King's College London

UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

B.Sc. EXAMINATION

CP/1490 Structure of Matter

Summer 2002

Time allowed: THREE Hours

Candidates must answer SIX parts of SECTION A,
and TWO questions from SECTION B.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper.
Where necessary, a College calculator will have been supplied.

TURN OVER WHEN INSTRUCTED

Permittivity of free space	ϵ_0	$= 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Planck constant	h	$= 6.63 \times 10^{-34}\,\mathrm{J\,s}$
Electronic charge	e	$= 1.60 \times 10^{-19}\,\mathrm{C}$
Speed of light in free space	c	$= 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Electron mass	m_e	$= 9.11 \times 10^{-31}\,\mathrm{kg}$
Atomic mass unit	u	$= 1.66 \times 10^{-27}\,\mathrm{kg}$
Neutron mass	m_n	$= 1.0087\,u$
Proton mass	m_p	$= 1.0077\,u$
Avogadro's number	N_A	$= 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Boltzmann constant	k_B	$= 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Gas constant	R	$= 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Acceleration due to gravity	g	$= 9.81\,\mathrm{m\,s^{-2}}$

SECTION A – Answer SIX parts of this section

1.1) An electron passes through crossed electric and magnetic fields. Explain how this could be used to measure the velocity of the electron. If the magnetic field is then removed, explain how the ratio of the charge to the mass of the electron, e/m_e, can then be derived from a measurement of the deflection of the electron.

[7 marks]

1.2) Neutrons with de Broglie wavelengths of 1 nm and 2 nm emerge simultaneously from an aperture and travel 10 m to a detector. What is the time interval between their arrivals at the detector?

[7 marks]

1.3) Carbon dating is used to determine the age of a fragment of cloth found in an ancient burial site. The ratio of the number of ^{14}C atoms to the number of ^{12}C atoms is found to be 7×10^{-13}. Determine the age of the cloth.
[^{14}C and ^{12}C are present in living matter in the atomic ratio 10^{-12} to 1 and the half life of ^{14}C is 5730 years].

[7 marks]

1.4) Explain why the heat capacity at constant volume of one mole of nitrogen gas at room temperature and pressure is $\frac{5}{2}R$ whereas that of one mole of helium gas is $\frac{3}{2}R$.

[7 marks]

1.5) How do dislocations in crystals allow materials to deform plastically? What effect does the presence of grain boundaries in polycrystalline materials have on the mechanical properties of materials?

[7 marks]

1.6) A mass of 1 kg is suspended by a copper wire of cross-sectional area 1 mm^2 and length 1 m. Calculate the strain energy in the wire.
[The Young modulus of copper is $1.3 \times 10^{11}\text{N m}^2$].

[7 marks]

1.7) Identify the fundamental forces and indicate the types of particles between which they act. Name the bosons which mediate each of the forces.

[7 marks]

1.8) The fission process induced by a low-energy neutron incident on a ^{235}U nucleus is:

$$^1\text{n} +^{235}\text{U} \rightarrow^{236}\text{U}^* \rightarrow^{141}\text{Ba} +^{92}\text{Kr} + 3^1\text{n}$$

The masses of ^{235}U, ^{141}Ba and ^{92}Kr are $235.044\,u$, $140.914\,u$ and $91.897\,u$, respectively. How much energy is produced in this reaction and in what form is this energy released?

[7 marks]

SECTION B – Answer TWO questions

2) Discuss the mechanisms leading to the emission of X-rays when a metal target is irradiated by high energy electrons. Sketch a typical graph of the intensity of radiation as a function of wavelength. Account for the form of this spectrum, indicating how it relates to the atomic structure of the metal target and the energy of the incident electrons.

[15 marks]

A beam of radiation of wavelength λ is incident with an angle θ between the beam direction and the planes of the crystal. Show that a strong reflection from crystal planes separated by a distance d could be observed for angles θ given by

$$n\lambda = 2d \sin\theta$$

where n is an integer.

[6 marks]

A crystal is illuminated by X-rays of wavelength 1.5×10^{-10} m, and a strong reflection is found for $\theta = 16.1°$. What is the separation of the atomic planes in the crystal?

[2 marks]

A parallel beam of electrons of energy 500 eV is directed at the same crystal. At what value of θ will a strong reflection of the electrons occur?

[7 marks]

3) In the Bohr model of hydrogen-like atoms, a single electron moves in a circular orbit around a nucleus of positive charge $+Ze$. The orbital angular momentum L of the electron is quantised, such that

$$L = m_e v r = \frac{nh}{2\pi} \quad ,$$

where m_e is the mass of the electron, v is its velocity, r is the radius of its orbit and n is an integer.

Write down the equation of motion for the electron and hence show that the radius of the electron orbit is given by

$$r_n = \frac{\epsilon_0 h^2 n^2}{\pi m_e Z e^2} \quad .$$

[12 marks]

Show that the energy of the electron in the nth state is given by

$$E_n = -\frac{Z^2 e^4 m}{8\epsilon_0^2 h^2 n^2} \quad .$$

[8 marks]

Evidence for very hot gas in giant clusters of galaxies has been found in X-ray observations of the Lyman α-line of hydrogen-like iron (i.e. iron ionised such that only one electron remains; the atomic number of iron $Z = 26$). The Lyman α-line involves transitions between the $n = 1$ and $n = 2$ energy levels. Estimate the photon energy (in eV) and the corresponding wavelength (in nm) of this line. Calculate the energy required to remove the remaining electron. Hence, provide an estimate for the temperature of the gas.

[10 marks]

4) State the basic assumptions underlying the kinetic theory of gases.

[6 marks]

Show that the pressure p of a gas is given at low pressures by

$$p = \frac{1}{3} nm < v^2 > \quad ,$$

where n is the number of gas atoms per unit volume, m is the mass of each atom and $< v^2 >$ is the mean-square velocity of the atoms.

[11 marks]

Show that the mean-square velocity of atoms in a gas of temperature T is given by

$$< v^2 > = \frac{3k_B T}{m} \quad .$$

Calculate the root-mean-square (rms) velocity $\sqrt{< v^2 >}$ of helium atoms at room temperature. Hence, determine the number of helium atoms per unit volume at atmospheric pressure (10^5 Pa).

[7 marks]

The mean free path length λ of atoms of diameter d in an ideal gas is given by the expression

$$\lambda = \frac{1}{\sqrt{2} n d^2} \quad .$$

Calculate the value of λ for helium gas at atmospheric pressure and room temperature and hence deduce the average time between collisions of helium atoms. [The diameter of a helium atom is 0.3 nm].

[6 marks]

5) The intermolecular potential energy $V(r)$ between neutral atoms or molecules, separated by a distance r, can be described by the Lennard Jones potential:

$$V(r) = -\epsilon \left[\left(\frac{a_0}{r}\right)^{12} - 2\left(\frac{a_0}{r}\right)^{6} \right] \quad .$$

Explain the physical origin of the first and second terms in the equation, including the meanings of the symbols ϵ and a_0.

[8 marks]

Derive an expression for the force $F(r)$ between the atoms or molecules and show that the force is zero when $r = a_0$.

[6 marks]

Sketch, as a function of r, the potential $V(r)$ and the force $F(r)$, indicating the separation a_0, the energy ϵ and the ranges of r over which the force is repulsive and attractive. Indicate the force and separation at which a perfect crystal would fracture and justify your reasoning.

[8 marks]

Describe qualitatively how the elastic modulus and the coefficient of thermal expansion of a material are related to the curves for $V(r)$ and $F(r)$.

[8 marks]