

IMPERIAL COLLEGE LONDON

MSc and EEE/ISE PART IV: M.Eng. and ACGI

Time allowed: 3:00 hours

Answer Question ONE, and ANY THREE of Questions 2 to 6
All questions carry equal marks.

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible

First Marker(s) :	E.M. Yeatman
Second Marker(s) :	K.D. Leaver

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge : $e = 1.6 \times 10^{-19} \text{ C}$

permittivity of free space : $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

relative permittivity of silicon : $\epsilon_r = 12$

Planck's constant : $h = 6.63 \times 10^{-34} \text{ J s}$

Boltzmann's constant : $k = 1.38 \times 10^{-23} \text{ J/K}$

speed of light : $c = 3.0 \times 10^8 \text{ m/s}$

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x} \tan(k_{1x}d/2) \text{ and } \kappa = -k_{1x} \cot(k_{1x}d/2)$$

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]
- a) A step index silica fibre has a core diameter of $6\text{ }\mu\text{m}$ and the cutoff wavelength for single mode operation is $1.520\text{ }\mu\text{m}$. Estimate the index difference of the fibre.
 - b) Raising the refractive index difference has what effect (if any) on the number of supported modes in step index multi-mode fibre?
 - c) What loss mechanism dominates in silica-based fibre at wavelengths in the range $1600 - 2000\text{ nm}$?
 - d) Estimate the spread in wavelengths, $\Delta\lambda$, emitted by a light emitting diode at room temperature with a nominal operating wavelength of 1310 nm .
 - e) Why is silicon not a suitable material for photodetectors for long-haul optical communication systems?
 - f) Which of the following is currently the highest bit rate in commercial optical transmitters – 10 Mbit/s , 400 Mbit/s , 1 Gbit/s , or 40 Gbit/s ? What is the main factor limiting this value?
 - g) Briefly explain the main performance advantage for optical communications of distributed feedback lasers over Fabry-Perot lasers.
 - h) What is the maximum possible slope efficiency of a semiconductor laser operating at a nominal wavelength of 780 nm ?
 - i) Briefly describe the main considerations in choosing suitable wavelengths for pump lasers in erbium-doped fibre amplifiers.
 - j) A passive 4-port coupler with negligible excess losses has an input of -10 dBm at one port, no signal at the other input port, and -20 dBm is emitted at one output port. What is the output power in dBm at the other output port and why?

2. a) A symmetric slab waveguide has a core thickness of $6 \mu\text{m}$. For a certain transverse electric (TE) mode, the transverse wavevector component in the core is $k_{1x} = 1.19 \times 10^6 \text{ m}^{-1}$. Find the order m of this mode, and determine the total number of TE modes supported by this guide. You may find it helpful to sketch a graph of the eigenvalue equations. [8]
- b) For the waveguide and mode of part (a), with a free-space wavelength of $1.50 \mu\text{m}$, the phase velocity is found to be $2.027 \times 10^8 \text{ m/s}$. Find the refractive index of the core, n_c , to 4 significant decimal places. [6]
- c) For the waveguide and wavelength described in (a) and (b), estimate the effective index n' , to 4 significant digits, of the lowest order ($m=0$) TE mode. [6]
3. a) Show that, at the wavelength λ where $d^2n/d\lambda^2 = 0$, the spread in pulse length due to material dispersion in a fibre is proportional to the third derivative of refractive index with respect to wavelength, i.e. $d^3n/d\lambda^3$. Derive an expression for the pulse broadening $\Delta\tau_g$ in this case. Note that the group delay $\tau_g = (L/c)(n - \lambda dn/d\lambda)$, where L is the fibre length. [6]
- b) If the spread in wavelength of a signal is dominated by the inherent source width $\sigma_{\lambda S}$, derive an expression for the received pulse width in the time domain σ_R as a function of the transmitted pulse width σ_o , the fibre length L , and the dispersion coefficient D . Derive an expression for the minimum of σ_o as a function of $\sigma_{\lambda S}$ for which the relative pulse spreading $\sigma_R/\sigma_o < 1.25$. [6]
- c) In a certain system the optical signal's spectral width is dominated by the input signal spectrum, and the inherent source width $\sigma_{\lambda S}$ is negligible. The input pulses are transform limited Gaussian, with variance σ_o . Show that for a certain fibre dispersion $D \cdot L$ there is a minimum received pulse width σ_R . Find an expression for the input pulse width σ_o at which this is obtained, and for the corresponding pulse spreading ratio σ_R/σ_o . [8]

4. a) A certain light emitting diode (LED) has radiative and non-radiative recombination lifetimes, τ_r and τ_{nr} respectively, in the ratio $\tau_{nr}/\tau_r = 5$. Calculate the internal quantum efficiency for this device. [2]
- b) Describe four factors that may limit the external quantum efficiency in an LED, and for each of these indicate a technique that can reduce the effect of this factor. [8]
- c) An LED is emitting into a step index, silica-based multi-mode fibre as shown in Fig. 4.1. The numerical aperture of the fibre is 0.10. Find the maximum emission angle θ from the LED active region for which light can be guided in the fibre, and the fraction of emitted power falling within this angle if the generated photons have a uniform directional distribution. [6]
- d) For the fibre described in (c), derive an expression for the limit on bit rate imposed by multi-mode dispersion, and sketch this as a function of fibre length. [4]

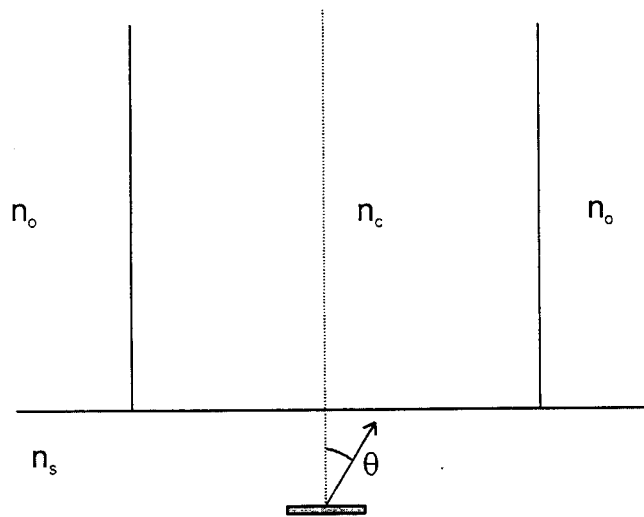


Figure 4.1

5. a) A certain receiver has unit quantum efficiency, and a noise equivalent power (NEP) of $5 \text{ pW}/\sqrt{\text{Hz}}$. Find the received optical power for which the shot noise and receiver noise are equal. [4]
- b) The receiver in (a) is used in a system with transmitted power of 10 mW, and fibre attenuation of 0.43 dB/km. On-off keying is used, the signal bandwidth is at the Nyquist limit, and the minimum optical SNR is 10 dB.
- Find an expression for the maximum bit rate B as a function of fibre length L , and simplify this expression for the two cases where shot noise or receiver noise dominate respectively. Using these approximations, sketch $\log(B)$ vs. L over a practical range of fibre lengths. [8]
- c) The system described in part (a) is extended by adding an optical amplifier at a suitable point along the fibre. The amplifier has 20 dB gain and a noise figures of 4 dB. Modify your expressions from (b) for maximum bit rate, and sketch $\log(B)$ vs. L . [8]

6. a) A silicon p-n photodiode (Fig. 6.1) has a depletion layer thickness of w , and p and n doping levels respectively of N_A and N_D , with $N_A = 5N_D = 10^{20} \text{ m}^{-3}$. The quantities w_p and w_n are the depleted widths in the p and n regions respectively. A reverse bias voltage V_b is applied. Find an expression for the full depletion width w as a function of V_b , and the value of V_b for which $w = 10 \text{ } \mu\text{m}$. [8]
- b) Neglecting Fresnel reflection, find an expression for the quantum efficiency η of the photodiode of (a) if the total p region thickness $h = 10 \text{ } \mu\text{m}$, and the absorption coefficient $\alpha = 10^5 \text{ m}^{-1}$. Hence find the value of V_b for which $\eta = 0.9$. [6]
- c) Assume that the resistance R encountered by the photocurrent I_{ph} is dominated by the resistance of the undepleted part of the p region, as the current flows laterally through it as shown in Fig. 6.1. If this resistance is in turn is inversely proportional to the thickness h_c of this region, find the value of h_c in terms of h for which the time constant RC of the diode is minimised. [6]

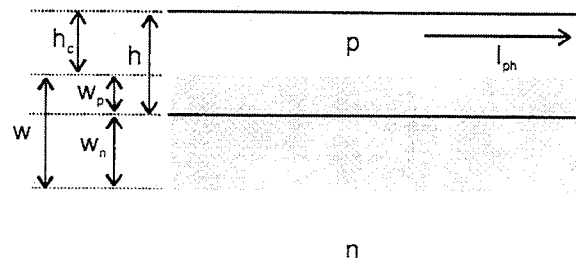


Figure 6.1

