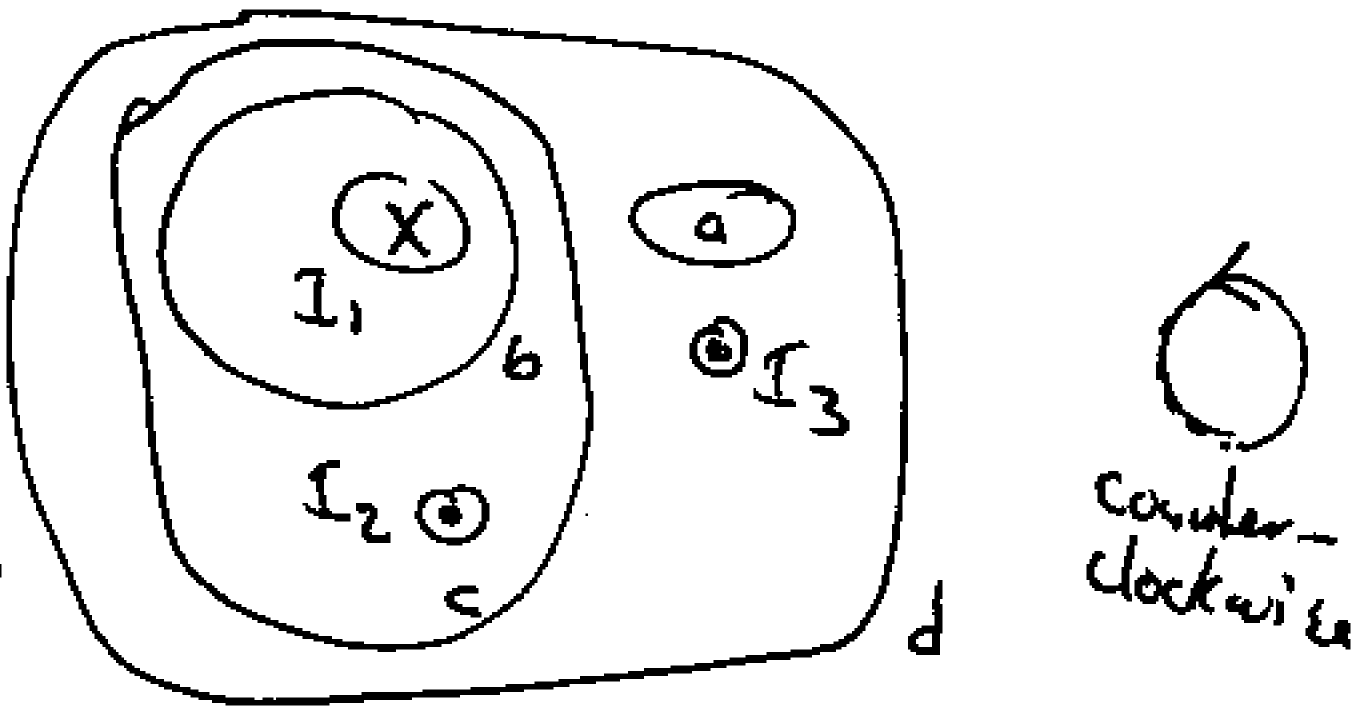
Electricity & Magnetism Classwork 7 Solutions

① Ampere's Law $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I_{encl}$
where direction of $+I$ based on R.H. rule for line integral. So all we need to do is add up algebraically the enclosed currents, with +ve being out of page:

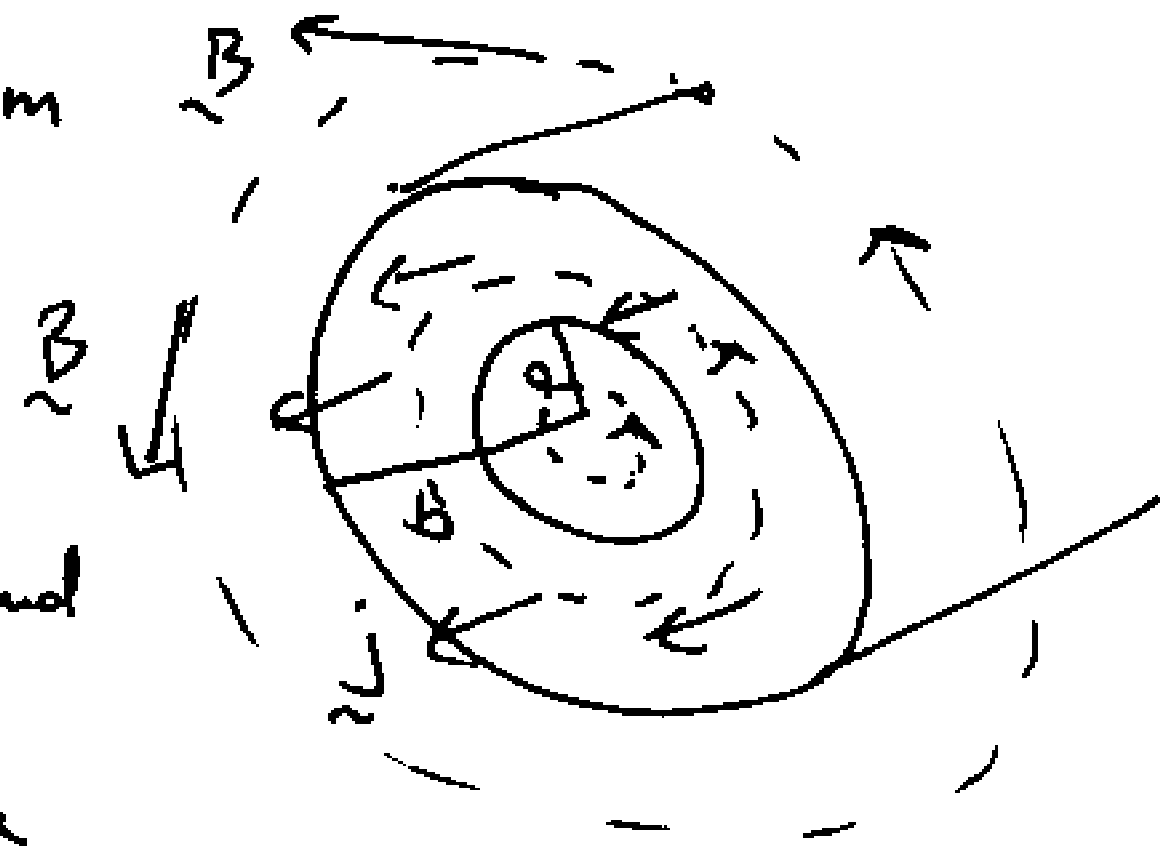


a) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = 0$ b) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = -\mu_0 I_1$ c) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 (I_2 - I_1)$

d) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 (I_2 + I_3 - I_1)$ $\qquad I_1 = 4A,\ I_2 = 6A,\ I_3 = 2A$ so

a) 0 ; b) $-4\pi \times 10^{-7} \times 4 = -5.02 \times 10^{-6}\ Tm$ c) $+4\pi \times 10^{-7}(2) = 2.51 \times 10^{-6}\ Tm$

d) $4\pi \times 10^{-7}(6+2-4) = +5.02 \times 10^{-6}\ Tm$



② $\underset{\sim}{j}$ uniform = current/area, so
$j(\pi b^2 - \pi a^2) = I \Rightarrow j = \dfrac{I}{\pi(b^2 - a^2)}$
Consider 3 Amperian circuits, RH sense around $\underset{\sim}{j}$. By symmetry $\underset{\sim}{B}$ tangent to these circles + depends only on r. Hence

a) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = 2\pi r B = 0 \Rightarrow B = 0 \quad (r < a)$

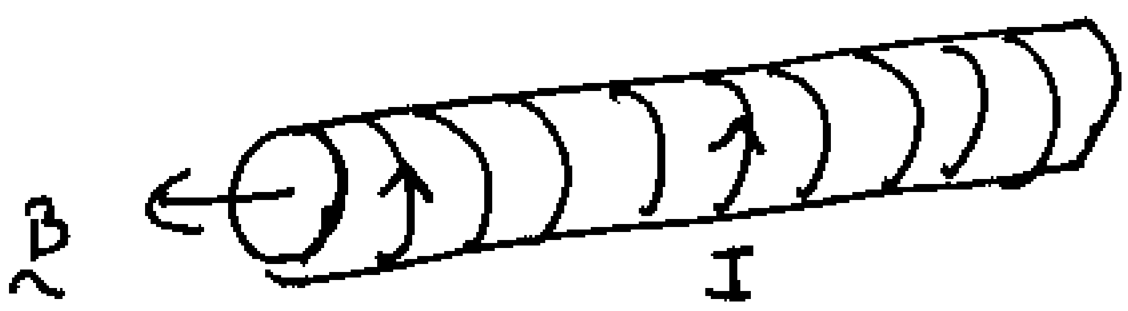
b) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = 2\pi r B = \mu_0(\pi r^2 - \pi a^2) j \Rightarrow B = \dfrac{\mu_0 I}{2\pi} \dfrac{(r^2 - a^2)}{r(b^2 - a^2)} \quad a < r < b$

c) $\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = 2\pi r B = \mu_0 I \quad \Rightarrow B = \dfrac{\mu_0 I}{2\pi} \dfrac{1}{r} \quad r > b$

③

a) From lecture a solenoid has

$B_0 = \mu_0 n I$



i) $= 4\pi \times 10^{-7} (60 \times 100)\ 0.15 = 1.13 \times 10^{-3}\ T$

ii) $B = \mu n I = K_m \mu_0 n I = B_0 + \mu_0 M \Rightarrow \mu_0 M = (K_m - 1)\mu_0 n I$

so $M = (K_m - 1) n I = 5199 (60 \times 100) 0.15 = 4.68 \times 10^6\ A/m$

iii) $B = K_m B_0 = 5200 \times 1.13 \times 10^{-3} = 5.88\ T$

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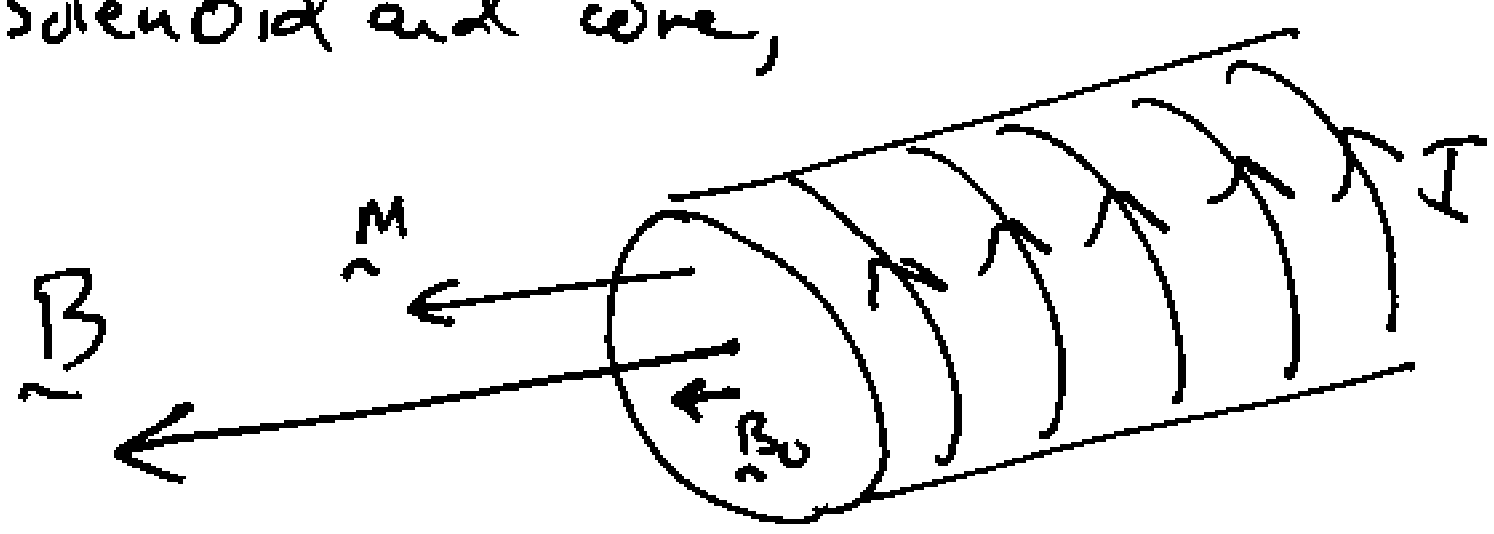
③ (cont)

b) All of $\underset{\sim}{B_0}$, $\underset{\sim}{B}$ and $\underset{\sim}{M}$ are parallel to the axis inside the solenoid and core,

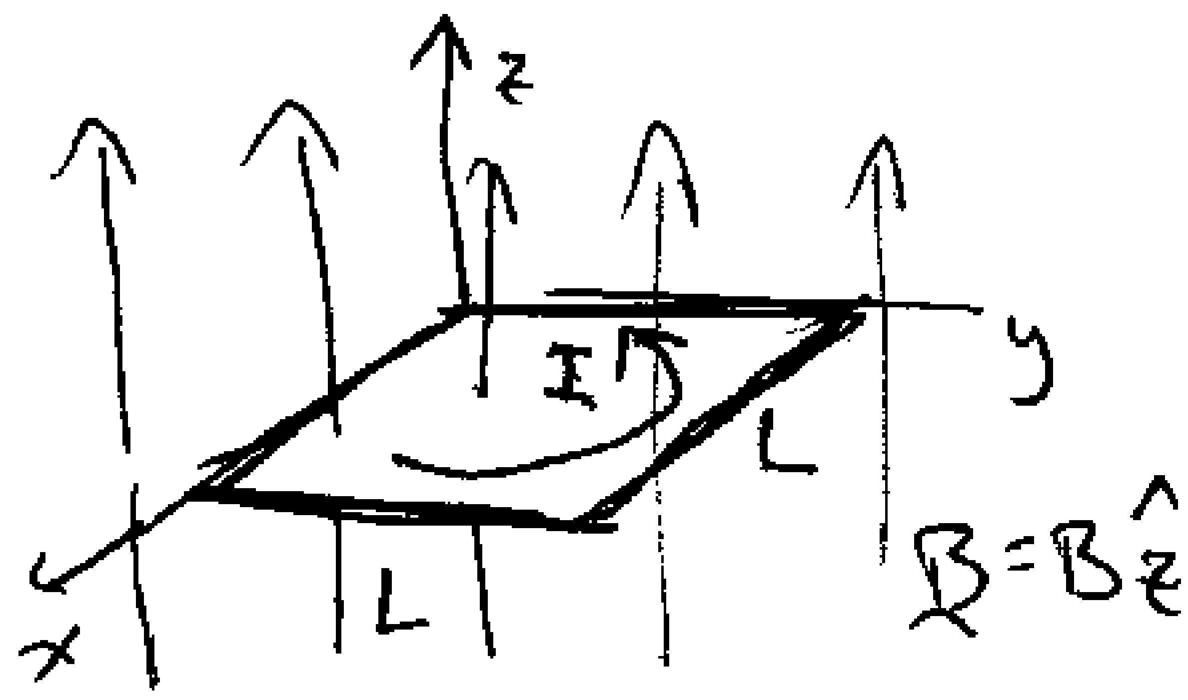
ie



and are uniform ($\approx$) within the core.

④

a) $\Phi_B = \iint \underset{\sim}{B} \cdot d\underset{\sim}{A}$



Choose $d\underset{\sim}{A} = dA\,\hat{z}$

Then $\Phi_B = \iint B\hat{z} \cdot dA\,\hat{z}$

$$= \iint B\,dA = B \iint dA = BL^2$$

So $\dfrac{d\Phi_B}{dt} = L^2 \dfrac{dB}{dt} = (0.2)^2 \dfrac{d}{dt}\left(10^{-3} e^{-t/3}\right) = (0.2)^2\, 10^{-3} \left(-\tfrac{1}{3}\right) e^{-t/3}$

Thus $\dfrac{d\Phi_B}{dt} = -1.33 \times 10^{-5} e^{-t/3}$ $\mathrm{T\,m^2/s}$

b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = +1.33 \times 10^{-5} e^{-t/3}$ so $\mathcal{E}$ is +ve in RH sense w.r.t. $d\underset{\sim}{A}$ $\Rightarrow$ I as shown

c) $\mathcal{E} = IR \Rightarrow I = \mathcal{E}/R = \dfrac{1.33 \times 10^{-5} e^{-t/3}}{100} = 1.33 \times 10^{-7} e^{-t/3}$ Amps