Paper Number(s): **ISE2.4**

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
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EXAMINATIONS 2000

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COMMUNICATIONS 2

Tuesday, May 2 2000, 2:00 pm

There are 4 questions on this paper. Answer 3 questions.

Time allowed: 2:00 hours.

Examiner(s):Prof L.F. Turner

(1) A speech signal is to be sampled in preparation for analogue-to-digital conversion. The probability density function associated with the sampled process is as shown in Figure 1 and a 4-bit uniform quantizer is to be used. The quantizer has a step-size $\triangle$ and the boundries of the quantization levels are as indicated in the figure. Derive from first principle an expression in terms of a, b and $\triangle$ for the minimum quantization noise associated with the level bounded by the sample values -$\triangle$ and 0. Use your expression to evaluate the numerical value of the quantization noise for the level. Also, use your derived expression to deduce the quantization noise in the situation in which the probability density function is constant within a level. Give your answer in terms of the quantization step size, $\triangle$, and the probability that the sampled value will fall within the level. [20]

(2) A speech signal $S(t)$, can be represented in the short term by $S(t) = \sum_{i=1}^{N} a_i cos(w_i t + \phi_i)$. The signal is to be used to amplitude modulate a carrier and envelope detection is to be used at the receiver.
If ϕ_i, $i = 1, ..., N$, has a probability density function that is uniform over the range 0 to 2π and is zero elsewhere, determine as a function of the a_i's the maximum percentage of the total transmitted power that can be placed in the side-frequencies (the sidebands) if distortion is to be avoided.

In a radio communication system a signal $Q(t)$ is used to amplitude modulate a carrier and envelope detection is to be used.

If the signal reaches the receiver by two paths, a direct path and one which introduces an attennuation k and delay τ relative to the direct path, derive an expression for the output of the envelope detector. [20]

(3) Derive an expression for the signal-to-noise power ratio at the output of a detector in terms of the corresponding signal-to-noise power ratio at the input to detector for

(i) Double-Sideband Suppressed Carrier Amplitude Modulation (DSB-SC), and
(ii) Single-Sideband Suppressed Carrier Amplitude Modulation (SSB-SC).

If there are differences between your two results explain why you think this occurs.

From the point of view of signal-to-noise ratios, compare the overall performance of the DSB-SC and SSB-SC systems. [20]

14.45

(4) (i) Describe and compare the circuit, message and packet switched methods of communication.

(ii) Describe and compare three methods by which a number of individuals can access and use a communication channel simultaneously.

(iii) It is proposed to store data in groups of 4 information digits in computer memory and to protect each group of digits by using a Hamming (7,4) code.

The code is to be used for the purpose of DETECTING errors.

Derive an expression for the overall probability of undetected codeword error as a function of the digit error probability, P. [20]

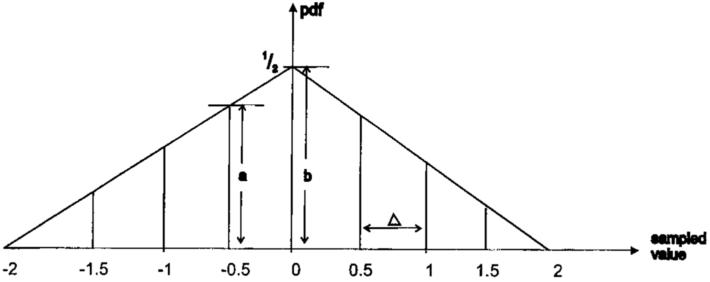
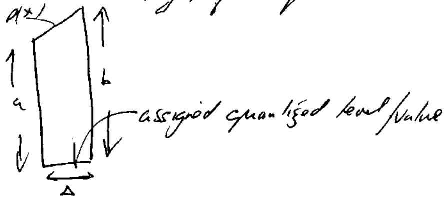


Figure 1

Solutions

marks schedule

Q1. Consider the pdf within the quantization level as indicated in Fig 1 of the question



We have to determine the optimum position for V the assigned quantized value

$$mse = \int_0^{\Delta} (x-V)^2 p(x)\,dx$$, where $p(x)$ is the pdf of the sampled value

Now the pdf has an equation $p(x) = a + \frac{(b-a)}{\Delta}.x$

hence the mse due to quantization is

$$\int_0^{\Delta} (x-V)^2 p(x)\,dx = \int_0^{\Delta} (x-V)^2 \left[a + \frac{(b-a)}{\Delta}x\right] dx$$

$$= \int_0^{\Delta} \left[ax^2 - 2aVx + aV^2 + \frac{(b-a)}{\Delta}x^3 - 2\frac{(b-a)}{\Delta}V.x^2 + \frac{(b-a)}{\Delta}V^2x\right] dx$$

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$$mse = \frac{a\Delta^3}{3} - 2aV\frac{\Delta^2}{2} + aV^2\Delta + \left(\frac{b-a}{\Delta}\right)\frac{\Delta^4}{4} - 2\left(\frac{b-a}{\Delta}\right)V\cdot\frac{\Delta^3}{3} + \left(\frac{b-a}{\Delta}\right)V^2\cdot\frac{\Delta^2}{2} \quad (1)$$

This has to be minimized by choice of position for V

$\frac{d(mse)}{dV} = 0$ yields minimum which is

$$-a\Delta^2 + 2a\Delta.V + (b-a)\Delta.V - 2\frac{(b-a)}{3}.\Delta^2 = 0$$

ie $$-a\Delta + 2aV + (b-a)V - 2\frac{(b-a)}{3}\Delta = 0$$

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from which we get

$$V = \frac{(a+2b)}{3(a+b)}.\Delta \quad (2)$$

NB: This is an essential part of the question and the assumption, based on a uniform/constant pdf that $V = \Delta/2$ will not be accepted.

Now the quantization noise power N_q = mse for the level is obtained by substituting for V in equation 1, using the result given in Eq 2.

If we do this we then obtain

Now (1) which can be re-written as

$$N_q = \left(\frac{a+3b}{12}\right)\cdot\Delta^3 - \frac{(a+2b)}{3}\Delta^2\cdot V + \left(\frac{a+b}{2}\right)\Delta V^2$$

on substituting for $V = \frac{(a+2b)}{3(a+b)}\cdot\Delta$ becomes

$$N_q = \left(\frac{a+3b}{12}\right)\Delta^3 - \frac{(a+2b)^2\Delta^3}{9(a+b)} + \frac{(a+b)(a+2b)^2\cdot\Delta^3}{18(a+b)^2}$$

$\boxed{6}$

$$\boxed{N_q = \left\{\left(\frac{a+3b}{12}\right)\cancel{\Delta^3} - \frac{(a+2b)^2}{18(a+b)}\right\}\Delta^3} \quad \text{—— (3)}$$

Numerical value of N_q : $\Delta = 1/2$; $a = 3/8$, $b = 1/2$ which are to be substituted into 3 $\boxed{1}$

Now if the pdf is constant within a level $a = b$ and hence N_q above simplifies to

$$\left(\frac{a}{3} - \frac{9a^2}{36a}\right)\Delta^3 = \frac{a}{12}\cdot\Delta^3 = \left(\frac{a\Delta}{12}\right)\cdot\Delta^2 = a\Delta\cdot\frac{\Delta^2}{12}$$

$\boxed{3}$

So the quantization noise in this case becomes $P_L\cdot\frac{\Delta^2}{12}$

Where $P_L = a\Delta$ is the probability of the sampled value falling in level L.

mark out of 20

Q2. Since envelope detection is to be used the transmitted signal has to be of the form

$$m(t) = A\left[1 + m\, s(t)\right]\cos\omega_c t,$$

where A is the amplitude of the carrier and m is the modulation index.

In order to avoid distortion $m\,s(t)$ must never be more ~~deg~~ negative than -1.

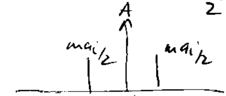
Now since the ϕ_is are random, in order to ensure that the above condition is satisfied it follows that since it is possible for $s(t)$ to have a maximum negative value of $-(a_1 + \dots + a_N)$ and hence we have to select m so that

$$m = \frac{1}{\sum_{i=1}^{N} a_i}$$

Now the carrier power is $\frac{A^2}{2}$

and the amplitude of the side frequencies at $(\omega_c + \omega_i)$ and $(\omega_c - \omega_i)$ are $\frac{m a_i}{2}$

ie



It follows that the power in side frequency at $\omega_c + \omega_i$ is $\frac{m^2 a_i^2}{8} A^2$

and at $\omega_c - \omega_i$ is $\frac{m^2 a_i^2}{8} A^2$

So the total power in the side frequencies is

$$\sum_{i=1}^{N} m^2 a_i^2/4 \, A^2$$ ie Sideband Power

$$= \frac{m^2}{4} (a_1^2 + \cdots + a_N^2) A^2$$

and the total transmitted power is

$$\frac{A^2}{2} + \frac{m^2}{4} \sum_{i=1}^{N} a_i^2$$

$\boxed{14}$

and hence the % power in the side frequencies is

$$\frac{\frac{m^2}{4} \sum_{i=1}^{N} a_i^2 . A^2}{\frac{A^2}{2} + \frac{m^2}{4} . A^2 \sum_{i=1}^{n} a_i^2} = \frac{\frac{m^2}{4} \left(\sum_{i=1}^{N} a_i^2 \right)}{\frac{1}{2} + \frac{m^2}{4} \sum_{i=1}^{m} a_i^2}$$

But $m = \frac{1}{\sum_{i=1}^{N} a_i}$

So we have % Power $= \dfrac{\frac{1}{4\left(\sum_{i=1}^{N} a_i\right)^2} . \sum_{i=1}^{N} a_i^2}{\frac{1}{2} + \frac{1}{4\left(\sum_{i=1}^{N} a_i\right)^2} . \sum_{i=1}^{N} a_i^2}$

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The transmitted signal is

$$A[1+m\,Q(t)]\cos\omega_c t$$

and the received signal is

$$R(t) = A[1+m\,Q(t)]\cos\omega_c t + KA[1+m\,Q(t-\tau)]\cos\omega_c(t-\tau)$$

$$= A[1+m\,Q(t)]\cos\omega_c t + KA[1+m\,Q(t-\tau)]\cos\omega_c\tau\,.\cos\omega_c t + KA[1+m\,Q(t-\tau)]\sin\omega_c\tau\,\sin\omega_c t$$

$$= \left\{A[1+m\,Q(t)] + KA[1+m\,Q(t-\tau)]\cos\omega_c\tau\right\}\cos\omega_c t + \left\{KA[1+m\,Q(t-\tau)]\sin\omega_c\tau\right\}\sin\omega_c t$$

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and the output from the envelope detector will be

$$\left[\left\{A[1+m\,Q(t)] + KA[1+m\,Q(t-\tau)]\cos\omega_c\tau\right\}^2 + \left\{KA[1+m\,Q(t-\tau)]\sin\omega_c\tau\right\}^2\right]^{1/2}$$

hence heavy, non-linear distortion introduced

(mark out of 20)

Q3/ Expression for $(S/N)_{out}$ as function of $(S/N)_{IN}$

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DSB-SC

The received signal and noise is

$$S(t)\cos w_c t + n(t) = S(t)\cos w_c t + x_c(t)\cos w_c t - x_s(t)\sin w_c t$$

The received signal power is $\overline{(S(t)\cos w_c t)^2} = \frac{1}{2}\overline{S^2(t)}$

and the received noise power $= N$

$$\therefore (S/N)_{IN} = \tfrac{1}{2}\overline{S^2(t)}/N$$

After demodulation and low-pass filtering we get at the output

$$\underline{[S(t)\cos w_c t + x_c(t)\cos w_c t - x_s(t)\sin w_c t]\cos w_c t}$$

which is low-pass filter

hence output $= \cancel{\frac{1}{2}S(t) + \frac{1}{2}}$

$$= \frac{1}{2}S(t) + \frac{1}{2}x_c(t)$$

$$\therefore (S/N)_{out} = \tfrac{1}{4}\overline{S^2(t)} \Big/ \tfrac{1}{4}\overline{x_c^2(t)} = \overline{S^2(t)}/N$$

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Hence $(S/N)_{out} = 2(S/N)_{IN}$

SSB-SC

Received signal $= S(t)\cos w_c t - S_q(t)\sin w_c t + n(t)$

$$= S(t)\cos w_c t - S_q(t)\sin w_c t + x_c(t)\cos w_c t - x_s(t)\sin w_c t$$

∴ Input Signal Power

$$= \overline{\left(s(t)\cos\omega_c t - s_q(t)\sin\omega_c t\right)^2}$$

$$= \tfrac{1}{2}\,\overline{s^2(t)} + \tfrac{1}{2}\,\overline{s_q^2(t)}$$

But since $s_q(t)$ is simply $s(t)$ Hilbert Transformed $\overline{s_q^2(t)} = \overline{s^2(t)}$

∴ Input signal power $= \overline{s^2(t)}$

Noise power $= N$

$$\therefore (S/N)_{IN} = \overline{s^2(t)}/N$$

After Demodulation and filtering we obtain at output

$$\tfrac{1}{2}s(t) + \tfrac{1}{2}x_c(t)$$

Hence $(S/N)_{out} = \tfrac{1}{4}\overline{s^2(t)} / \tfrac{1}{4}\overline{x_c^2(t)} = \overline{s^2(t)}/N$

$$\therefore (S/N)_{out} = (S/N)_{IN}.$$

[7]

Part 2 The reason for the 3dB gain with DSB-SC is that signal side frequency components at either side of carrier add on a voltage basis, but the noise components, being of random phase, add on a power basis ∴ 3dB gain

[2]

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Part 3

It appears that DSB-SC is better. This is in fact, so in so far as performance of detector is concerned but

If we send the same power (signal power) in the two cases then $\overline{S^2(t)}_{SSB-SC} = \overline{S^2(t)}_{DSB-SC}$ at the input to the detector

However, With SSB only half the bandwidth is involved as compared with DSB and hence we have that

$$(S/N)_{IN\,DSB} = \tfrac{1}{2}(S/N)_{IN\,SSB}$$

$\boxed{4}$

$\therefore$ After demodulation/detection the $(S/N)_{out\,DSB}$

$$= 2(S/N)_{IN\,DSB} = 2 \times \tfrac{1}{2}(S/N)_{IN\,SSB} = (S/N)_{IN}/_{SSB} = (S/N)_{out\,(SSB)}$$

$\therefore$ S/N wise the two systems overall, are the same.

Mark out of 20

Q4/

10/12

Part-1. This is base of textbook and lecture notes.

I expect points such as the following to be made

Circuit Switching (CS)

(i) Dedicated link for duration of session
(ii) Delay in setting up session
(iii) Small delays once session established
(iv) Inefficient use of channel capacity since link held even when 'silence' exists during session (not good for burst-type data)

$\boxed{3}$

Message Switching (MS)

(i) Long messages are passed from storage/switching centre to storage/switching centre.
(ii) Link only held during period when data active – hence high capacity utilisation
(iii) Delays can be long due to queueing and need to ~~rec~~ receive complete message before handing on.
(iv) Small % overhead (address info etc)

$\boxed{3}$

Packet Switching (PS)

(i) Message broken down into smaller packets
(ii) Again link only used when packet to be sent
(iii) Overheads greater than M.S
(iv) Delays less than MS, but greater than (CS)

$\boxed{3}$

Part 2

I expect a description and discussion of

(i) FDMA
(ii) TDMA
(iii) CDMA — code division multiple access.

$\boxed{5}$

Part 3.

The Hamming (7, 4) code has three check digits $C_1 C_2 C_3$ which are computed from the four information digits using the following parity-check equations

$$I_1 + I_2 + I_3 = C_1$$
$$I_1 + I_2 + I_4 = C_2$$
$$I_1 + I_3 + I_4 = C_3$$

The complete set of code words are

I_1	I_2	I_3	I_4	C_1	C_2	C_3
0	0	0	0	0	0	0
1	0	0	0	1	1	1
0	1	0	0	1	1	0
1	1	0	0	0	0	1
0	0	1	0	0	0	1
1	0	1	0	0	1	0
0	1	1	0	0	1	1
1	1	1	0	1	0	0
0	0	0	1	0	1	1
1	0	0	1	1	0	0
0	1	0	1	1	0	1
1	1	0	1	0	1	0
0	0	1	1	1	1	0
1	0	1	1	0	0	1
0	1	1	1	0	0	0
1	1	1	1	1	1	1

Now the only error patterns that will not be detected are those that are in fact equal to code words.

Examination of the set of code words shows that we have 7 weight 3 code words, 7 weight 4 codewords and 1 (one) weight 7 code word. $\boxed{6}$

So $\boxed{P_e = 7p^3(1-p)^4 + 7p^4(1-p)^3 + p^7}$

Mark out of 20