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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

MSc and EEE PART IV: MEng and ACGI

RADIO FREQUENCY ELECTRONICS

Wednesday, 5 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : S. Lucyszyn
Second Marker(s) : C. Papavassiliou

Special Information for Invigilators: none.

Information for Candidates: Filter curves and filter tables are attached at the back

The Questions

1. (a) From first-principles, show that the time-average power flow along a lossless transmission line is independent of the line length and is equal to the incident wave power minus the reflected wave power at the termination impedance. Assume the characteristic impedance is purely real.

[5]

(b) Define frequency dispersion in a transmission line. If a transmission lines is represented by a lumped-element model, show what happens to frequency dispersion when $RC = GL$.

[5]

(c) Lossless transmission lines can be represented by an infinite number of lumped series-*L*/shunt-*C* sections. From first principles, derive a self-consistent equation for characteristic impedance, in terms of L and C, and use this to derive the corresponding bandwidth for this solution.

[5]

(d) Calculate the input impedance, return loss and transmission loss at the cut-off frequency determined in 1(c).

[5]

2. (a) Describe, with the aid of a diagram, a branch-line coupler having a distributed-element implementation, indicating lengths and impedances. Indicate its main characteristics.

[5]

(b) Replace the distributed-element implementation in 2(a) with an equivalent lumped-element version. How does the performance compare with that in 2(a). With a coupler having an impedance of $Zo = 50\ \Omega$, calculate the components values for a design frequency of 1.8 GHz.

[5]

(c) Replace the lumped-element implementation in 2(b) with an equivalent lumped-distributed version. With a coupler having an impedance of $Zo = 50\ \Omega$ and line sections of $\phi = 45°$, calculate the components values for a design frequency of 1.8 GHz.

[5]

(d) Draw the topology of a balanced amplifier employing the coupler given in 2(a).

[5]

3. (a) Describe the various mechanisms associated with power loss in guided-wave structures.

[5]

(b) Describe the various modes that can be supported by CPW, GCPW, CBCPW and FGC transmission lines and, where possible, indicate how unwanted modes can be suppressed. Calculate the cut-off frequency for the TM_1 mode, given an FR4 substrate having a height of 1.524 mm and dielectric constant of 3.48.

[5]

(c) Define the quality factor of a dielectric, in terms of both energy and also propagation constant. From this latter definition, derive from first principles the relationship between the quality factor of a low loss dielectric in terms of both frequency and in loss tangent. The FR4 substrate in 3(c) has a loss tangent of 0.0037, calculate its quality factor.

[5]

(d) State the general equation for power flux density and also the equation for surface power density on a conducting surface. From first principles, prove that the power flux density at the surface of a metal is the same as the surface power density on a conducting surface.

[5]

4. (a) Given a square wave signal generator, with a clock frequency that can vary between zero and 1000 MHz, propose how this could be used to generate a sinusoidal signal at a frequency of 1800 MHz.

[4]

(b) Design a lumped-element *L-C* high-pass filter to meet the following specifications:

Pass band attenuation ripple	0.1 dB
-3 dB cut-off frequency:	1500 MHz
Stop band frequency:	600 MHz
Stop band attenuation:	> 70 dB
Source impedance, Zs:	50 Ω
Load impedance, Z_L:	50 Ω

[10]

(c) Determine the worst-case levels of return losses within both the pass band and the stop band for the filter in 4(b). How could the stop band return loss adversely effect the implementation of 4(a) and suggest a suitable topology for overcoming this problem?

[6]

5. (a) Explain the advantages of using *S*-parameters over other types of parameters and comment on which parameters commercial RF circuit simulators use to perform their analysis.

[5]

(b) Using appropriate diagrams describe the use of stability circles and indicate all the various conditions of stability. How do these conditions relate to Rollett's stability factor?

[5]

(c) In terms of power ratios, define Insertion Power Gain, Forward Transducer Power Gain, Operating Power Gain and Available Power Gain. Briefly explain the applications of each definition.

[5]

(d) Given a transistor with small-signal *S*-parameters of $S_{21} = 6.5 - j\,3.7$ and $S_{12} = 0.2 + j\,0.1$ and also a Rollett's stability factor, $K = 1.27$, calculate the Maximum Stable Gain and Maximum Available Gain. What are the roll-off frequency characteristics for these gains?

[5]

6. (a) Draw the simplified circuit of an RF valve amplifier and explain the operation of the thermionic device used. Compare and contrast the main features of this device with the modern day solid-state MESFET.

[4]

(b) To increase the frequency of operation, explain how the valve needs to be modified. Explain why the performance of triode valves deteriorates as frequency increases into the microwave spectrum.

[5]

(c) If the unwanted parasitics of a triode value consist of a grid-cathode capacitance of 20 pF and a cathode inductance of 10 nH, derive a general expression for the input resistance and calculate its value at 2 GHz, given a transconductance value of 2 mA/V. What will the effect be on the voltage gain of the amplifier?

[5]

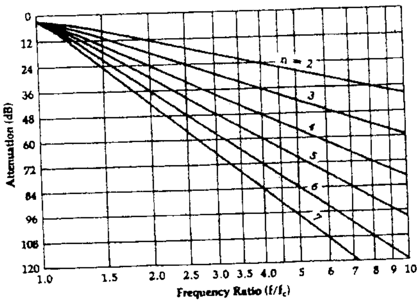
(d) Derive an expression for the Gain-Bandwidth-Product of the valve amplifier in 6(a) and calculate this for a 1 mA/V transconductance and a tank having a 10 pF capacitor. How many matched amplifiers would be needed to achieve a total power gain = 40 dB and bandwidth = 500 MHz?

[4]

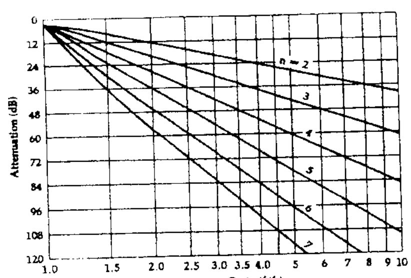
(e) Give examples of thermionic valves that could still be found within today's home, operating in the KHz, MHz and GHz frequency regions.

[2]

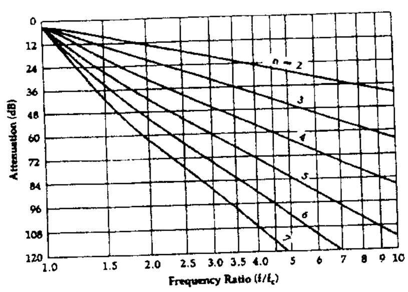
Filter tables



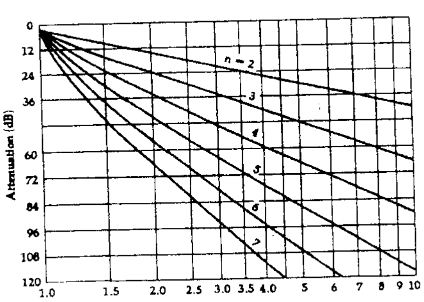
Attenuation characteristics for Butterworth filters.



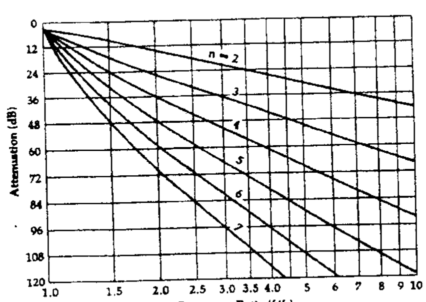
Attenuation characteristics for a Chebyshev filter with 0.01-dB ripple.



Attenuation characteristics for a Chebyshev filter with 0.1-dB ripple.

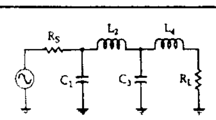


Attenuation characteristics for a Chebyshev filter with 0.5-dB ripple.

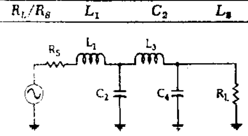


Attenuation characteristics for a Chebyshev filter with 1-dB ripple.

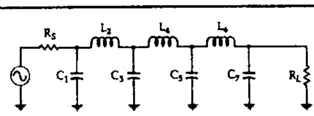
Chebyshev Low-Pass Prototype Element Values for 1.0-dB Ripple



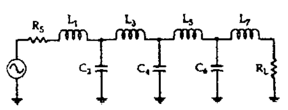
R_S/R_L	C_1	L_2	C_3	L_4
3.000	0.572	3.132		
4.000	0.365	4.600		
8.000	0.157	9.658		
∞	1.213	1.109		
1.000	2.216	1.088	2.216	
0.500	4.431	0.817	2.216	
0.333	6.647	0.726	2.216	
0.250	8.862	0.680	2.216	
0.125	17.725	0.612	2.216	
∞	1.652	1.460	1.108	
3.000	0.653	4.411	0.814	2.535
4.000	0.452	7.083	0.612	2.848
8.000	0.209	17.164	0.428	3.281
∞	1.350	2.010	1.488	1.106
R_L/R_S	L_1	C_2	L_3	C_4



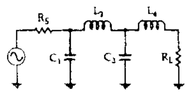
Chebyshev Low-Pass Prototype Element Values for 1.0-dB Ripple



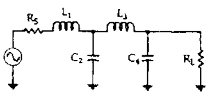
n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	2.207	1.128	3.103	1.128	2.207		
	0.500	4.414	0.565	4.653	1.128	2.207		
	0.333	6.622	0.376	6.205	1.128	2.207		
	0.250	8.829	0.282	7.756	1.128	2.207		
	0.125	17.657	0.141	13.961	1.128	2.207		
	∞	1.721	1.645	2.061	1.493	1.103		
6	3.000	0.679	3.873	0.771	4.711	0.969	2.406	
	4.000	0.481	5.644	0.476	7.351	0.849	2.582	
	8.000	0.227	12.310	0.198	16.740	0.726	2.800	
	∞	1.378	2.097	1.690	2.074	1.494	1.102	
7	1.000	2.204	1.131	3.147	1.194	3.147	1.131	2.204
	0.500	4.408	0.566	6.293	0.895	3.147	1.131	2.204
	0.333	6.612	0.377	9.441	0.796	3.147	1.131	2.204
	0.250	8.815	0.283	12.588	0.747	3.147	1.131	2.204
	0.125	17.631	0.141	25.175	0.671	3.147	1.131	2.204
	∞	1.741	1.677	2.155	1.703	2.079	1.494	1.102
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7



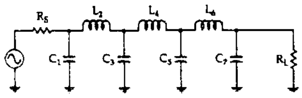
Butterworth Low-Pass Prototype Element Values



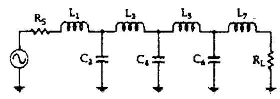
n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.111	1.035	1.835		
	1.250	0.849	2.121		
	1.429	0.697	2.439		
	1.667	0.566	2.828		
	2.000	0.448	3.346		
	2.500	0.342	4.095		
	3.333	0.245	5.313		
	5.000	0.156	7.707		
	10.000	0.074	14.814		
	∞	1.414	0.707		
3	0.900	0.808	1.633	1.599	
	0.800	0.844	1.384	1.926	
	0.700	0.915	1.165	2.277	
	0.600	1.023	0.965	2.702	
	0.500	1.181	0.779	3.261	
	0.400	1.425	0.604	4.064	
	0.300	1.838	0.440	5.363	
	0.200	2.669	0.284	7.910	
	0.100	5.167	0.138	15.455	
	∞	1.500	1.333	0.500	
4	1.111	0.466	1.592	1.744	1.469
	1.250	0.388	1.695	1.511	1.811
	1.429	0.325	1.862	1.291	2.175
	1.667	0.269	2.103	1.082	2.613
	2.000	0.218	2.452	0.883	3.187
	2.500	0.169	2.986	0.691	4.009
	3.333	0.124	3.883	0.507	5.338
	5.000	0.080	5.684	0.331	7.940
	10.000	0.039	11.094	0.162	15.642
	∞	1.531	1.577	1.082	0.383
n	R_L/R_S	L_1	C_2	L_3	C_4



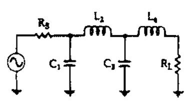
Butterworth Low-Pass Prototype Element Values



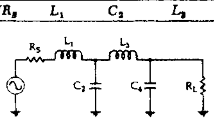
n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	0.900	0.442	1.027	1.910	1.756	1.389		
	0.800	0.470	0.866	2.061	1.544	1.738		
	0.700	0.517	0.731	2.285	1.333	2.108		
	0.600	0.586	0.609	2.600	1.128	2.552		
	0.500	0.686	0.496	3.051	0.924	3.133		
	0.400	0.838	0.388	3.736	0.727	3.965		
	0.300	1.094	0.285	4.884	0.537	5.307		
	0.200	1.608	0.186	7.185	0.352	7.935		
	0.100	3.512	0.091	14.095	0.173	15.710		
	∞	1.545	1.694	1.382	0.894	0.309		
6	1.111	0.289	1.040	1.322	2.054	1.744	1.335	
	1.250	0.245	1.116	1.126	2.239	1.550	1.688	
	1.429	0.207	1.236	0.957	2.499	1.346	2.062	
	1.667	0.173	1.407	0.801	2.858	1.143	2.509	
	2.000	0.141	1.653	0.654	3.369	0.942	3.094	
	2.500	0.111	2.028	0.514	4.141	0.745	3.931	
	3.333	0.082	2.656	0.379	5.433	0.552	5.280	
	5.000	0.054	3.917	0.248	8.020	0.363	7.922	
	10.000	0.026	7.705	0.122	15.786	0.179	15.738	
	∞	1.553	1.759	1.553	1.202	0.758	0.259	
7	0.900	0.299	0.711	1.404	1.489	2.125	1.727	1.296
	0.800	0.322	0.606	1.517	1.278	2.334	1.546	1.652
	0.700	0.357	0.515	1.688	1.091	2.618	1.350	2.028
	0.600	0.408	0.432	1.928	0.917	3.005	1.150	2.477
	0.500	0.480	0.354	2.273	0.751	3.553	0.951	3.064
	0.400	0.590	0.278	2.795	0.592	4.380	0.754	3.904
	0.300	0.775	0.206	3.671	0.437	5.761	0.560	5.258
	0.200	1.145	0.135	5.427	0.287	8.526	0.369	7.908
	0.100	2.257	0.067	10.700	0.142	16.822	0.182	15.748
	∞	1.558	1.799	1.659	1.397	1.055	0.656	0.223
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7



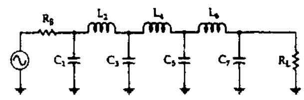
Chebyshev Low-Pass Element Values for 0.01-dB Ripple



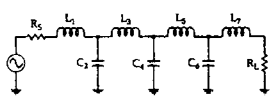
n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.101	1.347	1.483		
	1.111	1.247	1.595		
	1.250	0.943	1.997		
	1.429	0.750	2.344		
	1.667	0.609	2.750		
	2.000	0.479	3.277		
	2.500	0.363	4.033		
	3.333	0.259	5.253		
	5.000	0.164	7.650		
	10.000	0.078	14.749		
	∞	1.412	0.742		
3	1.000	1.181	1.821	1.181	
	0.900	1.092	1.660	1.480	
	0.800	1.097	1.443	1.806	
	0.700	1.160	1.228	2.165	
	0.600	1.274	1.024	2.598	
	0.500	1.452	0.829	3.164	
	0.400	1.734	0.645	3.974	
	0.300	2.216	0.470	5.280	
	0.200	3.193	0.305	7.834	
	0.100	6.141	0.148	15.390	
	∞	1.501	1.433	0.591	
4	1.100	0.950	1.938	1.761	1.046
	1.111	0.854	1.946	1.744	1.165
	1.250	0.618	2.075	1.542	1.617
	1.429	0.495	2.279	1.334	2.008
	1.667	0.398	2.571	1.128	2.461
	2.000	0.316	2.994	0.926	3.045
	2.500	0.242	3.641	0.729	3.875
	3.333	0.174	4.727	0.538	5.209
	5.000	0.112	6.910	0.352	7.813
	10.000	0.054	13.469	0.173	15.510
	∞	1.529	1.694	1.312	0.523
n	R_L/R_S	L_1	C_2	L_3	C_4



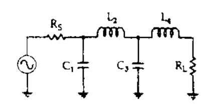
Chebyshev Low-Pass Element Values for 0.01-dB Ripple



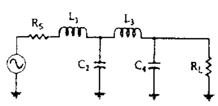
n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	0.977	1.685	2.037	1.685	0.977		
	0.900	0.880	1.456	2.174	1.641	1.274		
	0.800	0.877	1.235	2.379	1.499	1.607		
	0.700	0.926	1.040	2.658	1.323	1.977		
	0.600	1.019	0.863	3.041	1.135	2.424		
	0.500	1.166	0.699	3.584	0.942	3.009		
	0.400	1.398	0.544	4.403	0.749	3.845		
	0.300	1.797	0.398	5.772	0.557	5.193		
	0.200	2.604	0.259	8.514	.0.368	7.826		
	0.100	5.041	0.127	16.741	0.182	15.613		
	∞	1.547	1.795	1.645	1.237	0.488		
6	1.101	0.851	1.796	1.841	2.027	1.631	0.937	
	1.111	0.760	1.782	1.775	2.094	1.638	1.053	
	1.250	0.545	1.864	1.489	2.403	1.507	1.504	
	1.429	0.436	2.038	1.266	2.735	1.332	1.899	
	1.667	0.351	2.298	1.061	3.167	1.145	2.357	
	2.000	0.279	2.678	0.867	3.768	0.954	2.948	
	2.500	0.214	3.261	0.682	4.667	0.761	3.790	
	3.333	0.155	4.245	0.503	6.163	0.568	5.143	
	5.000	0.100	6.223	0.330	9.151	0.376	7.785	
	10.000	0.048	12.171	0.162	18.105	0.187	15.595	
	∞	1.551	1.847	1.790	1.598	1.190	0.469	
7	1.000	0.913	1.595	2.002	1.870	2.002	1.595	0.913
	0.900	0.816	1.362	2.089	1.722	2.202	1.581	1.206
	0.800	0.811	1.150	2.262	1.525	2.465	1.464	1.538
	0.700	0.857	0.967	2.516	1.323	2.802	1.307	1.910
	0.600	0.943	0.803	2.872	1.124	3.250	1.131	2.359
	0.500	1.080	0.650	3.382	0.928	3.875	0.947	2.948
	0.400	1.297	0.507	4.156	0.735	4.812	0.758	3.790
	0.300	1.669	0.372	5.454	0.546	6.370	0.568	5.148
	0.200	2.242	0.242	8.057	0.360	9.484	0.378	7.802
	0.100	4.701	0.119	15.872	0.178	18.818	0.188	15.652
	∞	1.559	1.867	1.866	1.765	1.563	1.161	0.456
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7



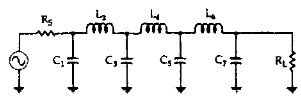
Chebyshev Low-Pass Prototype Element Values for 0.1-dB Ripple



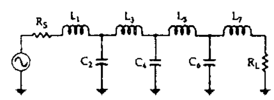
n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.355	1.209	1.638		
	1.429	0.977	1.982		
	1.667	0.733	2.489		
	2.000	0.560	3.054		
	2.500	0.417	3.827		
	3.333	0.293	5.050		
	5.000	0.184	7.426		
	10.000	0.087	14.433		
	∞	1.391	0.819		
3	1.000	1.433	1.594	1.433	
	0.900	1.426	1.494	1.622	
	0.800	1.451	1.356	1.871	
	0.700	1.521	1.193	2.190	
	0.600	1.648	1.017	2.603	
	0.500	1.853	0.838	3.159	
	0.400	2.186	0.660	3.968	
	0.300	2.763	0.486	5.279	
	0.200	3.942	0.317	7.850	
	0.100	7.512	0.155	15.466	
	∞	1.513	1.510	0.716	
4	1.355	0.992	2.148	1.585	1.341
	1.429	0.779	2.348	1.429	1.700
	1.667	0.576	2.730	1.185	2.243
	2.000	0.440	3.227	0.967	2.856
	2.500	0.329	3.961	0.760	3.698
	3.333	0.233	5.178	0.560	5.030
	5.000	0.148	7.607	0.367	7.614
	10.000	0.070	14.887	0.180	15.230
	∞	1.511	1.768	1.455	0.673
n	R_L/R_S	L_1	C_2	L_3	C_4



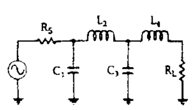
Chebyshev Low-Pass Prototype Element Values for 0.1-dB Ripple



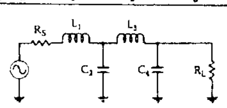
n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	1.301	1.556	2.241	1.556	1.301		
	0.900	1.285	1.433	2.380	1.488	1.488		
	0.800	1.300	1.282	2.582	1.382	1.738		
	0.700	1.358	1.117	2.868	1.244	2.062		
	0.600	1.470	0.947	3.269	1.085	2.484		
	0.500	1.654	0.778	3.845	0.913	3.055		
	0.400	1.954	0.612	4.720	0.733	3.886		
	0.300	2.477	0.451	6.196	0.550	5.237		
	0.200	3.546	0.295	9.127	0.366	7.889		
	0.100	6.787	0.115	17.957	0.182	15.745		
	∞	1.561	1.807	1.766	1.417	0.651		
6	1.355	0.942	2.080	1.659	2.247	1.534	1.277	
	1.429	0.735	2.249	1.454	2.544	1.405	1.629	
	1.667	0.542	2.600	1.183	3.064	1.185	2.174	
	2.000	0.414	3.068	0.958	3.712	0.979	2.794	
	2.500	0.310	3.765	0.749	4.651	0.778	3.645	
	3.333	0.220	4.927	0.551	6.195	0.580	4.996	
	5.000	0.139	7.250	0.361	9.261	0.384	7.618	
	10.000	0.067	14.220	0.178	18.427	0.190	15.350	
	∞	1.534	1.884	1.831	1.749	1.394	0.638	
7	1.000	1.262	1.520	2.239	1.680	2.239	1.520	1.262
	0.900	1.242	1.395	2.361	1.578	2.397	1.459	1.447
	0.800	1.255	1.245	2.548	1.443	2.624	1.362	1.697
	0.700	1.310	1.083	2.819	1.283	2.942	1.233	2.021
	0.600	1.417	0.917	3.205	1.209	3.384	1.081	2.444
	0.500	1.595	0.753	3.764	0.928	4.015	0.914	3.018
	0.400	1.885	0.593	4.618	0.742	4.970	0.738	3.855
	0.300	2.392	0.437	6.054	0.556	6.569	0.557	5.217
	0.200	3.428	0.286	8.937	0.369	9.770	0.372	7.890
	0.100	6.570	0.141	17.603	0.184	19.376	0.186	15.813
	∞	1.575	1.858	1.921	1.827	1.734	1.379	0.631
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7



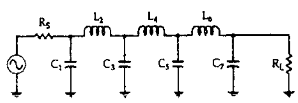
Chebyshev Low-Pass Prototype Element Values for 0.5-dB Ripple



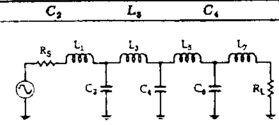
n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.984	0.983	1.950		
	2.000	0.909	2.103		
	2.500	0.564	3.165		
	3.333	0.375	4.411		
	5.000	0.228	6.700		
	10.000	0.105	13.322		
	∞	1.307	0.975		
3	1.000	1.864	1.280	1.834	
	0.900	1.918	1.209	2.026	
	0.800	1.997	1.120	2.237	
	0.700	2.114	1.015	2.517	
	0.500	2.557	0.759	3.436	
	0.400	2.985	0.615	4.242	
	0.300	3.729	0.463	5.576	
	0.200	5.254	0.309	8.225	
	0.100	9.890	0.153	16.118	
	∞	1.572	1.518	0.932	
4	1.984	0.920	2.586	1.304	1.826
	2.000	0.845	2.720	1.238	1.985
	2.500	0.516	3.766	0.869	3.121
	3.333	0.344	5.120	0.621	4.480
	5.000	0.210	7.708	0.400	6.987
	10.000	0.098	15.352	0.194	14.262
	∞	1.436	1.889	1.521	0.913
n	R_L/R_S	L_1	C_2	L_3	C_4



Chebyshev Low-Pass Prototype Element Values for 0.5-dB Ripple



n	R_S/R_L	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.000	1.807	1.303	2.691	1.303	1.807		
	0.900	1.854	1.222	2.849	1.238	1.970		
	0.800	1.926	1.126	3.060	1.157	2.185		
	0.700	2.035	1.015	3.353	1.058	2.470		
	0.600	2.200	0.890	3.765	0.942	2.861		
	0.500	2.457	0.754	4.367	0.810	3.414		
	0.400	2.870	0.609	5.296	0.664	4.245		
	0.300	3.588	0.459	6.871	0.508	5.625		
	0.200	5.064	0.306	10.054	0.343	8.367		
	0.100	9.556	0.153	19.647	0.173	16.574		
	∞	1.630	1.740	1.922	1.514	0.903		
6	1.984	0.905	2.577	1.368	2.713	1.299	1.796	
	2.000	0.830	2.704	1.291	2.872	1.237	1.956	
	2.500	0.506	3.722	0.890	4.109	0.881	3.103	
	3.333	0.337	5.055	0.632	5.699	0.635	4.481	
	5.000	0.206	7.615	0.406	8.732	0.412	7.031	
	10.000	0.096	15.186	0.197	17.681	0.202	14.433	
7	1.000	1.790	1.296	2.718	1.385	2.718	1.296	1.790
	0.900	1.835	1.215	2.869	1.308	2.883	1.234	1.953
	0.800	1.905	1.118	3.076	1.215	3.107	1.155	2.168
	0.700	2.011	1.007	3.384	1.105	3.416	1.058	2.455
	0.600	2.174	0.882	3.772	0.979	3.852	0.944	2.848
	0.500	2.428	0.747	4.370	0.838	2.289	0.814	3.405
	0.400	2.835	0.604	5.295	0.685	5.470	0.669	4.243
	0.300	3.546	0.455	6.867	0.522	7.134	0.513	5.635
	0.200	5.007	0.303	10.049	0.352	10.496	0.348	8.404
	0.100	9.456	0.151	19.649	0.178	20.631	0.176	16.665
	∞	1.646	1.777	2.031	1.789	1.924	1.503	0.895
n	R_L/R_S	L_1	C_2	L_3	C_4	L_5	C_6	L_7



A06

Radio Frequency Electronics (E4.18): 2004 Solutions

Model answer to Q 1(a): Bookwork and derivation exercise

The voltage and current on the line can be represented as :

$$V(z) = V_+\left(e^{-\gamma z} + \rho(0)e^{+\gamma z}\right)$$
$$I(z) = I_+\left(e^{-\gamma z} - \rho(0)e^{+\gamma z}\right)$$

$$\text{It can be found that}: V_+ = 0.5\left(V(0) + ZoI(0)\right) \quad \text{and} \quad V_- = 0.5\left(V(0) - ZoI(0)\right)$$

$$\therefore \text{incident wave power}, P_+ = \frac{|V_+|^2}{Zo} \quad \text{and} \quad \text{reflected wave power}, P_- = \frac{|V_-|^2}{Zo}$$

If Zo is taken to be purely real, the time-average power flow along the line is:

$$\mathrm{P}(z) = \mathrm{Re}\{V(z)I(z)^*\} = \mathrm{Re}\left\{V_+\left(e^{-\gamma z} + \rho(0)e^{+\gamma z}\right)I_+{}^*\left(e^{-\gamma z} - \rho(0)e^{+\gamma z}\right)^*\right\}$$

$$\text{where}, \rho(z) = \rho(0)e^{+2\gamma z} \equiv \rho(0)e^{+j2\beta z} \text{ for a lossless line}$$

$$\mathrm{P}(z) = \mathrm{Re}\left\{\frac{|V_+|^2}{Zo}(1+\rho(z))(1-\rho(z))^*\right\} = \mathrm{Re}\left\{\frac{|V_+|^2}{Zo}(1+\rho(z))(1-\rho(z)^*)\right\} = \frac{|V_+|^2}{Zo}\left(1-|\rho(z)|^2\right)$$

$$\text{but}, |\rho(z)| = |\rho(0)| \text{ for a lossless transmission line}$$

$$\therefore \mathrm{P}(z) = \frac{|V_+|^2}{Zo}\left(1-|\rho(0)|^2\right) = P_+\left(1-\frac{P_-}{P_+}\right) = \frac{|V_+|^2}{Zo}\left(1-|\rho(0)|^2\right) = P_+\left(1-\frac{P_-}{P_+}\right) = (P_+ - P_-)$$

This shows that, for a lossless transmission line, time-average power flow is independent of the line length and is equal to the incident wave power minus the reflected wave power.

[5]

Model answer to Q 1(b): Bookwork

The guided wavelength, λg, is defined as the distance between two successive points of equal phase on the wave at a fixed instance in time. The phase velocity of a wave is defined as the speed at which a constant phase point travels down the line. Frequency dispersion is said to occur when $\beta \neq \omega$.constant. Dispersion can occur when $vp = f(\omega)$, i.e. when $Dk = f(\omega)$. It can be shown that zero dispersion in a lossy line can also occur, but only when RC = GL:

$$\gamma^2 = (R + j\omega L)(G + j\omega C) \quad \text{and} \quad RC = GL$$

$$\therefore \alpha(\omega) = \alpha(0) = \sqrt{RG} \neq f(\omega) \quad \text{and} \quad \beta = \omega\sqrt{LC}$$

$$\text{also, Group Velocity}, Vg = \frac{\partial\omega}{\partial\beta} = \frac{1}{\sqrt{LC}} \equiv vp \neq f(\omega)$$

[5]

Model answer to Q 1(c): Bookwork

$$Zin = j\omega L + \frac{Zo\dfrac{1}{j\omega C}}{Zo + \dfrac{1}{j\omega C}} \equiv Zo$$

$$\therefore Zo = \frac{j\omega L}{2}\left(1 \mp \sqrt{1 - \frac{4}{\omega^2 LC}}\right)$$

$\therefore$ *Cut-off frequency,* $fc = \dfrac{1}{\pi\sqrt{LC}}$ representing the bandwidth, i.e. when $\dfrac{4}{\omega^2 LC} = 1$

$$Zo = \begin{cases} \sqrt{\dfrac{L}{C}} & when \quad \omega << \omega_c \quad i.e.\ purely\ real \\ Complex & when \quad 0 << \omega < \omega_c \\ j\sqrt{\dfrac{L}{C}} & when \quad \omega = \omega_c \quad i.e.\ purely\ imajinary \\ Imajinary & when \quad \omega \geq \omega_c \end{cases}$$

[5]

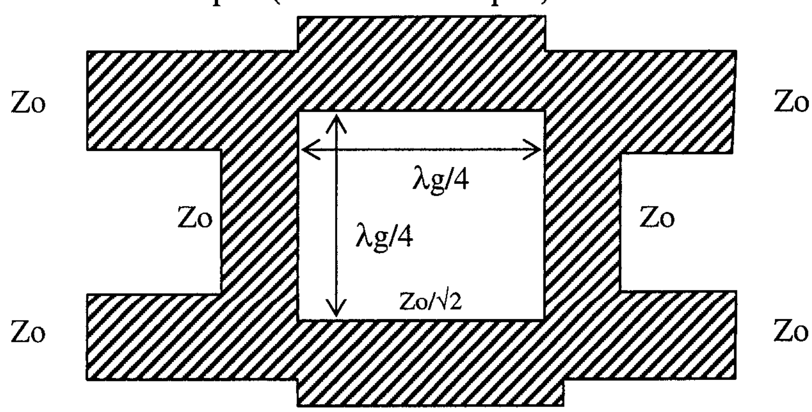
Model answer to Q 1(d): Computed Example

$$Zin(\omega_c) = jZo(\omega << \omega_c) \quad and \quad |\rho(\omega_c)|^2 = 1 \Rightarrow 0\ dB \quad and \quad |\tau(\omega_c)|^2 = 1 - |\rho(\omega_c)|^2 = 0 \Rightarrow -\infty\ dB$$

[5]

Model answer to Q 2(a): Bookwork

90° 3dB Directional Coupler (Branch-line Coupler)

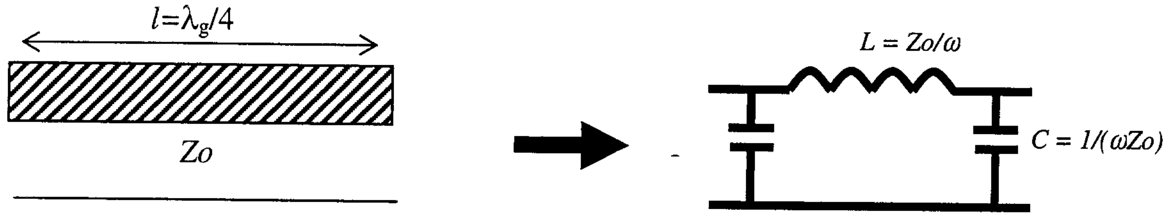


- Works on the interference principle, therefore, narrow fractional bandwidth (15% maximum)
- No bond-wires or isolation resistors required
- Wider tracks make it easier to fabricate and is, therefore, good for lower loss and higher power applications
- Simple design but large
- Meandered lines are possible for lower frequency applications

[5]

Model answer to Q 2(b): Bookwork and Computed Example

The lumped-element equivalent of a $\lambda_g/4$ transmission line is shown below.



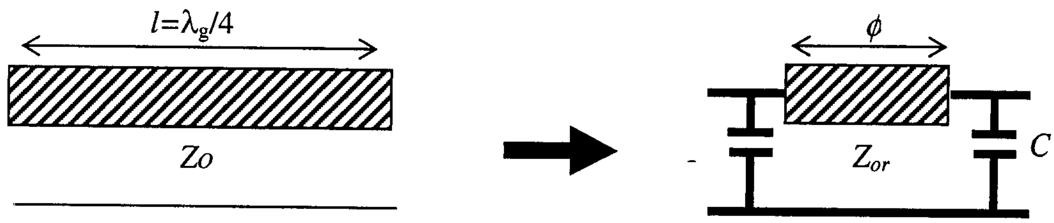
All the previous distributed-element couplers can be transformed into equivalent lumped-element couplers by simply replacing all the $\lambda_g/4$ lengths of transmission lines with the above π-network. Since lumped-element components have a lower Q-factor, when compared to distributed-element components, there is an insertion loss penalty. Also, because this π-network is clearly a low-pass filter, having a cut-off frequency, $f_c = \frac{1}{2\pi\sqrt{LC}}$, there is also a bandwidth penalty.

L = 4.42 nH and C = 1.77 pF for the Zo = 50 Ω sections of line
L = 3.13 nH and C = 2.50 pF for the Zo = 35.36 Ω sections of line

[5]

Model answer to Q 2(c): Bookwork and Computed Example

- Lumped-Distributed Couplers



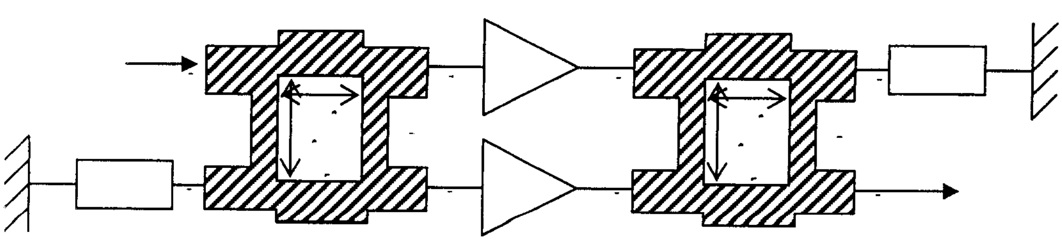
In this 'reduced-size' technique, each $\lambda_g/4$ line is replaced with the above π-network.

$$Z_{or} = \frac{Zo}{Sin\phi} \quad and \quad C = \frac{Cos\phi}{\omega Zo}$$

With $\phi = 45°$,
Z_{or} = 70.7 Ω and C = 1.25 pF for the Zo = 50 Ω sections of line
Z_{or} = 50 Ω and C = 1.77 pF for the Zo = 35.36 Ω sections of line

[5]

Model answer to Q 2(d): Solution given in class



[5]

Model answer to Q 3(a): Bookwork

Power Loss in Guided-Wave Structures

Transmission lines are generally realised using both conductor and dielectric materials, both of which should be chosen to have low loss characteristics. Energy is lost by joule's heating, multi-modeing and leakage. The former is attributed to ohmic losses, associated with both the conductor and dielectric materials. The second is attributed to the generation of additional unwanted modes that propagate with the desired mode. The latter is attributed to leakage waves that either radiate within the substrate (e.g. dielectric modes and surface wave modes) or out of the substrate (e.g. free space radiation and box modes).

[5]

Model answer to Q 3(b): Bookwork and Computed Example

Multi-Modeing in Conductor-Backed CPW Lines

With an ideal CPW line, only the pure-CPW (quasi-TEM) mode is considered to propagate. In the case of a grounded-CPW (GCPW) line, where the backside metallization is at the same potential as the two upper-ground planes (through the use of through-substrate vias), a microstrip like mode can also co-exist with the pure-CPW mode. With the conductor-backed CPW (CBCPW) line, where the backside metallization has a floating potential, parallel-plate line (PPL) modes can also co-exist. The significant PPL modes that are associated with CBCPW lines include the fundamental TEM mode (designated TM_0) found at frequencies from DC to infinity and the higher order TM_n modes that can only be supported above their cut-off frequency, $f_{cn} = nc/(2h\sqrt{\varepsilon_r})$. By inserting a relatively thick dielectric layer (having a lower dielectric constant than that of the substrate), between the substrate and the lower ground plane, the pure CPW mode can be preserved. This is because the capacitance between the upper and lower conductors will be significantly reduced and, therefore, there will be less energy associated with the parasitic modes. Alternatively, the parallel-plate line modes can also be suppressed by reducing the width of the upper-ground planes, resulting in finite ground CPW (or FGC). Finally, in addition to all the modes mentioned so far, the slot-line mode can also propagate if there is insufficient use of air-bridges/underpasses to equalise the potentials at both the upper-ground planes.

```
PURE-CPW + SLOT-LINE + MICROSTRIP + PARALLEL-PLATE (TEM + TMn)
|-------------CPW-------------|
|------------------GCPW and FGC----------------|
|--------------------------------------------CBCPW|-------------------------------------------|
```

For TM1, given a substrate having a height of 1.524 mm and dielectric constant of 3.48. the cut-off frequency will be 52.8 GHz.

[5]

Model answer to Q 3(c): Bookwork and Computed Example

Lossy Dielectric, Lossless Conductor

Here, electromagnetic energy flows through the dielectric and not inside the conductor (which merely acts as a guide).

$$Q = \omega \cdot \frac{Peak\ Energy\ Stored\ in\ either\ the\ E-field\ or\ H-field}{Total\ Energy\ Lost\ per\ Second\ (Average\ Power\ Loss)}$$

$$Q = \frac{\beta}{2\alpha}, \quad where \quad \beta = \frac{2\pi}{\lambda g} \quad \therefore Q = \frac{\pi}{\alpha \lambda g}$$

$$where,\ attenuation\ per\ unit\ guided-wavelength = \alpha\lambda g \quad [Np/\lambda g]$$

$$\{Note\ that,\ Power\ Attenuation = 10\log_{10}\left(e^{-2\alpha\lambda g}\right) = -20\alpha\lambda g \log_{10}(e) = -8.686\,\alpha\lambda g \quad [dB/\lambda g]\}$$

$$For\ a\ low\ loss\ dispersionless\ dielectric:\quad \alpha = \frac{\sigma'}{2}\sqrt{\frac{\mu}{\varepsilon'}} \neq f(\omega)\ \ and \quad \beta = \omega\sqrt{\mu\varepsilon'} = f(\omega)$$

$$therefore,\ \alpha\lambda g = \frac{\pi\sigma'}{\omega\varepsilon'} \quad but\ \ \sigma' = \omega\varepsilon''\ and\ tan\delta = \frac{\varepsilon''}{\varepsilon'} \quad \therefore Q = \frac{\omega\varepsilon'}{\sigma'} \equiv \frac{1}{tan\delta}$$

Note that the Q-factor of a dielectric increases proportionally with frequency! Therefore, components made with either minimal use or even no conductors at all can, in principle, have very high Q-factors (e.g. dielectric resonators).

Q-factor = 1/tanδ = 270

[5]

Model answer to Q 3(d): Bookwork

Lossless Dielectric, Lossy Conductor

The losses in the conductor result from a flow of electromagnetic energy, from the dielectric into the conductor.

$$\text{Surface Power Density, } P_S = |J_S|^2\,Rs \quad [W/m^2]$$
$$\text{where, } J_S = \text{surface current density [A/m]}$$

$$now,\ Power\ Flux\ Density,\ P_D(z) = \mathrm{Re}\{E_X(z)\,H_Y(z)^*\}$$

$$where\ \eta = \frac{E_X}{H_Y} \equiv Zs \quad \text{within a conductor}$$

$$\therefore P_D(z) = \mathrm{Re}\left\{E_X(z)\left(\frac{E_X(z)}{Zs}\right)^*\right\} = \mathrm{Re}\left\{\frac{|E_X(z)|^2}{Zs^*}\right\}$$

$$but,\ E_X(0) = ZsJs$$

$$\therefore P_D(0) = \mathrm{Re}\left\{\frac{|ZsJs|^2}{Zs^*}\right\} = \mathrm{Re}\left\{\frac{ZsZs^*|Js|^2}{Zs^*}\right\} = |J_S|^2\,Rs \equiv Ps \quad \text{Q.E.D.}$$

[5]

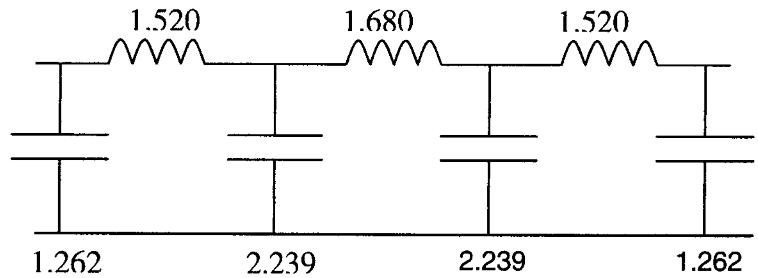
Model answer to Q 4(a): New application of theory

Given a square wave signal generator, with a clock frequency that can vary between zero and 1000 MHz, set the output to 600 MHz. Using a band-pass filter having a centre-frequency set to the third harmonic of the signal generator, the desired 1800 MHz sinusoidal signal can be extracted. Alternatively, a high-pass filter can be used, however, the 5^{th} and 7^{th} etc., harmonics will also be present but at much lower power levels.

[4]

Model answer to Q 4(b): Bookwork and Computed Example

From graphs provided, for Chebyshev filters with a 0.1 dB ripple, a 7^{th} order filter is required to achieve an out-of-band rejection of > 70 dB with an fc/f ratio of 1500/600 = 2.5.
From tables provided, with $R_S = R_L = Zo = 50$, the prototype low-pass filter and associated coefficients are given below:



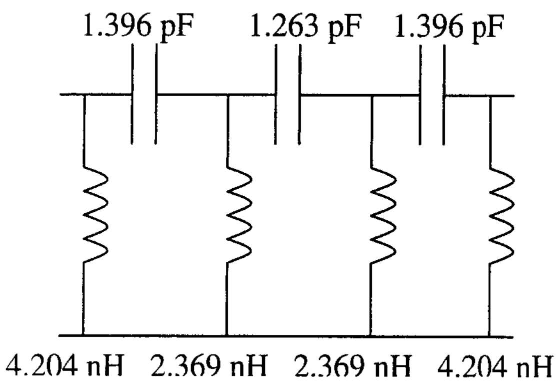
High-pass de-normalising:

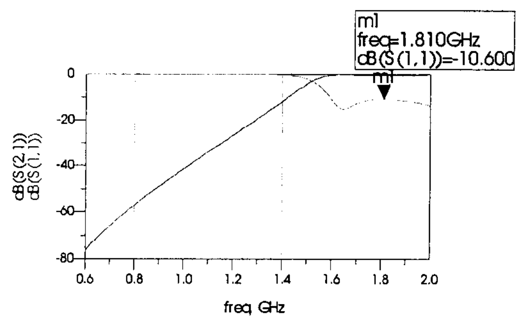
Shunt inductor:

$$L_P = \frac{R_L}{2\pi f_c Ln}$$

Series Capacitor:

$$C_S = \frac{1}{2\pi f_c CnR_L}$$





The slight deviation in the results from those predicted by theory are due to limited component tolerances used within the simulations.

[10]

Model answer to Q 4(c): New application of theory

The worst-case level of return loss within the pass band is 10 log[1- antilog(-0.1/10)] = -16.43 dB. This will have no effect within the pass band. However, well-below the pass band the return loss will be 0 dB. At the fundamental frequency of 600 MHz, all the signal will be reflected back by the filter and the square wave signal generator may not work properly (if at all). One possible solution is to insert a circulator between the square-wave generator and the filter, however, passive circulators tend to be narrow band in nature. However, a more practical solution is to use a balance topology for the filters, whereby two identical filters are embedded between two identical ultra-broadband 3 dB quadrature directional couples. This approach is inherently impedance matched across the bandwidth of the couplers.

[6]

Model answer to Q 5(a): Bookwork

Devices and networks are traditionally characterised using *Z*, *Y* or *h*-parameters. In order to measure these parameters directly, ideal open and short circuit terminations are required. These impedances can be easily realised at low frequencies. However, at microwave frequencies such impedances can only be achieved over narrow bandwidths (when tuned circuits are employed) and can also result in circuits that are conditionally stable (when embedded within a 'matched load' reference impedance environment) becoming unstable. Fortunately, scattering- (or *S*-) parameters can be determined at any frequency. To perform such measurements, the device under test (DUT) is terminated with matched loads. This enables extremely wideband measurements to be made and also greatly reduces the risk of instability; but only when the DUT is terminated with near ideal matched loads (this is irrespective of whether the measurement system is calibrated or not). *S*-parameter measurements also offer the following advantages:

1. any movement in a measurement reference plane along an ideal transmission line will vary the phase angle only (*c.f.* the complicated impedance transformation found with *z*-parameters)
2. for a linear device or network, voltage or current and measured power are related through the measurement reference impedance (normally 75 or 50 Ω for coaxial lines and 1 Ω for rectangular waveguides)
3. with some passive and reciprocal structures, ideal *S*-parameters can be deduced from spatial considerations, enabling the measurements of the structure to be checked intuitively.

While measurements are easier to perform using S-parameters, commercial RF simulators use Z- and Y-parameters to undertake the matrix calculations and then convert the results into S-parameters.

[5]

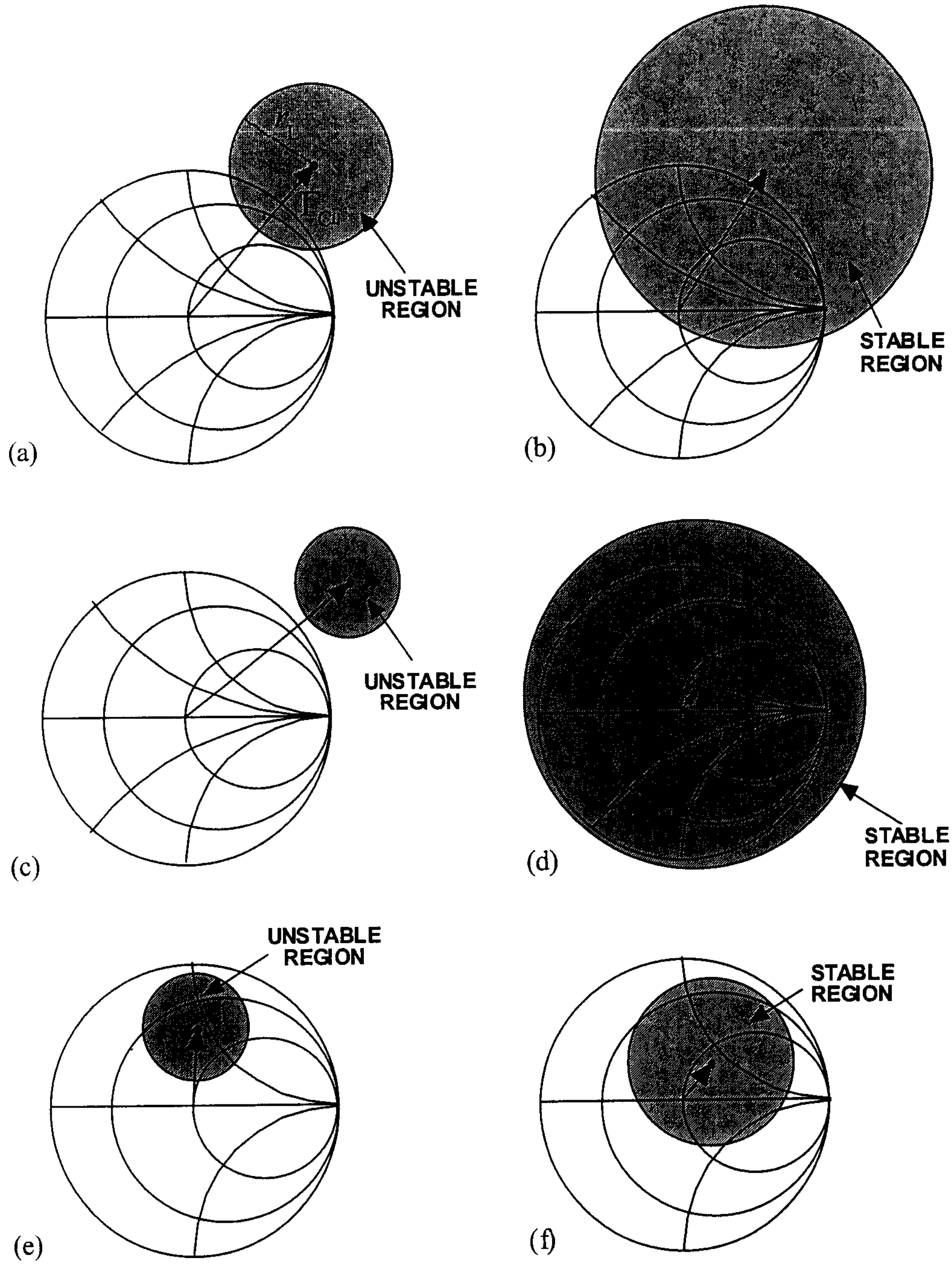
Model answer to Q 5(b): Bookwork

The "stability circle" represents a region on the Smith chart's ρ_L-plane, inside (if it does not encompass the Zo point) or outside (if it encompasses the Zo point) of which all load impedances will make the circuit's input reflection coefficient greater than unity, resulting in instability. The figure below shows the possible cases graphically. Regions (a), (b), (e) and (f) show conditionally stable regions, while (c) and (d) are unconditionally stable regions, for passive loads lying within the unit Smith chart.

It is very important for the designer to know straight away whether the transistor is unconditionally stable or not. Rollett's stability factor, *K*, gives an immediate indication of stability and this is given by:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} \quad \textit{and} \quad \Delta = S_{11}S_{22} - S_{12}S_{21}$$

If $K > 1$ then the transistor is unconditionally stable, and if $K < 1$ the stability depends on the position of the source and load impedances relative to the stability circles.



Stability circles on the Smith chart: (a) stability circle partially inside the Smith chart, (b) partially inside and encompassing the Zo point, (c) completely outside, (d) completely encompassing the Smith chart, (e) completely inside but not encompassing the Zo point and (f) completely inside and encompassing the Zo point

[5]

Model answer to Q 5(c): Bookwork

Given the following power levels, it is possible to define Power Gain in specific ways:

P_{AS} = Power Available from the Source to the Network

P_{DN} = Power Delivered to the Network

P_{AN} = Power Available from the Network to the Load

P_{DL} = Power Delivered to the Load

'Power Available' simply means the absolute maximum power that can be supplied (i.e. under complex conjugate matching). 'Power Delivered' simply means the actual power that is dissipated.

Insertion Power Gain

$$G_I = \frac{P_{DL}(DUT)}{P_{DL}(no\ DUT)}$$

In practice G_I is not very meaningful since a given level can be obtained with an infinite number of ρ_S or ρ_L values.

Forward Transducer Power Gain

$$G_{FT} = \frac{P_{DL}}{P_{AS}}$$

This is the most useful definition for characterising the advantage of employing an amplifier, as it gives the power gain relative to an ideal matching network that would perfectly match the load impedance to the source impedance.

Operating Power Gain

This represents G_{FT} when the input stage is matched.

$$G_{OP} = \frac{P_{DL}}{P_{DN}}$$

This definition is independent of ρ_S. In practice, G_{OP} is useful for power amplifier designs, since the output power of the amplifier depends on the output RF load line impedance.

Available Power Gain

This represents G_{FT} when the output stage is matched.

$$G_{AV} = \frac{P_{AN}}{P_{AS}}$$

This definition is independent of ρ_L. In practice, G_{AV} is useful for low noise amplifier designs, since the noise figure of the amplifier depends on ρ_S.

[5]

Model answer to Q 5(d): Bookwork and Computed Example

When the *K*-factor is less than unity, the device is conditionally stable and the point of maximum gain will be inside the unstable region. In this case we have to accept that the maximum transducer gain is not achievable due to instability. Therefore, the maximum gain that can be safely achieved is called the maximum stable gain (MSG) and is given by:

$$\mathrm{MSG} = \left|\frac{S_{21}}{S_{12}}\right| = 33.48 = 15.25 \text{ dB}$$

When K is greater than unity (and the magnitudes of S_{11} and S_{22} are both less than unity), the device is unconditionally stable and the maximum gain that can be achieved is called the maximum available gain (MAG), given by:

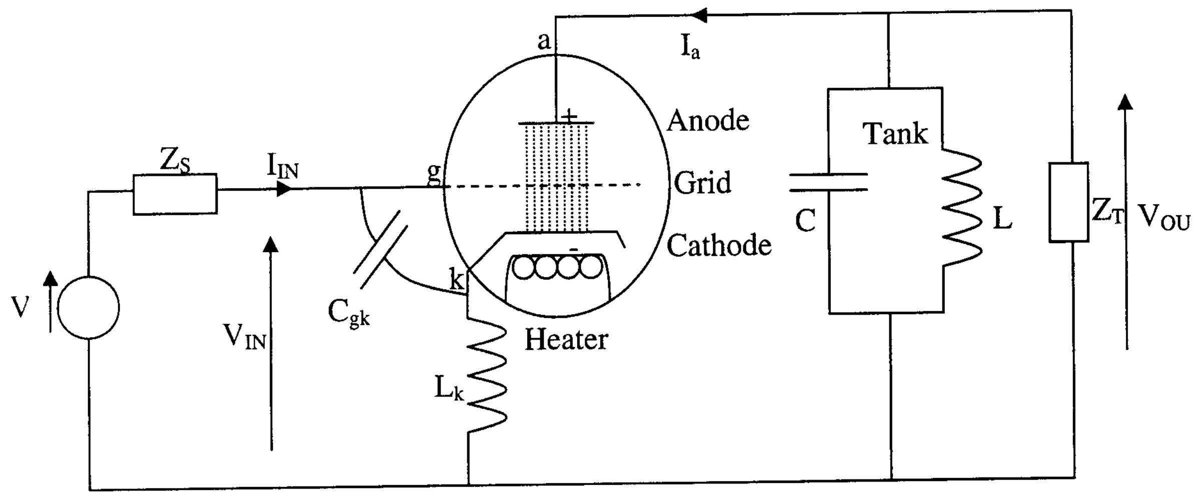
$$\mathrm{MAG} = \left|\frac{S_{21}}{S_{12}}\right|\left(K - \sqrt{K^2 - 1}\right) = 33.48 \text{ x } 0.487 = 16.31 = 12.12 \text{ dB}$$

Typically, a transistor is conditionally stable (K < 1) at low frequencies, and the maximum stable gain rolls off at 3 dB per octave. At a certain frequency K = 1, and beyond that the device is unconditionally stable (K > 1) and the maximum available gain rolls off at 6 dB per octave.

[5]

Model answer to Q 6(a): Bookwork

Triode Thermionic Valves (or Gridded Tubes)



- With the triode valve, a heater is required to heat the cathode electrode (which is negatively biased, with respect to ground). Within the vacuum or inert gas environment, electron that have absorbed enough thermal energy to overcome the work function of the cathode metal are emitted from the cathode and attracted to the positively biased plate electrode. The resulting flow of electrons can be modulated by controlling the grid-cathode potential. The GaAs MESFET is the modern day analog of the triode valve. With the MESFET, the gate-source potential controls the flow of electrons flowing from the source to drain electrodes. The triode value consumes a great deal of power for the heaters and required very large positive and negative potentials on the plate and cathode electrodes, respectively. The transconductance is generally low and, therefore, the GAIN-Bandwidth Product (GBP) will be low. In contrast, the MESFET has a higher transconductance and, therefore, higher GBP. Also, this device does not need to be heater or have large bias voltages.

[4]

Model answer to Q 6(b): Bookwork

- As frequency increases, the distance between the cathode, grid and plate must decrease. This is because the cathode-plate electron transit time must be kept well below the period of the RF cycle, otherwise:
  - The grid and plate signals may not be 180° out-of-phase, which can lead to problems with feedback oscillators.
  - The grid can take power from the driving source, since the grid voltage has time to change during the flow of a particular group of electrons from the cathode to the plate.

  As a result, at microwave frequencies, the internal dimensions are so small that:
  - The large parasitic capacitances could resonate with the lead inductances, within the band of operation.
  - Power levels are significantly reduced, since dielectric breakdown of dry air at one atmosphere is only ~ 3 x 10^6 V/m, and even much less in a vacuum.
  - Input impedance and, therefore, voltage gain are reduced.

[5]

Model answer to Q 6(c): Bookwork and Computed Example

If only C_{gk} and L_k are considered,

$I_{OUT} = g_m V_{gk}$ and, therefore, $V_{IN} \cong V_{gk} + j\omega L_k I_{OUT} = V_{gk}(1 + j\omega L_k g_m)$

$$But,\ V_{gk} = \frac{I_{IN}}{j\omega C_{gk}}$$

$$\text{Therefore, } Z_{IN} = \frac{V_{IN}}{I_{IN}} = \frac{1 + j\omega L_k gm}{j\omega C_{gk}}$$

$$\text{Therefore, } Y_{IN} = \frac{1}{Z_{IN}} = \frac{j\omega C_{gk}}{1 + j\omega L_k gm} \approx \omega^2 L_k C_{gk} g_m + j\omega C_{gk}$$

Since, $(1 \pm jA)^{-1} \approx (1 \mp jA)$ if $|A| << 1$

$$\therefore\ R_{IN} \cong \frac{1}{\omega^2 L_k C_{gk} g_m}$$

* With the tank at resonance, voltage gain = $\frac{V_{OUT}}{V_{IN}} = -g_m R_T$

But, $V_{IN} = \frac{V_S Z_{IN}}{Z_S + Z_{IN}}$ and, therefore, $\frac{V_{OUT}}{V_S} = \frac{-g_m R_T}{Z_S / Z_{IN} + 1}$

When fo = 2 GHz, L_k = 10 nH, C_{gk} = 20 pF and g_m =2 mA/V
Therefore, R_{IN} = 15.8 Ω, i.e. a near short circuit to a driving source impedance of Z_S = 50 Ω and, therefore, the overall voltage gain will be low.

[5]

Model answer to Q 6(d): Bookwork and Computed Example

Output Loaded-Quality factor, $Q_{OUT} = \frac{R_T}{|X_c|} = R_T \omega_0 C$

Therefore, output bandwidth, $\Delta\omega = \frac{\omega_0}{Q_{OUT}} = \frac{1}{R_T C}$

Therefore, for a matched amplifier, gain-bandwidth product, GBP $\cong g_m R_T \frac{\Delta\omega}{2\pi} = \frac{g_m}{2\pi C}$

For a power gain = 40 dB and bandwidth = 500 MHz it is impossible to meet this specification with the valve amplifier because each stage would have a GBP = 16×10^6 and, therefore, this represents a 30 dB attenuation.

[4]

Model answer to Q 6(e): Discussion in Lecture

In the KHz region, overly-priced Hi-Fi valve amplifiers are still commercially available.
In the MHz region, the cathode-ray tube found in domestic television receivers is still being used in the technologically backward western world.
In the GHz part of the spectrum, the 2.54 GHz magnetron is found in all domestic microwave ovens.

[2]