Yr 1 Electricity & Magnetism Problem sheet 3

1) Drude model $\sigma = \dfrac{ne^2\tau}{2m} = \dfrac{8.10^{28}.(1{\cdot}6.10^{-19})^2 5.10^{-14}}{2\text{-}9{\cdot}1.10^{-31}} = 5{\cdot}6.10^{7}\,\Omega^{-1}m^{-1}$

Resistivity of wire $A = \dfrac{l}{A\sigma} = \dfrac{10^4}{\pi.10^{-6}\,5{\cdot}6.10^{7}} = 57\,\Omega$

2) $500\,Vm^{-1} \rightarrow$ $\varepsilon_r = 5$ (i) Outside $\varepsilon_r = 1$ so $D = \varepsilon_0 E = 8{\cdot}854.10^{-12}.500 = 4{\cdot}43.10^{-9}\,Cm^{-2}$

(ii) D continuous across boundary $\Rightarrow D = 4{\cdot}43.10^{-9}\,Cm^{-2}$

(iii) Inside $E = D/\varepsilon_r\varepsilon_0 = 4{\cdot}43.10^{-9}/(5.\,8{\cdot}854.10^{-12}) = 100\,Vm^{-1}$

(iv) From $E = \dfrac{D}{\varepsilon_0} - \dfrac{P}{\varepsilon_0}$, $P = D - \varepsilon_0 E = 4{\cdot}43.10^{-9} - 100\times 8{\cdot}854.10^{-12} = 3{\cdot}5.10^{-9}\,Cm^{-2}$

(v) Draw pillbox around boundary $E_{outside}$ E_{inside} Apply Gauss's flux law $\int E.dA = Q/\varepsilon_0$ to unit area A $Q = \sigma A$

$\Rightarrow \sigma = \varepsilon_0(E_{outside} - E_{inside})$

$\sigma = 8{\cdot}854.10^{-12}(500 - 100) = 3{\cdot}5.10^{-9}\,Cm^{-2}$

OR use $\int \underset{\wedge}{P}.d\underset{\wedge}{A} = -Q_p$ where all the charge is polarisation charge $(Q_p = \sigma A)$

$P_{outside} = 0$, $P_{inside} = 3{\cdot}5.10^{-9}\,Cm^{-2}$, $\sigma = (P_{inside} - P_{outside}) = 3{\cdot}5.10^{-9}\,Cm^{-2}$

plate | air | dielec | plate
D_p | D_{air} | ε_r D_d | D_p

Divide into slabs, each with own D and E
In plate $D_p = 0$, $E_p = 0$ since it is a conductor
In air $E_{air} = D_{air}/\varepsilon_0$ ① In dielectric $E_d = \dfrac{D_d}{\varepsilon_0\varepsilon_r}$ ②

Free charge density σ (per unit area) on interface between plate and air $\Rightarrow D_{air} = \sigma$. Similarly at interface between dielectric and plate $D_d = \sigma$. This agrees with $D_{air} = D_d$ at interface between air and dielectric since there is no free charge there. Having found D throughout the system we can now find E. $E_{air} = \sigma/\varepsilon_0$, $E_d = \sigma/\varepsilon_r\varepsilon_0$ (from eq^n ① & ②)

Potential across capacitor (between plates) is $V = E_{air}\dfrac{x}{2} + E_d\dfrac{x}{2}$

$\Rightarrow V = \dfrac{\sigma x}{\varepsilon_0 2}\left(1 + \dfrac{1}{\varepsilon_r}\right)$ ③ $\Rightarrow V = \dfrac{\sigma x(\varepsilon_r + 1)}{2\varepsilon_r\varepsilon_0}$

Capacitance is $\dfrac{Q}{V} = \dfrac{\sigma A}{V} = \dfrac{2A\varepsilon_r\varepsilon_0}{x(\varepsilon_r+1)}$

Eq^n ③ says that most of the potential difference occurs across the air gap. – the '$\frac{1}{\varepsilon_r}$' in the bracket is small compared with the '1' in the bracket. Hence for a given charge density σ, V is made larger by the air gap. The air gap reduces the capacitance – so the air gap should be small for a practical capacitor. For comparison, a capacitor completely filled by a dielectric has $C = A\varepsilon_r\varepsilon_0/x$.

-) The velocity is perpendicular to the vertical mag. field, and $\underset{\sim}{v} \wedge \underset{\sim}{B}$ is directed across the tracks. Voltage is Bvl where l is gap between tracks $\Rightarrow$ $V = 4.10^{-4}.\ \dfrac{2.10^5}{3600}.\ 1{\cdot}4 = 0{\cdot}03$ volt.

5) No of turns per unit length $n = \dfrac{N}{l} = \dfrac{100}{0{\cdot}1} = 10^3$

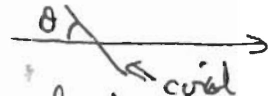
Air-filled $B = \mu_0 n I = 4\pi.10^{-7}.10^3.\ 0{\cdot}1 = 4\pi.10^{-5} = 1{\cdot}3.10^{-4}$ Tesla

$H = nI = 10^3.0{\cdot}1 = 100$ ampere m^{-1}

with $\mu_r = 10^4$, core, H is same, B is μ_r larger $\Rightarrow$ $B = 1{\cdot}3$ Tesla

Inductance $L = \dfrac{\mu_r \mu_0 N^2 A}{l}$ $\quad A = \pi.0{\cdot}02^2 \Rightarrow L = \dfrac{\mu_r\ 4\pi.10^{-7}.10^4.\pi.0{\cdot}02^2}{0{\cdot}1}$

. Air in centre $\Rightarrow L = 1{\cdot}6.10^{-4}$ H. With filled core $L = 1{\cdot}6$ H.

6) 

(i) Flux through one turn of coil is $\Phi_1 = B\pi R^2 \sin\theta$

Area of coil $= \pi R^2$ $\quad$ e.m.f. around one turn is $\dfrac{d\Phi_1}{dt} = \dfrac{d\theta}{dt} B\pi R^2 \cos\theta$

e.m.f. around N turns is $V = \dfrac{d\theta}{dt} NB\pi R^2 \cos\theta$

Coil rotates with ang. freq. ω, so $\dfrac{d\theta}{dt} = \omega$ and $\theta = \omega t + \phi$ for some ϕ

$V = \omega NB\pi R^2 \cos(\omega t + \phi)$

(ii) e.m.f. $= \omega NB\pi R^2$, $\omega = 2\pi/\tau$

Want e.m.f. of 1 Volt, so $\tau = \dfrac{2\pi^2 NBR^2}{\text{e.m.f.}} = 2\pi^2.\ 10^4.10^{-9}.10^6 = 200$ sec.

(iii) Source of energy is rotation of coil

(iv) There would be an e.m.f. across the wire, but any current along the wire needs to return in the reverse direction. If this current flowed through a second wire an opposing e.m.f. would cancel out that along the first wire.