E4.17
AO4

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

MSc and EEE PART IV: MEng and ACGI

HIGH PERFORMANCE ANALOGUE ELECTRONICS

Thursday, 6 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

Corrected Copy

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E Rodriguez-Villegas
Second Marker(s) : D. Haigh

1. (a) Figure 1.1 shows the architecture of a Hartley receiver, where LOB is the output of the local oscillator, $\phi 1$ and $\phi 2$ are the phase shifts of the phase shifters and IF1, IF2 and LOA are the intermediate outputs. Assuming that the incoming RF signal is RF=$A\cos(\omega_A t+\phi_A)+B\cos(\omega_B t+\phi_B)$, explain how the system works by answering the following questions:

(i) Write expressions for IF1 and IF2.

(ii) Write expressions for the signals at the output of the filters.

(iii) Write an expression for the signal at the output of the phase shifter ϕ_2.

(iv) Write an expression for IF Out.

(v) If $\phi_1=90^o$, what should be the value of ϕ_2 to avoid signal distortion and ensure image rejection?

[10]

(b) Of all the frequencies that must be rejected by a superheterodyne receiver, why is the image frequency so important?

[5]

(c) A superheterodyne receiver is tuned to receive an RF signal of 612MHz, and the local oscillator frequency ($f_L=\omega_L/2\pi$) is at 660MHZ. Calculate the IF and image frequency.

[5]

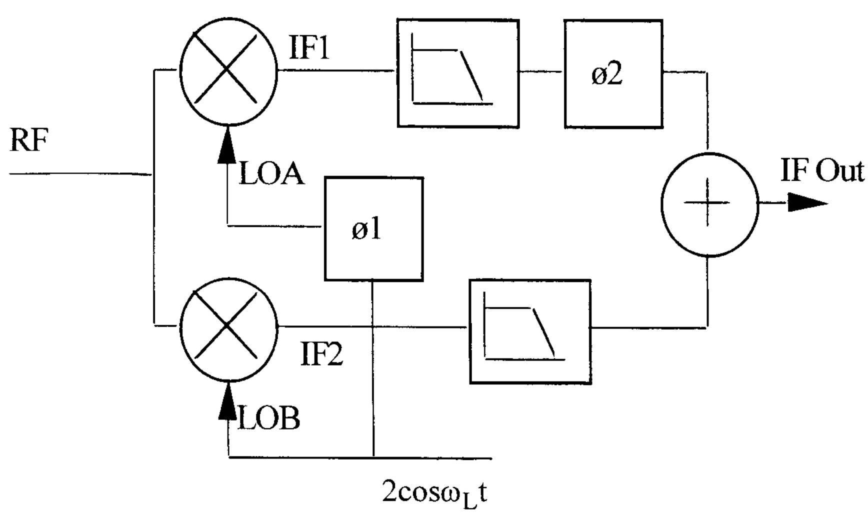

Figure 1.1

2. For the differential pair in Figure 2.1:

(a) Find the value of $Iout = Id1 - Id2$ when $Is1 = 50\ \mu A$ and $Vd = 0.1V$. Assume that $\beta_{M1} = \beta_{M2} = \beta_1 = 0.1\ mA/V^2$ (subscript Mi refers to transistor Mi).

[10]

(b) If another differential pair is cross-coupled to the circuit in Figure 2.1 (see Figure 2.2) where the value of β for the new transistors is $\beta_{M3} = \beta_{M4} = \beta_2 = 0.2\ mA/V^2$, explain how you would choose the value of $Is2$ to get a linear transfer characteristic. What is the value of the transconductance for this circuit?

[10]

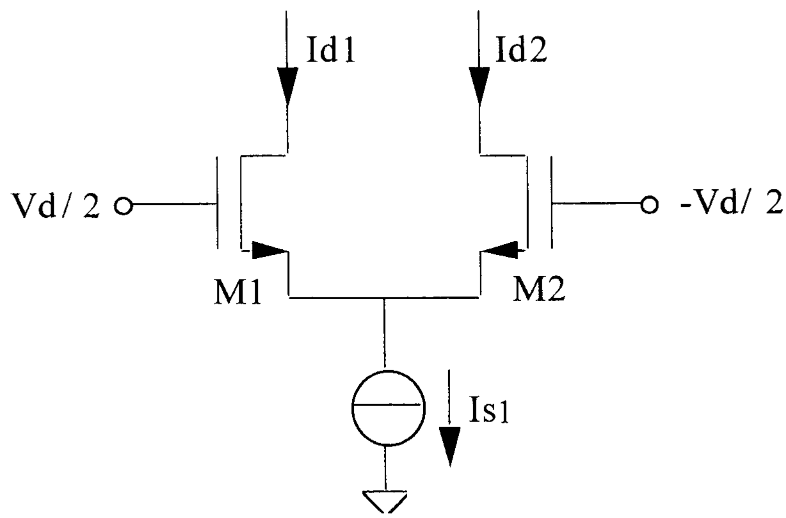

Figure 2.1

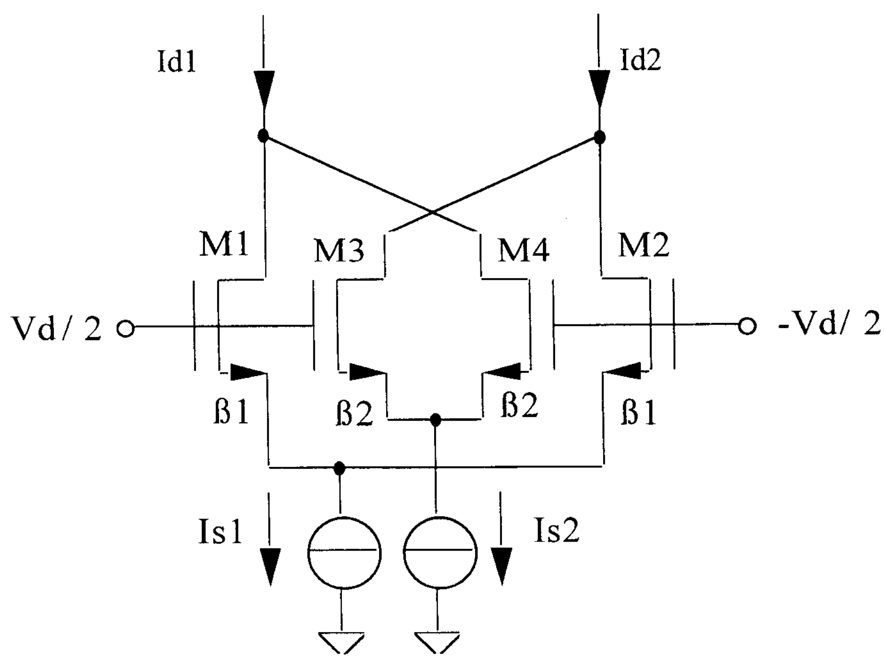

Figure 2.2

3. (a) Enumerate the three main sources of noise in a MOS transistor. Give expressions for the power spectral density of noise associated to each one of them. Which one of them is least significant compared to the others and why?

[6]

(b) Draw a small signal equivalent for noise in an nMOS transistor. For the equivalent noise model referred to the input at the gate, as shown in Figure 3.1, find expressions for vn^2 and in^2 .

[7]

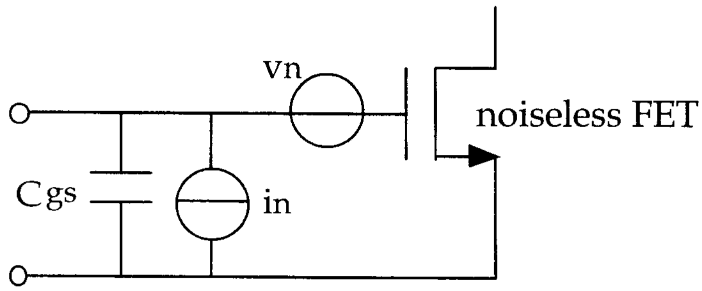

Figure 3.1

(c) Find the equivalent noise power at the input of the following system in a bandwidth between 1MHz and 20MHz. You can give the final solution as a function of Boltzmann's constant, the temperature, the transconductance of the transistor and fc. Justify any assumptions.

[7]

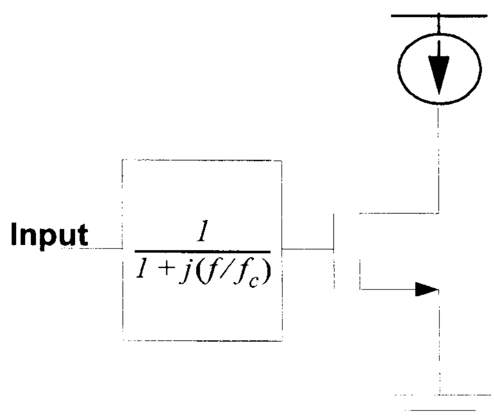

Figure 3.2

4. (a) The circuit in Figure 4.1 is an integrator circuit based on a linearised transconductor. Derive expressions for *id1-id2*, *id3-id4* and *iout=(id1-id2)-(id3-id4)*.

[10]

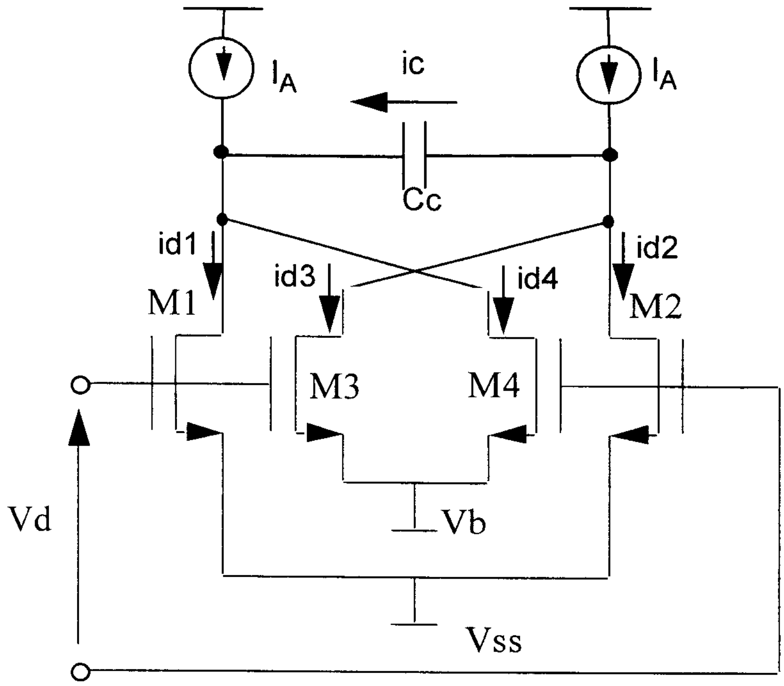

Figure 4.1

(b) What is the current flowing through the integrating capacitance *ic*? Find a differential equation for the voltage across the capacitance (*Vout*).

[5]

(c) Sketch the voltage across the capacitance (*Vout*) versus the time from 0 to 20 μs, giving a value for the maximum and the minimum *Vout* if the differential input *Vd* is 1V from 0 to 10 μs and -1V from 10 μs to 20 μs (see Figure 4.2). Assume that *Cc*=100 pF, β=0.1 mA/V^2, *Vb*=1 V and *Vss*=0 V. Assume that initially *Vout*=0.

[5]

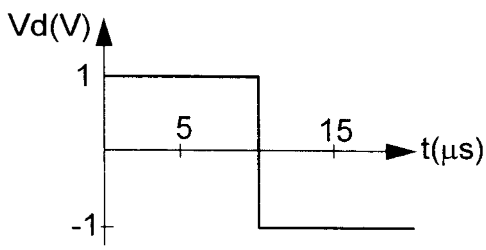

Figure 4.2

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5. (a) Figure 5.1 shows a Gilbert multiplier. Explain how it works by answering the following questions:

(i) The Gilbert multiplier is a modification of the circuit in Figure 5.2. Find an expression for the output current (*Iout=Io1-Io2*) for the circuit in Figure 5.2 as a function of *Va*, the thermal voltage *Vt*, *Iq1* and *Iq2*.

(ii) Assuming that *Va<5mV* and the tail currents can be written as *Iq1=Iq+gmVb* and *Iq2=Iq-gmVb*, what is the value of Iout in Figure 5.2 as a function of *Va*, *Vb*, *gm* and *Vt*?

(iii) Assuming that *Vb*<5 mV and based on (i) and (ii) give an expression for the value of *Iout=Io1-Io2* in the Gilbert multiplier (Figure 5.1) as a function of *Iq*, *Va*, *Vb* and the thermal voltage *Vt*.

[10]

(b) Explain why the circuit in Figure 5.3 is an improved version of the circuit in Figure 5.1. Why can we not use degeneration resistors in the top differential pairs as well?

[10]

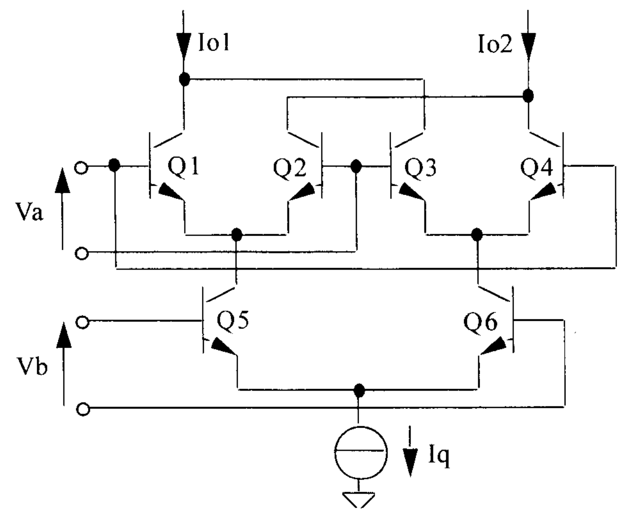

Figure 5.1

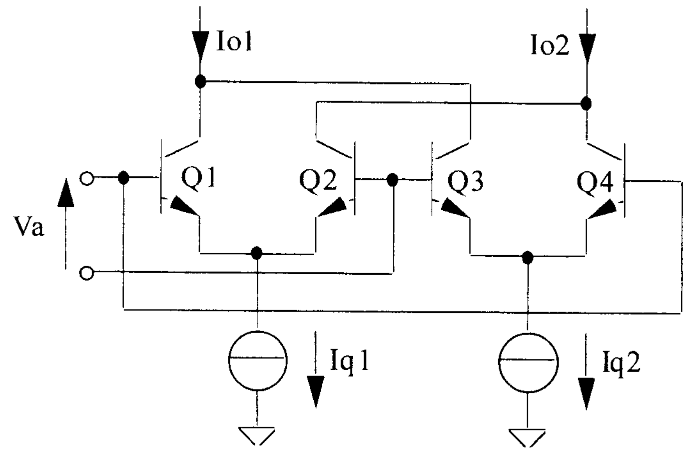

Figure 5.2

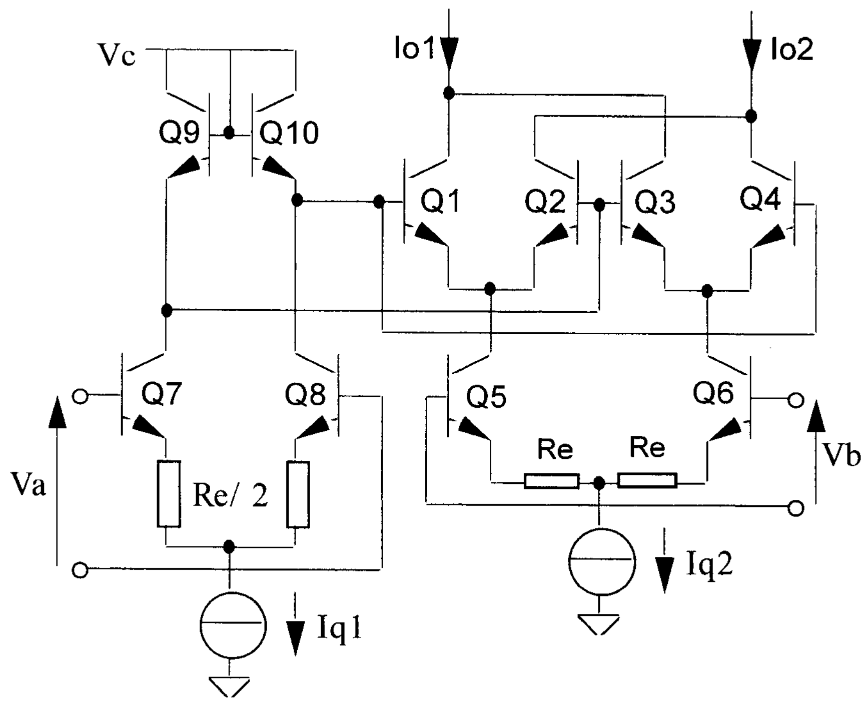

Figure 5.3

6. (a) Find the transfer function for the system in Figure 6.1.

[4]

(b) How would you choose the coefficients in the block diagram ($\tau 1$,$\tau 2$,Q, $k0$,$k1$ and $k2$) to implement a bandpass filter with resonant frequency ωo=1 rad/sec, quality factor Q =2 and gain K=1?

[3]

(c) How would you choose the coefficients in the block diagram ($\tau 1$,$\tau 2$,Q, ko,$k1$ and $k2$) to implement a high pass filter with resonant frequency ωo=1 rad/sec, quality factor Q =20 and gain K =1?

[3]

(d) Write the ladder state equations for the LC ladder filter in Figure 6.2. Draw the signal flow graph. Draw the scaled signal flow graph.

[5]

(e) Show how the circuit can be implemented using five integrators and two amplifiers.

[5]

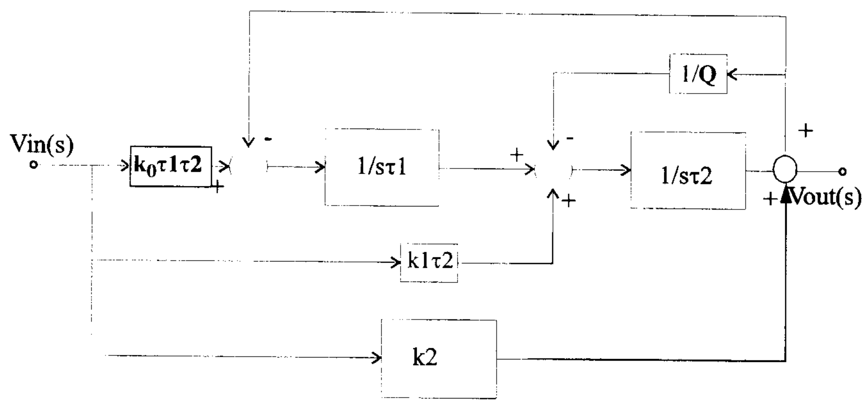

Figure 6.1

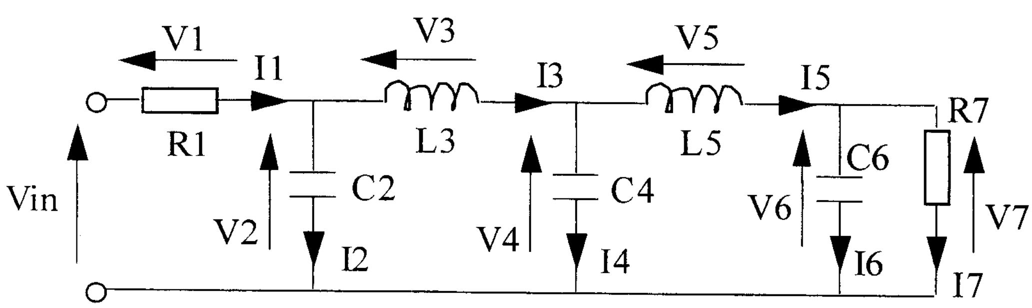

Figure 6.2

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1.

(a) **(theory)**

Local oscillator :

$$LOA = 2\cos(\omega_L t - \text{ø}1) \qquad LOB = 2\cos\omega_L t$$

RF signal :

$$A\cos(\omega_A t + \text{ø}_A) + B\cos(\omega_B t + \text{ø}_B)$$

where $\omega_A = (\omega_L - \omega_{IF})$ is the wanted signal, and $\omega_B = (\omega_L + \omega_{IF})$ is the image.

After mixing { Recall $2\cos X\cos Y = \cos(X-Y) + \cos(X+Y)$ and $\cos(-X)=\cos(X)$ }

- $IF1 = 2A\cos(\omega_A t + \text{ø}_A)\cos(\omega_L t - \text{ø}1) + 2B\cos(\omega_B t + \text{ø}_B)\cos(\omega_L t - \text{ø}1)$

$$= A\cos((\omega_L - \omega_A)t - \text{ø}1 - \text{ø}_A) + A\cos((\omega_L + \omega_A)t - \text{ø}1 + \text{ø}_A) + B\cos((\omega_B - \omega_L)t + \text{ø}_B + \text{ø}1) + B\cos((\omega_B + \omega_L)t + \text{ø}_B - \text{ø}1)$$

- $IF2 = 2A\cos(\omega_A t + \text{ø}_A)\cos\omega_L t + 2B\cos(\omega_B t + \text{ø}_B)\cos\omega_L t$

$$= A\cos((\omega_L - \omega_A)t - \text{ø}_A) + A\cos((\omega_L + \omega_A)t + \text{ø}_A) + B\cos((\omega_B - \omega_L)t + \text{ø}_B) + B\cos((\omega_B + \omega_L)t + \text{ø}_B)$$

Lowpass filter removes sum components:

- $IF1 = A\cos((\omega_L - \omega_A)t - \text{ø}_A - \text{ø}1) + B\cos((\omega_B - \omega_L)t + \text{ø}_B + \text{ø}1)$

$$= A\cos(\omega_{IF} t - \text{ø}_A - \text{ø}1) + B\cos(\omega_{IF} t + \text{ø}_B + \text{ø}1)$$

- $IF2 = A\cos((\omega_L - \omega_A)t - \text{ø}_A) + B\cos((\omega_B - \omega_L)t + \text{ø}_B)$

$$= A\cos(\omega_{IF} t - \text{ø}_A) + B\cos(\omega_{IF} t + \text{ø}_B)$$

After phase shift – ø2:

- $IF1 = A\cos(\omega_{IF} t - \text{ø}_A - \text{ø}1 - \text{ø}2) + B\cos(\omega_{IF} t + \text{ø}_B + \text{ø}1 - \text{ø}2)$

- $IF2 = A\cos(\omega_{IF} t - \text{ø}_A) + B\cos(\omega_{IF} t + \text{ø}_B)$

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Adding signals IF1 and IF2 :

$$IFOut = A\cos(\omega_{IF}t - \varnothing_A - \varnothing 1 - \varnothing 2) + A\cos(\omega_{IF}t - \varnothing_A) + B\cos(\omega_{IF}t + \varnothing_B + \varnothing 1 - \varnothing 2) + B\cos(\omega_{IF}t + \varnothing_B)$$

$$= 2A\cos\left(\frac{\phi 1+\phi 2}{2}\right)\cos\left(\omega_{IF} - \phi_A - \frac{\phi 1+\phi 2}{2}\right) + 2B\cos\left(\frac{\phi 1-\phi 2}{2}\right)\cos\left(\omega_{IF} - \phi_B + \frac{\phi 1-\phi 2}{2}\right)$$

To avoid signal distortion, we require $\cos\frac{\varnothing 1+\varnothing 2}{2} = 1$ i.e. $\frac{\phi 1+\phi 2}{2} = 2n\pi$

To ensure image rejection, we require $\cos\frac{\varnothing 1-\varnothing 2}{2} = 0$ i.e. $\frac{\phi 1-\phi 2}{2} = \frac{(2n+1)\pi}{2}$

e.g. $\varnothing 1 = 90°$ $\varnothing 2 = -90°$

[10]

(b) **(bookwork)**

Because the image frequency will be downconverted/upconverted to the same frequency the frequency of interest is upconverted/downconverted.

[5]

(c) **(new computed example)**

The image frequency is 708MHz and the IF frequency is 48MHz.

[5]

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2. (new computed example)

Assuming matched transistors in saturation:

$$Vd = Vgs1 - Vgs2 = \frac{\sqrt{Id1} - \sqrt{Id2}}{\sqrt{\beta}} \qquad Is = Id1 + Id2$$

Solving these simultaneous equations gives a solution for Id1 and Id2:

$$Id1 = \frac{Is}{2} + \frac{Is}{2}\sqrt{\frac{2\beta Vd^2}{Is} - \frac{\beta^2 Vd^4}{Is^2}} \qquad Id2 = \frac{Is}{2} - \frac{Is}{2}\sqrt{\frac{2\beta Vd^2}{Is} - \frac{\beta^2 Vd^4}{Is^2}}$$

Iout = Id1 - Id2

$$= Is\sqrt{\frac{2\beta Vd^2}{Is} - \frac{\beta^2 Vd^4}{Is^2}} = Vd\ \sqrt{2\beta Is}\sqrt{1 - \frac{Vd^2\beta}{2Is}}$$

This gives for a value for the output current of 9.95 μA.

[10]

(b) **(new computed example)**

For a single transistor:

$$Id = Ido + \left(\frac{\partial Ic}{\partial Vin}\right)Vin + \left(\frac{\partial^2 Id}{\partial Vin^2}\right)\frac{Vin^2}{2} + \left(\frac{\partial^3 Id}{\partial Vin^3}\right)\frac{Vin^3}{6} + \ldots\ldots$$

where the derivatives are evaluated at the quiescent point Id = Ido

For the circuit above:

Iout = (Id1 – Id2) – (Id3 – Id4)

$$\text{Thus} \quad Iout = \frac{Vd}{2}\left(\left(\frac{\partial Id}{\partial Vd}\right)_1 + \left(\frac{\partial Id}{\partial Vd}\right)_2 - \left(\frac{\partial Id}{\partial Vd}\right)_3 - \left(\frac{\partial Id}{\partial Vd}\right)_4\right)$$

$$+ \frac{Vd}{48}\left(\left(\frac{\partial^3 Id}{\partial Vd_3}\right)_1 + \left(\frac{\partial^3 Id}{\partial Vd^3}\right)_2 - \left(\frac{\partial^3 Id}{\partial Vd^3}\right)_3 - \left(\frac{\partial^3 Id}{\partial Vd^3}\right)_4\right)$$

where $\left(\frac{\partial^n Id}{\partial Vd^n}\right)_m$ is the n^{th} derivative of the m^{th} transistor

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Since M1 and M2 are matched, and M3 and M4 are matched, then

$$\left(\frac{\partial^n Id}{\partial Vd^n}\right)_1 = \left(\frac{\partial^n Id}{\partial Vd^n}\right)_2 \text{ and } \left(\frac{\partial^n Id}{\partial Vd^n}\right)_3 = \left(\frac{\partial^n Id}{\partial Vd^n}\right)_4$$

To eliminate the third harmonic term, the transistors should be scaled appropriately so that $\left(\frac{\partial^3 Id}{\partial Vd^3}\right)_1 = \left(\frac{\partial^3 Id}{\partial Vd^3}\right)_3$ however we must ensure that $\left(\frac{\partial Id}{\partial Vd}\right)_1$ and $\left(\frac{\partial Id}{\partial Vd}\right)_3$ don't also cancel.

First order derivative $\frac{\partial Id}{\partial Vd} = \sqrt{\beta Is}$ Third order derivative $\frac{\partial^3 Id}{\partial Vd^3} = \sqrt{\beta^3 / 8Is}$

For cancellation of the third order coefficients:

$$(\beta 1/\beta 2)^{3/2} = \sqrt{(Is1/Is2)} \quad \text{thus} \quad \frac{(W1/L1)^3}{(W2/L2)^3} = \frac{Is1}{Is2}$$

Hence, the value of Is2 should be 400μA.

The total transconductance will then be:

$$gm = \left(\frac{\partial Id}{\partial Vd}\right)_1 - \left(\frac{\partial Id}{\partial Vd}\right)_2 = \sqrt{Is/\beta 1} - \sqrt{Is/\beta 2}$$

gm=-0.7mS

[10]

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3. (a) **(bookwork)**

In a MOSFET, there are three main sources of noise:

(i) *Thermal noise* due to the resistance of the channel:

$$ind^2 = \frac{8kTgm\Delta f}{3} \; A^2$$

This noise source can also be represented by an equivalent channel resistance rd = 3/2gm.

(ii) *Flicker (1/f) noise* in series with the gate:

$$vng^2 = \frac{k_f \Delta f}{CoxWLf} \; V^2$$

k_f is a flicker noise coefficient which is process dependent. Note that the 1/f noise is inversely proportional to gate area, thus bigger devices are less noisy.

(iii) *Shot noise* due to the gate-source leakage current:

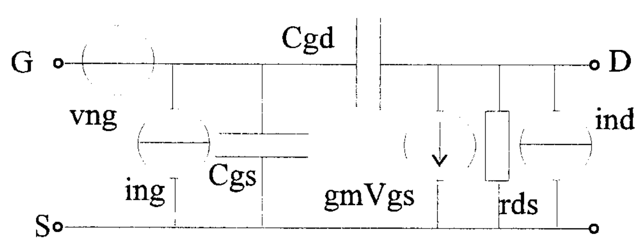
$ing^2 = 2qIg\Delta f \; A^2$ (often neglected since negligible)

The shot noise is negligible because in modern processes the current at the gate is 0.

[6]

(b) **(bookwork)**

Equivalent noise model:



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$$vn^2 = \frac{ind^2}{gm^2} + vng^2 = \frac{8kT}{3gm} + \frac{k_f}{WLCox\,f} \; V^2$$

$$in^2 = ing^2 = 2qIg\Delta f \; A^2$$

[7]

(c) **(new computed example)**

The only important noise is thermal. The flicker noise is negligible for frequencies in the range of MHz. Hence:

$$Noise = \int_{10^6}^{(20\cdot 10^6)} \frac{8kT}{3g_m}\left(1+\frac{f^2}{f_c^2}\right)df = \frac{8kT}{3g_m}\left[19\cdot 10^6 + \frac{8\cdot 10^{21}}{3f_c^2}\right]$$

[7]

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4.

(a) **(bookwork)**

It's a linearised transconductor

Assuming all devices have equal ß:

$$Id1 - Id2 = \beta\, Vd(Vc1 - Vth) \qquad Id3 - Id4 = \beta Vd\,(Vc2 - Vth)$$

where $Vc1 = \dfrac{Vgs1 + Vgs2}{2}$ and $Vc2 = \dfrac{Vgs3 + Vgs4}{2}$

Thus $Iout = (Id1 - Id2) - (Id3 - Id4)$

$$= \beta\, Vd\,(Vc1 - Vc2) = \beta\, Vd\,(Vb - Vss)$$

[10]

(b) **(new theory)**

$$i_C = \frac{i_{out}}{2} \qquad C_C \frac{dV_{out}}{dt} = \frac{i_{out}}{2}$$

[5]

(c) **(new theory and computed example)**

$$V_{out} = \frac{1}{C_C}\int_{t_1}^{t_2} \frac{\beta}{2} \cdot (Vb - Vss) \cdot Vd \cdot dt$$

Solving that equation for the first 10µs:

$$V_{out} = 5 \cdot 10^5 \cdot t$$

This gives a maximum value of 5V. Hence, between 10µs and 20µs:

$V_{out} = 5 - 5 \cdot 10^5 \cdot (t - 10^{-5})$ and the minimum value for t=20µs will be 0.

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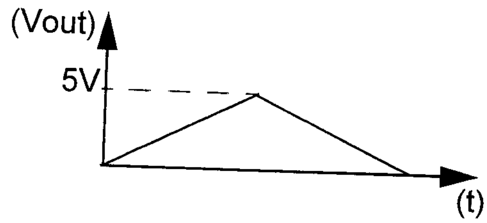

[5]

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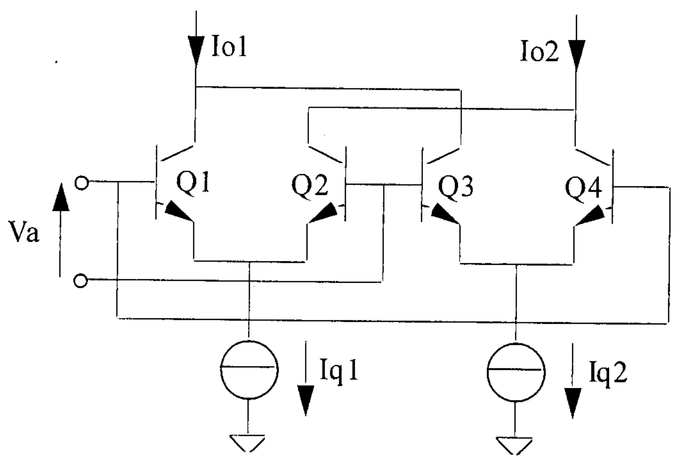
5.

(a) **(theory)**

For the sake of clarity, let us draw the Gilbert multiplier as follows:



The analysis of that circuit yields,

Io1 – Io2 = (Ic1 – Ic2) + (Ic3 – Ic4)

= Iq1 tanh (Va/2Vt) + Iq2 tanh (-Va/2Vt)

= (Iq1 – Iq2) tanh (Va/2Vt)

= (Iq1 – Iq2) (Va/2Vt) (if Va < 5 mV)

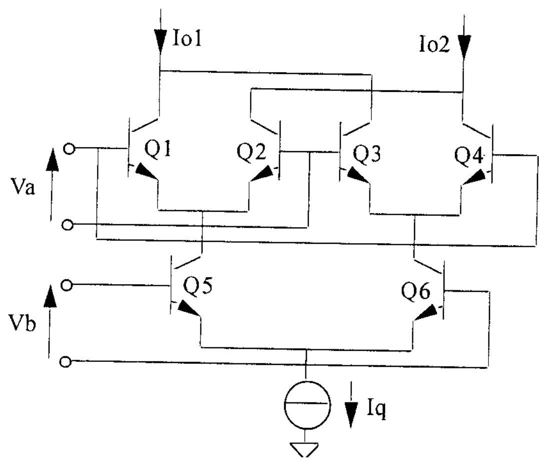
If the lower current sources are driven differentially with a small signal input Vb:

Iq1 = Iq + gmVb and Iq2 = Iq – gmVb

then

Iout = Io1 – Io2 = 2gmVb (Va/2Vt) = VaVb (gm/Vt)

To generate the balanced currents Iq1 and Iq2 :



Iq1 = Ic5 = (1 + X)Iq/2 Iq2 = Ic6 = (1 – X)Iq/2

where X = tanh (Vb/2Vt)

For small input signals (Vb << Vt) then $X \cong Vb/2Vt$:

$$Iq1 = \frac{Iq + gmVb}{2} \qquad Iq2 = \frac{Iq - gmVb}{2} \qquad \text{where } gm = \frac{Iq}{2Vt}$$

Thus $Iout \cong \frac{Iq}{4Vt^2} VaVb$

[10]

(b) **(bookwork)**

The double-balanced multiplier gives linear multiplication for small input signal levels only. The linear input dynamic range of the second input Vb applied to the bottom transistors (Q5, Q6) can be extended by using emitter degeneration (linear compression). However we do not want to use emitter degeneration in the top transistors (Q1-Q4) because the gain of these devices will no longer be dependent on the tail current (i.e. we will have lost the ability to multiply!).

The dynamic range of Va can be extended by pre-distorting the input signal :

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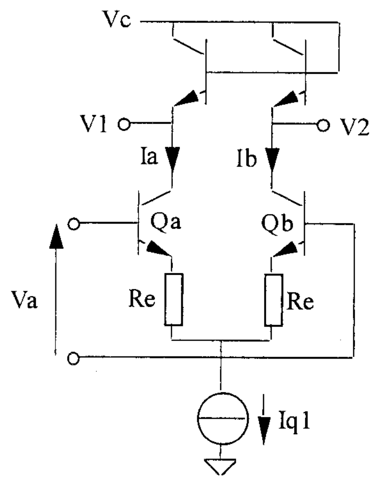
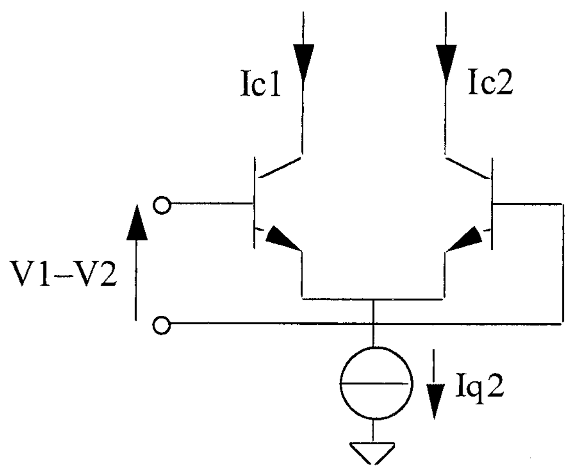


The differential voltage (V1 – V2) is taken as the input to the (mixing) differential pair



$V1 - V2 = Vt \ln (Ic1/Is) - Vt \ln (Ic2/Is) = Vt \ln (Ic1/Ic2)$

But $V1 - V2 = Vt \ln(Ib/Ia)$

Thus $\frac{Ic1}{Ic2} = \frac{Ib}{Ia} = \frac{(1-X)}{(1+X)}$

$$Ic1 = (1-X)\frac{Iq2}{2}, \quad Ic2 = (1+X)\frac{Iq2}{2}$$

where $X = \frac{Va}{Iq1\ Re}$

$$Iout = (Ic2 - Ic1) = \frac{Iq2\ Va}{Iq1\ Re}$$

The linear range is extended by using the degeneration resistor Re. Multiplication can be achieved by using a second input to control the tail current Iq2. Again, a balanced arrangement should be used to eliminate the amplified term, and leave the multiplication term.

[10]

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Paper Code: E4.17	Second Examiner:	David Haigh

6.

(a) **bookwork**

$$\frac{Vout(s)}{Vin(s)} = \frac{k_2 s^2 + k_1 s + k_0}{s^2 + s/(\tau_2 Q) + 1/(\tau_1 \tau_2)}$$

[4]

(b) **(new computed example)**

$Q=2$, $\tau_2=1$, $k_1=0.5$, $k_2=0$, $k_0=0$, $\tau_1=1$

[3]

(c) **(new computed example)**

$Q=20$, $\tau_1=1$, $\tau_2=1$, $k_2=1$, $k_1=0$, $k_0=0$

[3]

(d) **(bookwork)**

V1 = Vin – V2
I1 = V1/R1

I2 = I1 – I3
V2 = I2/sC2

V3 = V2 – V4
I3 = V3/sL3

I4 = I3 – I5
V4 = I4/sC4

V5 = V4 – V6
I5 = V5/sL5

I6 = I5 – I7
V6 = I6/sC6

V7 = V6
I7 = V7/R7

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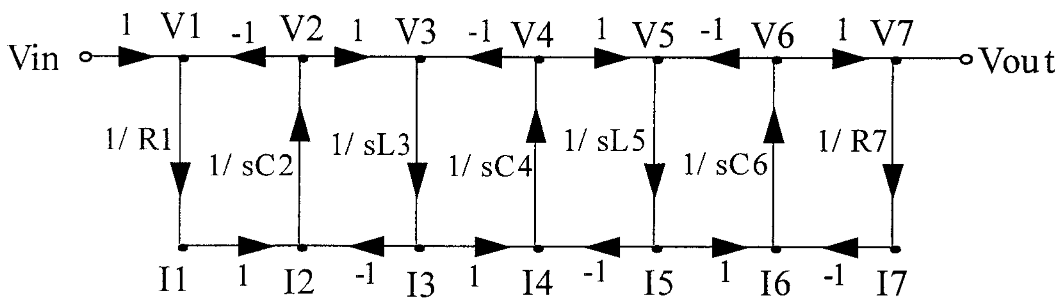
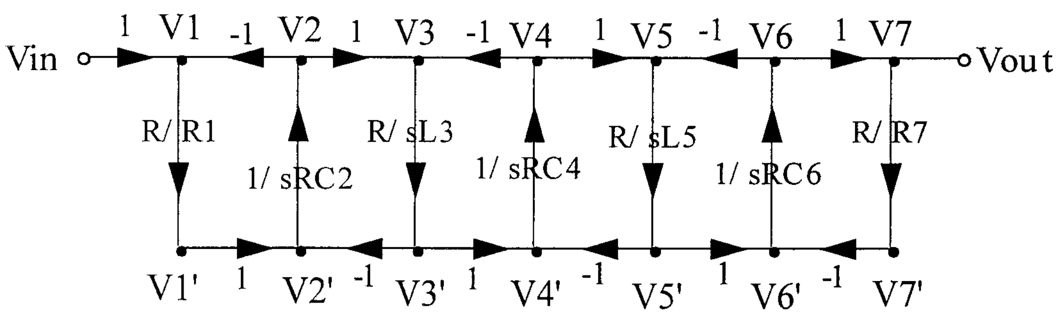
Signal flow graph:

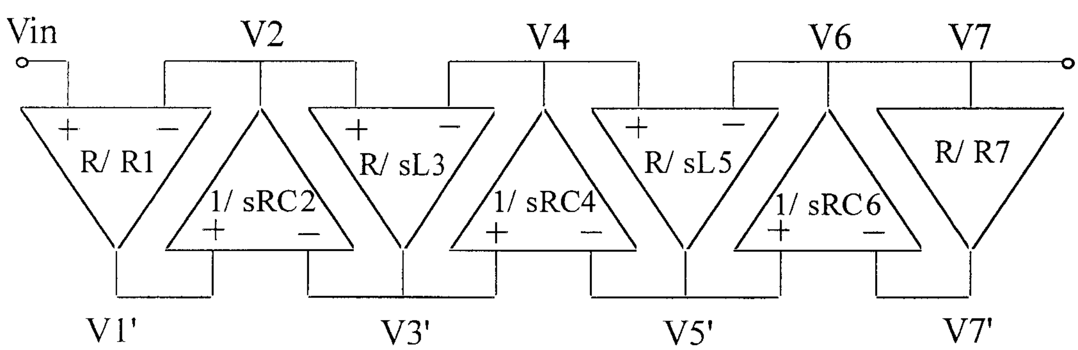


Scaled signal flow graph:



[5]

(e) **(bookwork)**



[5]