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IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERIN~
EXAMINATIONS 2005

MSc and EEE/ISE PART IV: MEng and ACGI

Corrected Copy
• Eqn (5.1)
• 4(d)
• 1(h)

OPTICAL COMMUNICATION

Wednesday, 18 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions 2 to 6

All questions carry equal marks.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : E.M. Yeatman
Second Marker(s) : A.S. Holmes

Special instructions for invigilators: None.

Information for Candidates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3.0 \times 10^{8}$ m/s

The eigenvalue equations for TE modes in a symmetric slab waveguide of thickness d are

$$\kappa = k_{1x}\tan(k_{1x}d/2) \quad \text{and} \quad \kappa = -\, k_{1x}\cot(k_{1x}d/2)$$

1 You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, but you should show any calculations you use to arrive at your answers, and give a brief (one or two lines) explanation where appropriate. All parts have equal value. [20]

a) If a certain optical communication link has an optical SNR of 8 dB, what is the equivalent electrical SNR for the link?

b) A p-i-n photodiode detects a constant optical flux of 10^{13} photons/sec at a wavelength of 1.3 μm. Assuming the quantum efficiency $\eta = 1$, calculate the photocurrent.

c) What is the maximum possible phase velocity of a guided wave supported by a step index fibre of core and cladding indices 1.52 and 1.50 respectively?

d) A cavity is formed by two flat parallel mirrors in air, 10.0 μm apart. Find the wavelength closest to 1.50 μm for which this cavity is resonant.

e) A p-i-n photodiode has p^+ and intrinsic layer thicknesses of 2.0 and 8.0 μm respectively, and an attenuation coefficient of 2.5×10^5 m^{-1} at the operating wavelength. Neglecting surface reflection, calculate the fraction of incident photons which is absorbed in the intrinsic layer.

f) Arrange the following sources in order of typical spectral width, from broadest to narrowest: Fabry-Perot laser diode, incandescent light bulb, distributed feedback laser, light emitting diode.

g) A glass step index fibre has an index difference of 0.007. Estimate its numerical aperture.

h) A semiconductor laser has a slope efficiency of 1.2 ~~A/W~~ W/A. Calculate the ~~minimum~~ maximum nominal output wavelength.

i) A certain erbium doped fibre amplifier has an effective bandwidth of 15 nm. Estimate the equivalent bandwidth in GHz.

j) Briefly explain how a heterostructure can be used to increase the quantum efficiency in a light emitting diode.

2. A symmetric slab waveguide as shown in Fig. 2.1 has a core thickness d = 16 μm, and core and cladding indices of n_1 = 1.47 and n_2 = 1.46 respectively. For a free-space wavelength of 1.50 μm, a certain mode of the waveguide has an effective index of n' = 1.46526.

a) State the boundary conditions that must be satisfied at the core-cladding interfaces for TE modes, with respect to the electric field distribution E(x). Show that the effective index given above is consistent with these conditions. [10]

b) Sketch the field profile E(x) for the mode having this value of n', and state its mode number m. [5]

c) Determine the total number of TE modes supported by the guide at this wavelength. [5]

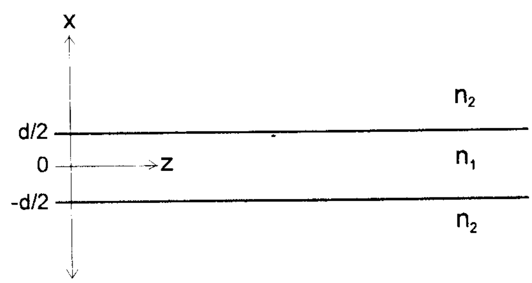


Figure 2.1

3. A certain Fabry-Perot laser diode has a cavity length of L = 350 μm, and a nominal operating wavelength of 850 nm.

a) Assuming a semiconductor refractive index of 3.5, estimate the spacing, in GHz, of the longitudinal modes. [5]

b) If the end facets are simply flat semiconductor-air interfaces, and the attenuation coefficient for the lasing modes in the laser is approximately 2.5×10^3 m^{-1} at the operating wavelength, estimate the external quantum efficiency. [5]

c) Two adjacent longitudinal modes of this laser are propagated in a fibre, and are found to have a phase velocity difference of 450 m/s. Calculate the dispersion coefficient of the fibre for the laser wavelength. [5]

d) When diode lasers are modulated with high bit-rate on-off keying signals, there is often some unwanted finite output power for the zero bits. Explain why this should be so, and what unwanted effect this has on the system performance. [5]

4. A laser of peak output power 6 dBm, with nominal wavelength $\lambda_o = 1530$ nm and spectral width $\Delta\lambda = 2$ nm, emits into a fibre with an attenuation of 0.25 dB/km and dispersion coefficient 8 ps/nm·km. It is modulated with an on-off keying format at a bit rate B. The receiver has quantum efficiency $\eta = 0.8$ and input resistance $R = 10$ kΩ.

a) Assume that the SNR is dominated by thermal noise in the receiver. Hence, derive an expression for the maximum bit rate as a function of fibre length, for an optical SNR of 12. Neglect dispersion, and state any other approximations or assumptions made. [5]

b) Assume that the SNR is dominated by shot noise in the receiver. Hence, derive an expression for the maximum bit rate as a function of fibre length, for an optical SNR of 12. Neglect dispersion, and state any other approximations or assumptions made. [5]

c) Derive an expression for the bit rate at which the dispersion time is equal to one quarter of the bit time 1/B, as a function of fibre length. [5]

d) On a single graph of bit rate vs. ~~time~~ length, sketch the three relations derived in (a), (b) and (c) above, using appropriate scales and ranges for your two axes. Briefly discuss the significance of this graph for the operation of this optical link. [5]

5. a) How are group and phase velocity each defined for a propagating electromagnetic wave? What is the main physical significance of group velocity? [6]

b) In the wavelength range of interest, the refractive index of a sample of silica glass can be approximated by

$$n(\lambda_o) \cong D_o + D_1 \lambda_o^{-2} - D_2 \lambda_o^{2} \qquad (5.1)$$

where $D_o = 1.45$, $D_1 = 0.003\ \mu m^2$ and $D_2 = 0.0032\ \mu m^{-2}$. Find expressions for the phase and group velocities v_p and v_g in this material, as functions of D_o, D_1, D_2, and λ_o and c. Hence show that v_g is always $< v_p$ in this case. [8]

c) For the glass described above, find the wavelength of zero material dispersion. [6]

6. a) A silicon p-i-n photodiode has intrinsic layer thickness $w_i = 8$ μm, and p and n doping levels respectively of $N_A^+ = 2\times10^{21}$ m^{-3} and $N_D^+ = 10^{21}$ m^{-3}. The intrinsic layer doping level is labelled as N_D^-. The electron and hole velocities can be approximated as linearly proportional to applied field, with mobilities (drift velocity per unit electric field) of 0.125 m^2/Vs and 0.05 m^2/Vs respectively, up to a saturation velocity of 10^5 m/s (for both electrons and holes). Find the applied electric field amplitude at which the electrons reach their saturation velocity. Hence, find the value of N_D^- such that the electron velocity reaches the saturation value at the p-i junction, and drops 10% below this value across the intrinsic region. Calculate also the corresponding applied voltage needed to reach this condition. [10]

b) For the structure and conditions of part (a), find the maximum time for a carrier pair to be swept out of the depletion region for a photon absorbed in the intrinsic region (you may neglect the propagation in the depleted parts of the n$^+$ and p$^+$ regions). You may find the following integral useful:

$$\int \frac{dx}{C + ax} = \frac{1}{a}\ln(C + ax)$$

[5]

c) Explain why the photodiode is a superior detector to the simple photoconductor for optical communications. [5]

Optical Comms 2005 Solutions

1

a) Since $P_e \propto P_o^2$, $SNR_e = 2 SNR_o = \underline{16\,dB}$

b) For $\eta = 1$ detector produces 1 electron per photon, so $I_{ph} = \dot{N}e = 10^{13}_{s^{-1}} \times 1.6\times10^{-19}\,C$
$I_{ph} = 1.6\,\mu A$. Wavelength is irrelevant.

c) $v_p = c/n'$, minimum $n' = 1.50$, $\therefore$ minimum
$v_p = 3\times10^8/1.50 = \underline{2.0\times10^8\,m/s}$

d) Cavity resonates for $L = m\lambda/2$ with m an integer. For $\lambda = 1.5$, $m = 2(10)/1.5 = 13.33$, closest integer m is 13, giving $\lambda = 2(10)/13 = 1.538\,\mu m$

e) $\eta = e^{-w_1\alpha} - e^{-(w_1+w_2)\alpha} = e^{-2\times.25} - e^{-10\times.25} = \underline{0.524}$

f) incandescent bulb, LED, F-P laser, DFB laser

g) $NA = \sqrt{n_1^2 - n_2^2} \cong \sqrt{2n\cdot\Delta n}$ Taking $n \cong 1.5$,
$NA \cong \sqrt{3\Delta n} = \sqrt{3\times.007} = 0.145$

h) $S = 1.2\,A/W = \eta e\lambda/hc$. Minimum λ is at $\eta = 1$, $\lambda = hcS/e = \dfrac{6.63\times10^{-34}\times3\times10^8\times1.2}{1.6\times10^{-19}} = 1.40\,\mu m$

i) The EDFA band is around 1530 nm.
Using $\Delta\lambda/\lambda \cong \Delta f/f$, and $f = c/\lambda$,
$\Delta f \cong c\cdot\Delta\lambda/\lambda^2 = 3\times10^8\times15\times10^{-9}/(1530\times10^{-9})^2 = 1922\,GHz$

j) If the p^+ layer is grown with a composition of higher bandgap, it can be made transparent and so absorption loss is reduced.

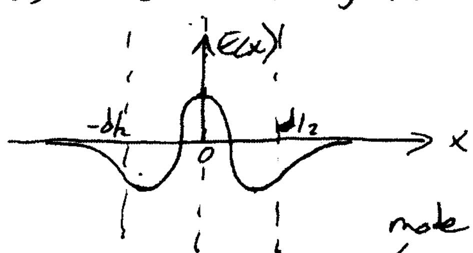
2

a) Boundary conditions on $E(x)$ are that $E(x)$ and $dE(x)/dx$ are continuous.
We can check one boundary ($d/2$) only.
For $x > d/2$, $E_2(x) = C\exp(-Kx)$
For $|x| \le d/2$, $E_1(x) = A\cos(k_{1x}x)$ even modes
or $A\sin(k_{1x}x)$ odd modes.

For even modes, continuity of $E(x)$ requires:
$$A\cos(k_{1x}d/2) = C\exp(-Kd/2) \quad (i)$$
and of dE/dx requires:
$$-k_{1x}A\sin(k_{1x}d/2) = -KC\exp(-Kd/2) \quad (ii)$$
We are free to choose A and C to satisfy the first condition, so we can insert this into (ii) to get:
$$K = k_{1x}\tan(k_{1x}d/2)$$

Now, $k_{1x}^2 + (n'k_0)^2 = (n_1k_0)^2$
$\therefore\ k_{1x} = k_0\sqrt{n_1^2 - n'^2} = (2\pi/\lambda_0)\sqrt{1.47^2 - 1.46526^2}$
$= 0.49398\ \mu m^{-1}$
and $k_{1x}d/2 = 3.95184$
and $K = k_0\sqrt{n'^2 - n_2^2} = (2\pi/1.5)\sqrt{1.46526^2 - 1.46^2}$
$= 0.51969\ \mu m^{-1}$
$k_{1x}\tan(k_{1x}d/2) = 0.49398\tan(3.95184) = 0.5192$
Agrees! (close enough).

b)



Since $k_{1x}d/2 \gtrsim \pi$, Each side is just over a half cycle.
mode number $m = 2$
(= no. of zero crossings)

c) modes are supported if:
$$d \ge \frac{1}{2}\frac{m\lambda}{NA}$$
so $m_{max} = \mathrm{int}[2 \cdot NA \cdot d/\lambda]$
$= \mathrm{int}[2\sqrt{1.47^2 - 1.46^2} \cdot 16/1.50]$
$= 3$
Number of modes is $\underline{\underline{4}}$, ie $m = 0, 1, 2, 3$.

③ a) Long. modes have $L = m\lambda_o/2n$

$$m\lambda_m = (m+1)\lambda_{m+1} = 2nL$$

$$\Delta\lambda = \lambda_m - \lambda_{m+1} = \frac{2nL}{m} - \frac{2nL}{m+1}$$

At the nominal wavelength, $m = \frac{2(3.5)350}{0.85} = 2882$

$$\Delta\lambda = 2(3.5)(350)\left(\frac{1}{2882} - \frac{1}{2883}\right) = 2.95\times10^{-4}\ \mu m$$
$$= 0.295\ nm$$

$\Delta f = c\cdot\Delta\lambda/\lambda^2$ (see ① i))

$$= \frac{3\times10^8 \times 0.295\times10^{-9}}{(0.85\times10^{-6})^2} = 1.22\times10^{11} = \underline{122\ GHz}.$$

b) $R_1 = R_2 = \left(\frac{3.5-1}{3.5+1}\right)^2 = 0.309$

$$\eta = \frac{\ln(1/R_1R_2)}{\ln(1/R_1R_2) + 2\alpha L}$$ ← by memory, or derive using fraction of photons escaping through mirrors.

$$= \frac{\ln(1/.309^2)}{\ln(1/.309^2) + 2(2.5\times10^3\times350\times10^{-6})}$$

$$= \underline{0.573}$$

c) $\Delta\tau = D\cdot L\cdot\Delta\lambda = \left|\Delta\left(\frac{L}{v}\right)\right|$

$$\Delta\left(\frac{1}{v}\right) = -\Delta v/v^2$$

$$D\cdot \cancel{L}\cdot\Delta\lambda = \cancel{L}\,\Delta v/v^2$$

$$v = \frac{c}{n} \cong \frac{c}{1.5} \qquad D \cong \frac{\Delta v\,(1.5)^2}{\Delta\lambda\, c^2}$$

$$= \frac{450\,(1.5)^2}{0.295\times10^{-9}\,(3\times10^{8})^2} \doteq 3.8\times10^{-5}\ \frac{s}{m^2}$$

$$= 38\ \text{ps/nm}\cdot\text{km}$$

d) When photocurrent is switched off, output power is rapidly extinguished by stimulated emission on the order of the laser round-trip time. However, once the photon density drops to a low level this mechanism ceases to dominate, and the final carriers recombine by the much slower spontaneous recombination. This keeps a low intensity output during the off bits if B is high enough. The result is a reduced signal Q and thus a worse BER.

④

a) For thermal noise only, $(I^*)^2 = \frac{4kT}{R}$

$$SNR_o = \frac{I_{ph}}{\sqrt{(I^*)^2 \Delta f}} \qquad I_{ph} = \mathcal{R}\Phi_R$$

$\Delta f \cong B/2$ (assume)

$$12 = \frac{\Phi_T \exp(-\alpha L)\mathcal{R}}{\sqrt{2kTB/R}}$$

$$\mathcal{R} = \eta e\lambda/hc = \frac{0.8(1.6\times10^{-19})(1.53\times10^{-6})}{6.63\times10^{-34}\times 3\times10^{8}} = 0.985$$

$$\frac{2kTB}{R} = \frac{\Phi_T^2 \exp(-2\alpha L)\mathcal{R}^2}{144}$$

$$B = \frac{I_T^2 \mathcal{R}^2 R \exp(-2\alpha L)}{288\, kT}$$

$\Phi_T = 6\text{ dBm} = 4\text{ mW}$ Assume $T \cong 300\text{K}$

$$B = \frac{(4\times10^{-3})^2(10^4)(0.985)^2}{288(1.38\times10^{-23})(300)}\exp\left(-\frac{0.25}{4.34}L\right)$$

$$= 1.3\times10^{17}\exp(-0.058L)$$

b) For shot noise, $(I^*)^2 = 2eI_{ph}$

$$SNR_o = \frac{I_{ph}}{\sqrt{2eI_{ph}\Delta f}} = \sqrt{\frac{I_{ph}}{eB}}$$

$$12^2 = \frac{4\times10^{-3}\times 0.985\exp(-.058L)}{1.6\times10^{-19}\; B}$$

$$B = 1.7\times10^{14}\exp(-0.29L)$$

c) Dispersion time $\tau_D = D\cdot L\cdot\Delta\lambda = 0.25/B$

$$B = \frac{0.25}{8\times10^{-12}\times 2}L^{-1} = 1.56\times10^{10}L^{-1}$$

5)

a) $v_p = \omega/k \qquad v_g = \frac{d\omega}{dk}$

v_g describes the speed that pulses propagate

b) $k = nk_0 = n\omega/c$

$v_g = \frac{1}{dk/d\omega} \qquad \frac{1}{v_g} = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}$

$\frac{dn}{d\omega} = \frac{dn}{d\lambda_0}\frac{d\lambda_0}{d\omega} \qquad \omega = k_0 c = \frac{2\pi c}{\lambda_0}$

$\frac{d\lambda_0}{d\omega} = \frac{-2\pi c}{\omega^2} = -\frac{\lambda_0}{\omega}$

$\frac{dn}{d\omega} = -\frac{\lambda_0}{\omega}\frac{dn}{d\lambda_0}$

$\frac{1}{v_g} = \frac{1}{c}(n - \lambda_0 dn/d\lambda_0)$

$n = D_0 + D_1\lambda_0^{-2} - D_2\lambda_0^2$

$\frac{dn}{d\lambda_0} = -2D_1\lambda_0^{-3} - 2D_2\lambda_0$

$\frac{1}{v_g} = \frac{1}{c}(n + \lambda_0(2D_1\lambda_0^{-3} + 2D_2\lambda_0))$

$= \frac{1}{c}(D_0 + D_1\lambda_0^{-2} - D_2\lambda_0^{-2} + 2D_1\lambda_0^{-2} + 2D_2\lambda_0^2)$

$v_g = \frac{c}{D_0 + 3D_1\lambda_0^{-2} + D_2\lambda_0^2}$

$v_p = \frac{c}{n} = \frac{c}{D_0 + D_1\lambda_0^{-2} - D_2\lambda_0^2}$

since $v_g = \frac{c}{n + 2D_1\lambda_0^{-2} + 2D_2\lambda_0^2}$

and the denominator is always $> n$, $v_g < v_p$

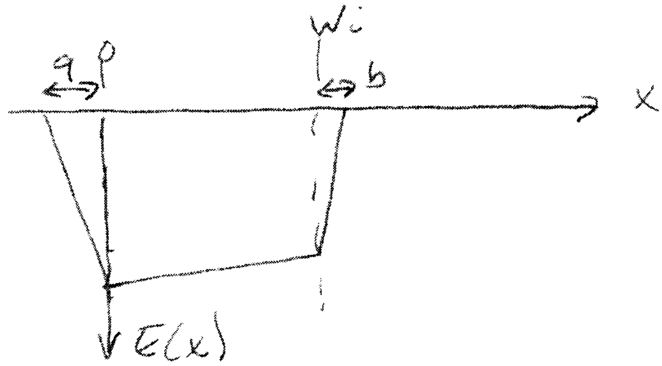
c) $dn/d\lambda = -2D_1\lambda_0^{-3} - 2D_2\lambda_0$

$\frac{d^2n}{d\lambda^2} = 6D_1\lambda_0^{-4} - 2D_2 = 0$ at zero dispersion

$3D_1/D_2 = \lambda_0^4 \qquad \lambda_0 = \sqrt[4]{3\times.003/.0032}$

$= 1.295$ mm

3 a)



$$E_{sat} = \frac{V_{dsat}}{\mu} = \frac{10^5}{0.125} = 8\times10^5\ V/m \quad \text{(electrons)}$$

$$= \frac{10^5}{.05} = 2\times10^6\ V/m \quad \text{(holes)}$$

We want

$E(x=a) = 8\times10^5$ V/m

$E(x=w_i) = 7.2\times10^5$ V/m

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon} \quad \therefore \rho = \varepsilon_0\varepsilon_r \Delta E/W_i = \frac{12\times8.85\times10^{-12}\times8\times10^4}{8\times10^{-6}}$$

(in intrinsic region) $= 1.06\ C/m^3$

But $\rho = e N_D^- \quad \therefore N_D^- = \frac{1.06}{1.6\times10^{-19}} = \underline{6.6\times10^{18}\ m^{-3}}$

To get voltage need a and b.

$$\frac{E_{max}}{a} = \frac{eN_A^+}{\varepsilon_r\varepsilon_0} \quad \therefore \quad a = \frac{8\times10^5}{2\times10^{21}} \Big/ \frac{1.6\times10^{-19}}{12\times8.85\times10^{-12}} = 0.27\ \mu m$$

$$b = \frac{\varepsilon_r\varepsilon_0\times0.9\,E_{max}}{eN_D^+} = \frac{12\times8.85\times10^{-12}\times0.9\times8\times10^5}{1.6\times10^{-19}\times10^{21}} = 0.48\ \mu m$$

$$V = -\int E\,dx = 0.95\,E_{max}W_i + \tfrac{1}{2}E_{max}\,a + \tfrac{1}{2}\times0.9E_{max}\,b$$

$$= 8\times10^5\left(0.95(8) + 0.5(.27) + .5\times0.9\times.48\right)\times10^{-6}$$

$$= \underline{6.36\ V}$$

b) Largest time will be for holes to return from bottom of intrinsic region.

$$E(x) = E_{max}(1 - x/80) \quad (x \text{ in } \mu m)$$

$$V_d = \mu_h E(x) = 0.05(8\times10^5)(1 - x/8)$$

$$= 4\times10^4(1 - x/80) = 5\times10^2(80-x) \ \mu m/s$$

$$= 5\times10^8(80-x) \ m/s$$

$$dt = dx/v(x) \quad \therefore \ \Delta t = +\int_8^0 \frac{dx}{v(x)}$$

$$\Delta t = \frac{-1}{5\times10^8}\int_0^8 \frac{dx}{80-x} = \frac{-1}{5\times10^8}\Big[\ln(80-x)\Big]_0^8$$

$$= \frac{\ln(80/72)}{5\times10^8} = 0.21 \ ns$$

c) In a photoconductor response speed can be traded off against responsivity in the design, using geometry, but the main disadvantage for communications is the large dark current