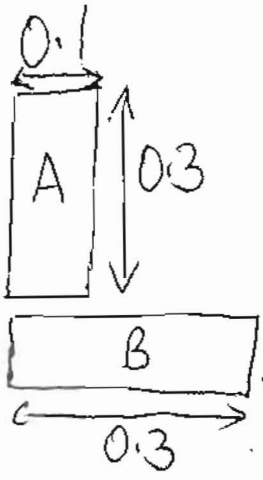
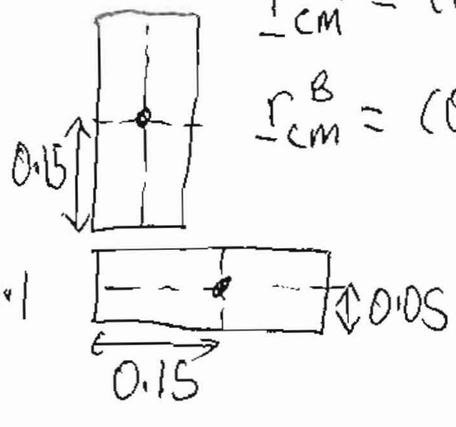
MECHANICS PROBLEM SHEET 5, ANSWERS

1 (i)





$\underline{r}_{cm}^{A} = (0.05, 0.25)$

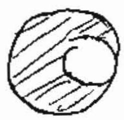
$\underline{r}_{cm}^{B} = (0.15, 0.05)$

$m_A = m_B = m$, say

CoM of the L is at CoM of those 2 parts

$$x_{cm} = \frac{m x_{cm}^{A} + m x_{cm}^{B}}{2m} = \frac{1}{2}(0.05 + 0.15) = 0.1$$

$$y_{cm} = \frac{1}{2}(0.25 + 0.05) = 0.15$$

(ii) A = the disc with hole  (centre at origin)

B = the disc radius 0.5m without hole  (centre at origin)

C = disc of radius 0.25m [centre at (0.25, 0)]

B = A + C i.e. $r_{cm}^{B} = \dfrac{m_A \underline{r}_{cm}^{A} + m_C \underline{r}_{cm}^{C}}{m_A + m_C}$

But $\underline{r}_{cm}^{B} = 0$, $m_B = \left(\dfrac{0.5}{0.25}\right)^2 m_C = 4m_C$, $m_A = m_B - m_C = 3m_C$

$\therefore 3m_C \underline{r}_{cm}^{A} + m_C \underline{r}_{cm}^{C} = 0 \quad \therefore \underline{r}_{cm}^{A} = -\frac{1}{3}\underline{r}_{cm}^{C}$

$\therefore$ Coord's of CoM are $(-0.083, 0)$

(iii) Taking origin at centre of sun

$$r_{cm}^{\text{Sun-Earth}} = \frac{M_{sun} r_{sun} + M_{earth} r_{earth}}{M_{sun} + M_{earth}}$$

0 (at origin)

Sun-Earth distance.

$$= \frac{5.98\times10^{24}\times 1.49\times10^{11}}{1.99\times10^{30}+5.98\times10^{24}} = 4.48\times10^{5}\,m \simeq 450\,km$$

(about the distance from London to Lands' End]

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2. As it rolls "up" the incline its CoM gets lower

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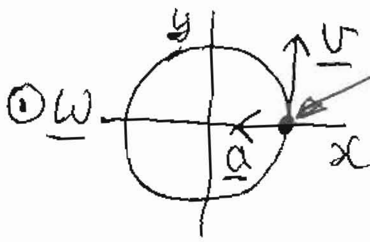
3 (i) ω = ang speed $= 2\pi/T = 2\pi/5 = 1.26\ rad\,s^{-1}$

v = speed $= r\omega = 0.2\times1.26 = 0.25\ ms^{-1}$

$|\underline{a}|$ = mag of accel $= \frac{v^2}{r} = \frac{0.25^2}{0.2} = 0.32\ ms^{-2}$

(ii) 

at this point $\underline{a}$ is in $-x$ dim $\rightarrow \underline{v}$ is in $+y$ dirn., i.e. particle is moving in $+y$ dim at that instant.

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4 $\frac{dW}{dt} = \underline{F}.\underline{v} = m\underline{a}.\underline{v} = 0$ since $\underline{a}$ is $\perp$ to $\underline{v}$

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5 (i) Momentum cons

$$\left.\begin{aligned} &x\text{-comp: } mu_A = mv_A\cos\alpha + mv_B\cos\beta \\ &y\text{-comp: } 0 = mv_A\sin\alpha - mv_B\sin\beta \\ &\text{KE cons: } \tfrac{1}{2}mu_A^2 = \tfrac{1}{2}mv_A^2 + \tfrac{1}{2}mv_B^2 \end{aligned}\right\} \text{cancel } m\text{'s}$$

(ii) $u_A^2 = v_A^2 + v_B^2 \rightarrow u_A = \sqrt{4+1} = \sqrt{5}\ ms^{-1}$

y comp of mom cons: $\sin\beta = (v_A/v_B)\sin\alpha = 2\sin\alpha$

x comp of mom cons: $\cos\beta = \underbrace{u_A}_{\sqrt{5}} - \underbrace{v_A}_{2}\cos\alpha$

$\therefore \sin^2\beta = 4\sin^2\alpha$ & $\cos^2\beta = 5 - 4\sqrt{5}\cos\alpha + 4\cos^2\alpha$

$\therefore \underbrace{\sin^2\beta + \cos^2\beta}_{1} = 5 - 4\sqrt{5}\cos\alpha + \underbrace{4\cos^2\alpha + 4\sin^2\alpha}_{4}$

$\therefore 4\sqrt{5}\cos\alpha = 8 \rightarrow \cos\alpha = \frac{2}{\sqrt{5}} \rightarrow \alpha = 26.6^\circ$

$\sin\beta = 2\sin\alpha = 0.894 \rightarrow \beta = 63.4^0$

(iii) Square the comp's of mom cons

$$v_A^2\cos^2\alpha + 2v_Av_B\cos\alpha\cos\beta + v_B^2\cos^2\beta = u_A^2$$
$$v_A^2\sin^2\alpha - 2v_Av_B\sin\alpha\sin\beta + v_B^2\sin^2\beta = 0$$

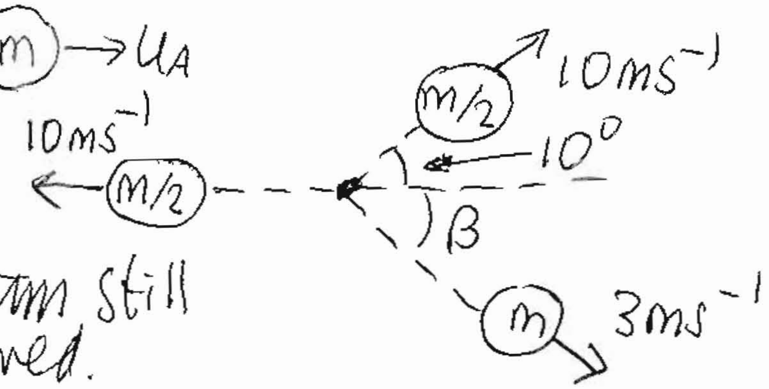
Add & use $\sin^2\alpha + \cos^2\alpha = 1$ & $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha+\beta)$

$\rightarrow v_A^2 + 2v_Av_B\cos(\alpha+\beta) + v_B^2 = u_A^2 = v_A^2 + v_B^2$ (ke cons)

$\therefore 2v_Av_B\cos(\alpha+\beta) = 0$ Assuming neither v_A nor v_B are zero $\rightarrow \cos(\alpha+\beta) = 0 \Rightarrow \alpha+\beta = 90^0$

(iv) before (m) $\rightarrow u_A$

after



Momentum still conserved.

y comp: $\frac{m}{2} \times 10 \times \sin 10^0 = m \times 3 \times \sin\beta$

$\therefore \sin\beta = \frac{5}{3}\sin 10^0 = 0.289 \rightarrow \beta = 16.8^0$

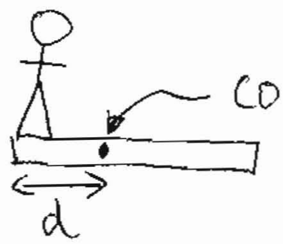
x comp: $mu_A = -\frac{m}{2}\times 10 + \frac{m}{2}\times 10\times\cos 10^0 + m\times 3\times\cos\beta$

$\therefore u_A = -5 + 5\cos 10^0 + 3\cos 16.8^0 = 2.8ms^{-1}$

$\therefore K_{init} = \frac{1}{2}\times 0.17\times(2.80)^2 = 0.66J$

$K_{fin} = \frac{1}{2}\left(\frac{0.17}{2}\right)\times 10^2 + \frac{1}{2}\left(\frac{0.17}{2}\right)\times 10^2 + \frac{1}{2}\times 0.17\times 3^2 = 9.27J$

$\therefore \Delta K = K_{fin} - K_{init} = 8.60J$ = energy released

6 

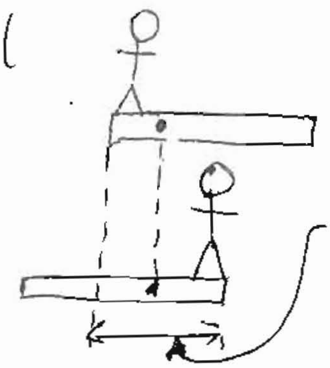
COM of plank/woman system

$$d = \frac{M_{woman} x_{woman} + M_{plank} + x_{plank}^{centre}}{M_{woman} + M_{plank}}$$

$$= \frac{50 \times 0 + 20 \times 10}{50 + 20} = \frac{20}{7} = 2.86\,m$$

initial

final



While she is walking no ext force act $\therefore$ COM stays fixed

distance moved wrt pond $= 2d = 5.71\,m$

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7 (i) $V^2 = \underline{V} \cdot \underline{V} = (\underline{V}_A - \underline{V}_B) \cdot (\underline{V}_A - \underline{V}_B) = V_A^2 - 2\underline{V}_A \cdot \underline{V}_B + V_B^2$

(ii) $V_{cm}^2 = \frac{(m_A \underline{V}_A + m_B \underline{V}_B)}{M} \cdot \frac{(m_A \underline{V}_A + m_B \underline{V}_B)}{M}$

$$= \frac{1}{M^2} (m_A^2 V_A^2 + 2 m_A m_B \underline{V}_A \cdot \underline{V}_B + m_B^2 V_B^2)$$

(iii) $K'_{tot} + K_{cm} = \frac{1}{2} \frac{m_A m_B}{M} (V_A^2 - 2\underline{V}_A \cdot \underline{V}_B + V_B^2)$

$$+ \frac{1}{2} M \frac{1}{M^2} (m_A^2 V_A^2 + 2 m_A m_B \underline{V}_A \cdot \underline{V}_B + m_B^2 V_B^2)$$

$$= \frac{1}{2M} \{ m_A m_B V_A^2 - 2 m_A m_B \underline{V}_A \cdot \underline{V}_B + m_A m_B V_B^2 + m_A^2 V_A^2 + 2 m_A m_B \underline{V}_A \cdot \underline{V}_B + m_B^2 V_B^2 \}$$

$$= \frac{1}{2M} \{ m_A \underbrace{(m_A + m_B)}_{M} V_A^2 + m_B \underbrace{(m_A + m_B)}_{M} V_B^2 \} = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

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8 (i) Gravity & normal contact force from bucket (both down)

(ii) At highest pt the accel of the bucket in its circular motion is downwards. If this accel $> g$ the water stays in bucket.