIX parts of this section

1.1) In the Thomson model of the atom, electrons were thought to be embedded in a cloud of positive charge. Describe briefly an experiment which demonstrated that this was not correct and how its interpretation showed that, instead, the mass must be concentrated in a very small nucleus within the atom.

[7 marks]

1.2) In the Bohr model of the hydrogen atom the energy E_n of the nth electron orbital is given by

$$E_n = -\frac{13.6}{n^2}\ \mathrm{eV} \quad .$$

Draw an energy level diagram and indicate on the diagram the three lowest energy transitions to the $n = 1$ level. Determine the wavelengths of the three lines in the emission spectrum of hydrogen corresponding to these transitions.

[7 marks]

1.3) Calculate the speed of a proton whose de Broglie wavelength is 10^{-12} m. Determine the electric potential through which this proton would have to be accelerated to acquire this speed.

[7 marks]

1.4) Briefly explain the origin of Bremsstrahlung radiation emitted when a metal target is irradiated by high energy electrons. Derive an expression relating the minimum wavelength of the Bremsstrahlung radiation to the potential difference through which the electrons are accelerated.

[7 marks]

1.5) Sketch a graph showing the variation of mass number A with atomic number Z for stable nuclei. Indicate on the graph the line $A = 2Z$. Briefly explain the form of the graph.

[7 marks]

1.6) Show that the frequency of oscillation ν of a molecule is related to the form of the inter-atomic potential $V(r)$ about the equilibrium separation, $r = a_0$, by the expression,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{(\mathrm{d}^2V/\mathrm{d}r^2)_{r=a_0}}{m}} \quad ,$$

where m is the reduced mass of the molecule.

[7 marks]

1.7) A volume contains helium gas at a temperature of 80 K and a pressure of 10^5 Pa. Calculate (i) the number of helium atoms per unit volume in the container and (ii) the average kinetic energy of the atoms.

[7 marks]

1.8) Calculate the binding energy per nucleon, in MeV, of the ^{197}Au nucleus. The atomic number of gold is 79 and the atomic mass is 196.967 u.

[7 marks]

SECTION B – Answer TWO questions

2)

a) Describe the motion of a negatively charged particle travelling with a velocity $\mathbf{v}$ when:

i) an electric field $\mathbf{E}$, and

ii) a magnetic field $\mathbf{B}$,

is applied in a direction perpendicular to the initial direction of motion. In each case, write down an expression for the force acting on the particle and draw a sketch indicating the direction of the force, relative to the direction of travel and the field direction.

[10 marks]

b) For a particle with charge q and mass m travelling with velocity $\mathbf{v}$ perpendicular to an electric field $\mathbf{E}$, derive an expression for the deflection of the particle after it has travelled a distance x in the original direction of motion.

[4 marks]

c) Draw a labelled diagram of Thomson's apparatus for measuring the charge to mass ratio e/m of the electron. Explain how the magnetic field is achieved and how the electrons are accelerated.

[8 marks]

d) Explain how Thomson was able to use this apparatus to determine the velocity $\mathbf{v}$ of the electrons and their charge to mass ratio.

[8 marks]

3)

a) Describe the mechanisms responsible for:

i) the emission of an alpha-particle from a nucleus in the alpha decay process, and

[12 marks]

ii) the emission of an electron from a nucleus in the beta decay process.

[8 marks]

b) ^{223}Ra decays by alpha emission with a half-life of 11.4 days. Determine how many helium atoms are created in 7 days from an initially pure sample of 1 g of ^{223}Ra.

[10 marks]

4)

a) Sketch a graph of stress versus strain for a ductile metal wire up to its fracture point. Label on the graph the yield point and the region over which Hooke's law is obeyed.

[4 marks]

A metal wire is placed under a stress τ_{max}, such that the wire suffers plastic deformation. Indicate on your graph the variation of stress versus strain both as the stress is increased, from zero to τ_{max}, and then as the stress is reduced from τ_{max} back to zero. Label on the graph the permanent strain.

[4 marks]

b) Describe the physical processes that give rise to the form of the graph.

[12 marks]

c) A suspension bridge is supported by 100 steel suspension rods of diameter 2 cm. Calculate (i) the number of cars the bridge can carry simultaneously before a permanent strain is induced in the suspension rods, and (ii) the strain in the rods when there are 100 cars on the bridge. You may assume the mass of each car is 1000 kg, and the mass of the bridge is 5×10^5 kg. The Young modulus for steel is $2 \times 10^{11}\,\mathrm{N\,m^{-2}}$ and the yield strength of steel is $2.5 \times 10^8\,\mathrm{N\,m^{-2}}$.

[10 marks]

5)

a) A one-dimensional ionic crystal is made up of equally spaced ions with alternating charges of $+e$ and $-e$. The potential energy $V(r)$ of an ion in such a crystal of inter-ionic spacing r is given by:

$$V(r) = \left[A\left(\frac{a_0}{r}\right)^{12} - \frac{\alpha e^2}{4\pi\epsilon_0 r} \right] \quad .$$

A is a positive constant and a_0 is the equilibrium separation of adjacent ions. Explain the physical origins of the first and second terms in the equation and of the Madelung constant α.

[8 marks]

b) Sketch $V(r)$, indicating the position of a_0.

[4 marks]

c) Derive an expression for the constant A given a_0 and α. Hence obtain an expression for the minimum potential energy.

[10 marks]

For this one-dimensional ionic crystal, show that $\alpha = 2\ln 2$.

[8 marks]

[You may require the identity, $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$]