Mechanics Classwork III. Answers

1 (i) $V_{0x} = V_0 \cos\theta$ (ii) $V_{0z} = V_0 \sin\theta$

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2. $E = K + U = \text{const}$, $K = \frac{1}{2}mv^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_z^2$, $U = mgz$

But $v_x = \text{const} = V_{0x}$ $\therefore \frac{1}{2}mv_z^2 + U = E - \frac{1}{2}mv_x^2 = \text{const}$

At launch $(z=0)$: $v_z = V_{0z}$ & $U = 0$
At highest pt $(z = H)$: $v_z = 0$ & $U = mgH$

$\therefore \frac{1}{2}mV_{0z}^2 + 0 = 0 + mgH$ $\therefore H = \dfrac{V_{0z}^2}{2g} = \dfrac{V_0^2 \sin^2\theta}{2g}$

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3 $z = -Ax^2 + Bx$
$z = 0 \rightarrow x(B - Ax) = 0$
$\rightarrow x = 0$ (launch) or $x = B/A$ (hits ground)

$\therefore R = \dfrac{B}{A} = \tan\theta \times \dfrac{2V_0^2 \cos^2\theta}{g}$

But $\tan\theta \times \cos^2\theta = \dfrac{\sin\theta}{\cos\theta} \times \cos^2\theta = \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

$\therefore R = \dfrac{V_0^2 \sin 2\theta}{g}$

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4 Particle moves in x dim with const vel V_{0x}

$\therefore T = \dfrac{R}{V_{0x}}$ Use: $R = \dfrac{2V_0^2 \sin\theta\cos\theta}{g}$ & $V_{0x} = V_0\cos\theta$

$\rightarrow T = \dfrac{2V_0 \sin\theta}{g}$

5 The maximum value of $\sin x$ $(=1)$ occurs when $x = 90^\circ$.
$\therefore$ R_{max} occurs when $2\theta = 90^\circ \Rightarrow \theta = 45^\circ$

[If you want to do this the hard way:
$\frac{dR}{d\theta} = \frac{V_0^2}{g} 2\cos 2\theta = 0$ when $\cos 2\theta = 0$ i.e. $2\theta = 90^\circ \ldots$]

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6 For $\theta = 45^\circ$: $R = \frac{V_0^2}{g}$, $H = \frac{V_0^2}{4g} = \frac{R}{4}$, $T = \frac{2V_0}{g}\sin 45^\circ = \frac{V_0\sqrt{2}}{g}$.

(i) $R = 1.18 \times 10^3 m \rightarrow V_0 = (Rg)^{\frac{1}{2}} = (1.18 \times 10^3 \times 9.81)^{\frac{1}{2}} = 108 ms^{-1}$

(ii) $H = \frac{R}{4} = 295 m$

(iii) $T = \frac{V_0\sqrt{2}}{g} = 15.5 s$

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7. $H = \frac{V_0^2}{2g}\sin^2\theta$. The maximum value of $\sin^2 x$ $(=1)$ occurs when $x = 90^\circ$ i.e. H_{max} occurs when $\theta = 90^\circ \rightarrow H_{max} = \frac{V_0^2}{2g}$

For Second Amendment $\rightarrow H_{max} = 590 m$
