Paper Number(s): **E4.06**
AS3
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IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

MSc and EEE/ISE PART IV: M.Eng. and ACGI

OPTICAL COMMUNICATION

Monday, 14 May 10:00 am

There are SIX questions on this paper.

Answer Question ONE, and ANY THREE of Questions Two to Six.

All questions carry equal marks.

Time allowed: 3:00 hours

Corrected Copy

Examiners: Yeatman,E.M. and Leaver,K.D.

Special instructions for invigilators: None.

Information for Candidiates:

Numbers in brackets in the right margin (e.g. [5]) indicate maximum marks for each section of each question.

The following constants may be used:

electron charge :	$e = 1.6 \times 10^{-19}$ C
permittivity of free space :	$\varepsilon_o = 8.85 \times 10^{-12}$ F/m
relative permittivity of silicon :	$\varepsilon_r = 12$
Planck's constant :	$h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant :	$k = 1.38 \times 10^{-23}$ J/K
speed of light :	$c = 3 \times 10^8$ m/s

1. You should attempt all parts of this question. Short answers only are required; there is no need to re-state the questions in your answer book, or to give derivations or proofs, but you should show any calculations you use to arrive at your answers, and give a brief (one or two lines) explanation where appropriate. All parts have equal value.

a) Total internal reflection occurs when the angle of incidence (with respect to the normal) is greater than a critical angle θ_c. For TE polarisation, is θ_c greater than, equal to or less than that for TM polarisation?

b) A certain guided mode has a wave vector magnitude of 7×10^6 m^{-1} for a free space wavelength of 1.50 μm. Find the effective index of this mode.

c) In a dielectric waveguide, higher order modes have a higher, lower, or equal phase velocity compared to lower order modes?

d) For a certain optical signal intensity, would an avalanche photodiode be expected to produce more, less, or approximately the same photocurrent as a p-i-n photodiode?

e) A Fabry-Perot semiconductor laser diode with a free-space wavelength of 1.5 μm has a cavity length of 300 μm. Estimate the separation in nm between adjacent longitudinal modes. Assume a refractive index for the semiconductor of 3.5.

f) A laser diode with a free-space wavelength of 1550 nm is found to have a slope efficiency of 0.64 W/A. What is its quantum efficiency?

g) A laser diode with a free-space wavelength of 1.5 μm has an output power of 3 dBm. How many photons per second is this?

h) A longitudinal grating is required to act as a back-reflector in a silica fibre for signals of free-space wavelength 1330 nm. What should the grating period be?

i) An LED has a refractive index 3.6 and an output wavelength of 0.78 μm (in air). What thickness and index values would be suitable for an anti-reflection coating for this diode?

j) A passive four-port coupler has inputs labelled 1 and 3 and outputs labelled 2 and 4. If a signal entering at port 1 drops 10% of its power to port 4, then what fraction of power for a signal at the same wavelength entering at port 3 will be dropped to port 4? Ignore excess losses.

2\. a) For TE modes in symmetric slab waveguides, what are the boundary conditions on the electric field at the core-cladding interfaces? [4]

b) Using these boundary conditions, derive the eigenvalue equations for the TE modes. [8]

c) A certain symmetric slab waveguide has a core thickness $d = 2\lambda_o$, where λ_o is the free space wavelength. The lowest order TE mode in this guide has an electric field magnitude $E(x)$ such that $E(d/2) = E(0)/\sqrt{2}$, where $x = 0$ and $x = d/2$ are the centre and edge of the waveguide core respectively. If the core index $n_1 = 1.50$, find the effective index of this mode. [8]

3\. a) Explain what is meant by shot noise in optical detection. Give expressions for the spectral density of shot and thermal noise. [4]

b) For what value of photocurrent will the shot and thermal noise be of equal magnitude for an optical receiver with an effective input resistance of 100 kΩ? [2]

c) A laser diode of nominal wavelength 1550 nm transmits 10 mW into a fiber with propagation losses of 0.5 dB/km. The light at the other end of the fibre is detected by a receiver as in part (b) with quantum efficiency $\eta = 0.8$. How long will the fibre be for the case where shot and thermal noise are equal? [4]

d) For the case described in (c), what is the maximum bit rate at which the signal to noise ratio (in terms of the photocurrent) will be at least 12? Assume a receiver bandwidth of half the bit rate B, and ignore noise sources other than thermal and shot noise. [4]

e) At the bit rate determined in (d), Gaussian pulses are transmitted with an rms width σ_o equal to 0.2/B. If the pulses are transform limited, such that $\sigma_\omega \sigma_o = ½$, find the rms pulse width at the receiver if the dispersion coefficient D = 20 ps/nm·km. [6]

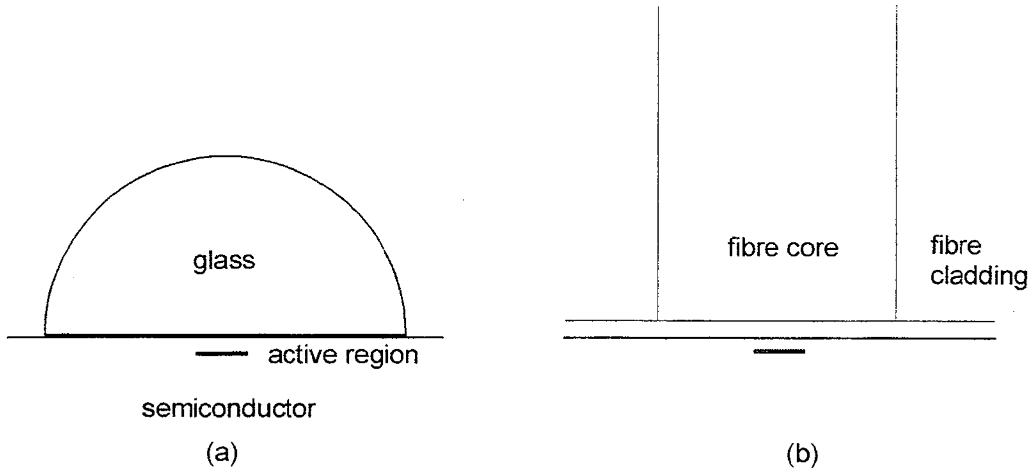


Figure 1.

4. a) A semiconductor LED has a refractive index of 3.6. If photons are emitted from the active region in all directions equally, calculate the fraction of them escaping from the diode surface. Neglect Fresnel reflection and absorption between the active region and the surface. [4]

b) The LED is now covered by a glass hemisphere of index 1.5, as illustrated in Fig. 1(a). Assuming the hemisphere is large compared to the width and distance from the surface of the active region, find the fraction of power escaping into the air. You can neglect internal absorption, but should include Fresnel reflection at the two boundaries, although the reflection coefficient may be approximated as that at normal incidence. [4]

c) The hemisphere is now replaced by silica multi-mode fibre whose axis is normal to the surface, as illustrated in Fig. 1(b). Neglecting Fresnel reflection and absorption, find the fraction of generated power which is propagated in guided modes of the fibre. The core diameter is much larger than the active region, and the fibre has an index difference of 0.01. [6]

d) Briefly explain how a heterostructure can be used to increase the external efficiency of an LED. [4]

5. a) An optical cable consists of three 10 km sections joined by connectors. These connectors each have losses of 1 dB, and back-reflections of –20 dB. What fraction of the power lost at the connector is reflected back along the fibre? [4]

b) Describe the operation of four-port couplers, and how they can be used in a passive optical network. Indicate the relationships between the various coupling ratios within one such coupler. [8]

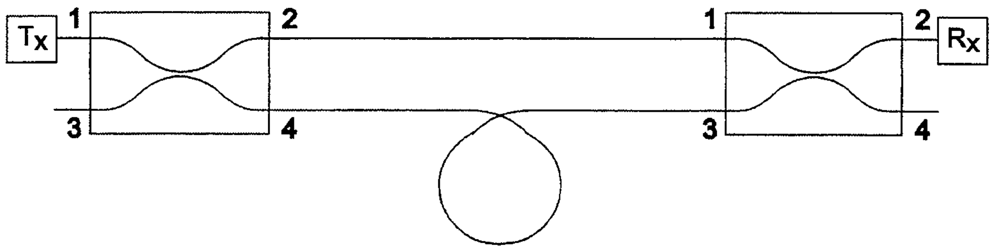


Figure 2

c) A single optical link is configured as shown in Fig. 2, to provide a protection path in case of failure of one of the fibres between the transmitter (T_X) and receiver (R_X). The two passive couplers employed are identical. The upper and lower fibre lengths between the couplers are L_A and L_B respectively, $\Delta L = L_B - L_A$, and the attenuation coefficient is α in both fibres. Find an expression for P_{21}, the fraction of power incident at port 1 exiting on port 2 of the coupler, such that equal power from each of the two paths arrives at the receiver R_X. Neglect excess losses in the couplers. What limitation should be placed on ΔL? [8]

6. a) Describe the structure and operating principles of erbium doped fibre amplifiers. Use diagrams where appropriate. Indicate the main performance criteria, and give typical values for these. Include a definition of amplified spontaneous emission (ASE) and discuss its significance. [8]

b) The ASE noise spectral density can be approximated as: [12]

$$(I_A^*)^2 = 4\, e^2 G\, (G-1) S_o \lambda / hc$$

where G and S_o are the amplifier gain and input power respectively. Show that if a high gain amplifier is placed before the receiver, the resulting signal-to-noise ratio (SNR) in terms of photocurrent is worse by a factor of $\sqrt{2}$ than without the amplifier, if the latter case is dominated by shot noise. State any assumptions or approximations used. Hence, deduce the rate at which SNR degrades with the number of amplifiers n in a particular link.

Describe the most general conditions under which an optical amplifier will improve the SNR.

Optical Communication 2001 : Solutions

① a) Equal to. θ_c depends on phase matching, not on polarisation

b) $k = 7\times10^6\,m^{-1} = n'k_o = n'2\pi/\lambda_o$

$n' = k\lambda_o/2\pi = 7\times10^6 \times 1.5\times10^{-6}/2\pi = \underline{1.67}$

c) Higher order modes, being closer to cutoff, have more light in the cladding where the index is less, therefore have a higher phase velocity.

d) In an APD there is more than one electron produced per photon, so photocurrent is higher.

e) $L = m\lambda/2 = m\lambda_o/2n \therefore \lambda_o = 2nL/m$

$\Delta\lambda = 2nL\left(\frac{1}{m} - \frac{1}{m+1}\right) \cong \frac{2nL}{m^2} = \frac{\lambda_o^2}{2nL} = \frac{1.5^2\,\mu m^2}{2(3.5)300\,\mu m} = \underline{1.1\,nm}$

f) Slope efficiency $= \eta(hc/\lambda)/e = 0.64$ W/A

$\eta = \frac{.64\,(1.6\times10^{-19})\,1.55\times10^{-6}}{6.63\times10^{-34}\times3\times10^8} = \underline{0.80}$

g) $3\,dBm = 2\,mW = (hc/\lambda)\dot{N}$

$\dot{N} = (2\times10^{-3})(1.5\times10^{-6})/(6.63\times10^{-34}\times3\times10^8) = \underline{1.5\times10^{16}}$ photons/sec.

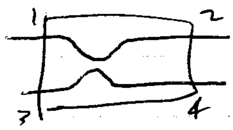
h) Grating spatial frequency $K = 2k = 2n'k_o$ for back-reflection. $K = 2\pi/\Lambda$ where Λ = grating period

$\therefore \Lambda = \lambda_o/2n' \quad n' \cong 1.5$ (silica)

$\Lambda = 1330/3 = \underline{443\,nm}$

i) AR coating has $n = \sqrt{n_1 n_2} = \sqrt{3.6\times1} = \underline{1.90}$

thickness $= \lambda/4 = \lambda_o/4n = 0.78/(4\times1.9) = \underline{0.103\,\mu m}$

j) 

P_{ij} = fraction of input at i leaving at j

$P_{12} = P_{34}$ (symmetry)

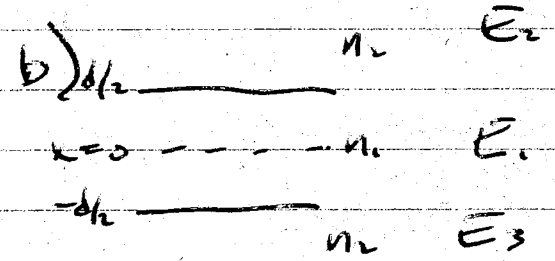
$P_{12} + P_{14} = 1$ (power cons)

$P_{14} = 0.1 \quad \therefore P_{12} = 0.9 \quad \therefore \quad P_{34} = 0.9 = \underline{90\%}$

2. a) Boundary conditions are that fields should be phase matched, ie $k_{1z} = k_{2z}$, and that magnitude and slope of E should be continuous, ie $E_1(x=d/2) = E_2(x=d/2)$ and

$$dE_1/dx\,(x=d/2) = dE_2/dx\,(x=d/2)$$

b)



$$E_1(x) = A\exp jk_{1x}x + B\exp -jk_{1x}x$$
$$E_2(x) = C\exp -Kx$$
$$E_3(x) = D\exp Kx$$
where $K = |k_{2x}| = |k_{3x}|$

Since guide is symmetric, modes will be symmetric or anti-symmetric. For the former:

$C = D \quad A = B$

$E_1(x) = A'\cos(k_{1x}x) \qquad A' = 2A$

(i) $A'\cos(k_{1x}d/2) = C\exp(-Kd/2)$

$dE_1/dx = -k_{1x}A'\sin(k_{1x}x)$

$dE_2/dx = -KC\exp(-Kx)$

(ii) $k_{1x}A'\sin(k_{1x}d/2) = KC\exp(-Kd/2)$

Divide (ii) by (i)

$k_{1x}\tan(k_{1x}d/2) = K$ ← 1st eigenvalue eq'n

For antisymmetric, $A = -B$, $C = -D$

$E_1(x) = A''\sin(k_{1x}x) \qquad A'' = 2jA$

$dE_1/dx = k_{1x}A''\cos(k_{1x}x)$

(iii) $A''\sin(k_{1x}d/2) = C\exp(-Kd/2)$

(iv) $k_{1x}A''\cos(k_{1x}d/2) = -KC\exp(-Kd/2)$

Divide (iv) by (iii)

$k_{1x}\cot(k_{1x}d/2) = -K$ ← 2nd eigenvalue eq'n

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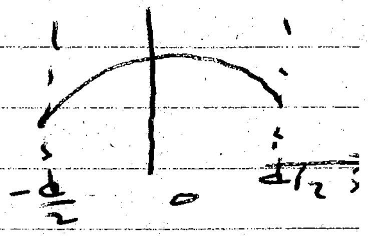
2 c) For lowest order, $m=0$, and we have $E_1(x) = A' \cos(k_{1x} x)$

then $E_1(d/2) = A' \cos(k_{1x} d/2)$

$E_1(0) = A'$

$\therefore \cos(k_{1x} d/2) = 1/\sqrt{2}$



$k_{1x} d/2 = \pi/4$

$d/\lambda_0 = 2 \quad \therefore k_0 d = 4\pi$

$k_{1x}/k_0 = \sqrt{n_1^2 - n'^2}$ by definition, but

$k_{1x} d/2 = (k_{1x}\lambda_0/2)(d/\lambda_0) = k_{1x}\lambda_0 = 2\pi k_{1x}/k_0 = \pi/4$

$k_{1x}/k_0 = 1/8 = \sqrt{n_1^2 - n'^2}$

$n' = \sqrt{(1.5)^2 - (1/8)^2} = 1.495$

3. a) Shot noise: since photon detection is a discrete process, and individual arrival times are random noise (statistical) results.

$(I_{sh}^*)^2 = 2eI_{ph}$ where I_{ph} = photocurrent

$(I_{th}^*)^2 = 4kT/R$

b) $\frac{4kT}{10^4 R} = 2eI_{ph}$; $I_{ph} = \frac{2kT}{Re} = \frac{2(1.38\times10^{-23})(300)}{10^5(1.6\times10^{-19})}$

$= 0.52\ \mu A$

c) $\Phi_R = \frac{hc}{\lambda}\frac{I_{ph}}{e}\frac{1}{\eta} = \frac{6.63\times10^{-34}\times3\times10^8}{1.55\times10^{-6}}\frac{\times5.2\times10^{-7}}{1.6\times10^{-19}}\times\frac{1}{0.8}$

$= .52\ \mu W$

prop loss $= 10\log\left(\frac{10\times10^{-3}}{5.2\times10^{-7}}\right) = 42.8$ dB

$L = 42.8/.5 = 85.6$ km

3 d) $SNR = 12 = \frac{I_{ph}}{[2eI_{ph} + 2eI_{ph}]^{1/2} \Delta f^{1/2}}$

$$12 = \left[\frac{I_{ph}}{4e\Delta f}\right]^{1/2}$$

$$\Delta f = \frac{I_{ph}}{4e \times 12^2} = \frac{0.52\times10^{-6}}{4\times1.6\times10^{-19}\times12^2} = 5.6\times10^{9}\ Hz$$

$$B = 2\Delta f = 11\ Gbit/s$$

e) $\sigma_\omega = \frac{1}{2\sigma_0} = \frac{1}{2(1/5B)} = 2.5B$

$$\left|\frac{\sigma_\omega}{\omega}\right| \simeq \left|\frac{\sigma_\lambda}{\lambda}\right| \qquad \sigma_\lambda = \frac{\lambda}{\omega}\sigma_\omega = \frac{\lambda^2}{2\pi c}\sigma_\omega$$

$$\sigma_\lambda = \frac{(1.55\times10^{-6})^2}{2\pi\times3\times10^8} \times 2.5\times11\times10^9 = 3.5\ pm$$

$$\Delta\sigma = D\cdot L\cdot\sigma_\lambda = 20\ ps/nm\cdot km \times 3.5\times10^{-3}\ nm \times 85.6\ km$$
$$= 6\ ps$$

$$\sigma_0 = \frac{1}{5B} = 18\ ps$$

$$\sigma = \sqrt{\sigma_0^2 + \Delta\sigma^2} = \sqrt{18^2+6^2} = 19\ ps.$$

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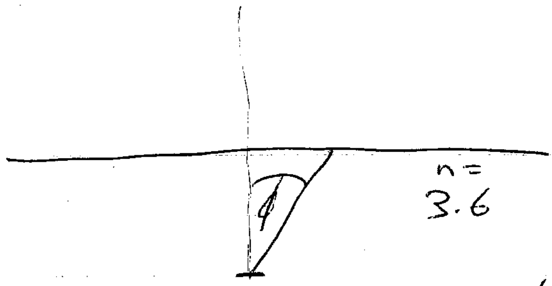
4 a)



critical angle is $\sin^{-1}\left(\frac{1}{3.6}\right) = 16.1°$

Fraction escaping = (solid angle for $\phi \leq 16.1°$)/4π

$$\Omega = \int_0^{\phi_c} 2\pi \sin\phi \, d\phi = 2\pi \left[\cos\phi\right]_{\phi_c}^{0} = 2\pi(1 - \cos 16.1°)$$

$= .246$

fraction escaping = $.246/4\pi = 0.020 = 2\%$

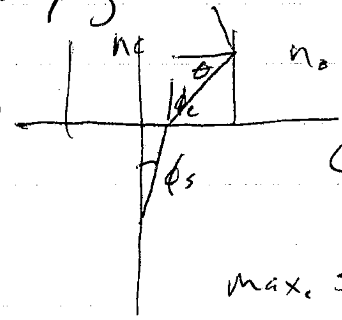
b) Critical angle is now $\sin^{-1}(1.5/3.6) = 24.6°$
fraction within critical angle $f = \frac{1}{2}(1 - \cos 24.6°)$
$= .045$

At semicond.-glass boundary:
$$R_{sg} = \left(\frac{3.6 - 1.5}{3.6 + 1.5}\right)^2 = .17$$

At glass air, $R_{ga} = \left(\frac{1.5 - 1}{1.5 + 1}\right)^2 = .04$

Neglecting multiple reflections in glass, total fraction escaping $= .045 \times (1 - .17) \times (1 - .04) = 3.6\%$

c)



Critical angle in fibre:
$\sin\theta_c = n_o/n_c$
max. $\sin\phi_c = \cos\theta_c = \sqrt{1 - (n_o/n_c)^2}$
$= \sqrt{n_c^2 - n_o^2}/n_c$
max $\sin\phi_s = \frac{n_c \sin\phi_c}{n_s} = \frac{\sqrt{n_c^2 - n_o^2}}{n_s}$

4 c) (continued)

Max. $\sin\phi_s = \frac{NA}{\sin\theta_s}$ $\quad NA = \sqrt{2n\,\Delta n}$ estimate $n = 1.46$

$NA = \sqrt{2 \times 1.46 \times .01} = 0.17$

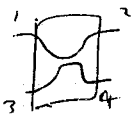
max $\sin\phi_s = \frac{0.17}{3.6}$ $\quad \phi_s(\max) = 2.7°$

fraction launched into guided modes =
$\frac{1}{2}(1 - \cos 2.7°) = .00056 = .06\,\%$

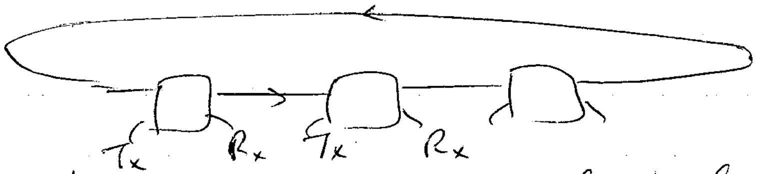
d) By using a heterostructure, we can make the region above the active region using a wider bandgap material, so as to provide a low resistance channel for the injected current without absorbing the photons travelling towards the surface.

5 a) −20 dB represents 1% of power
($-20 = 10 \log .01$). 1 dB loss corresponds to
$-1 = 10 \log_{10}(P_1/P_0)$ $\quad 10^{-0.1} = P_1/P_0 = 0.794$, ie
20.6 % loss. So the back reflection is 1/20 of the total loss.

b)



should define P_{21}, P_{41}, and indicate
$P_{21} = P_{43}$, $P_{21} + P_{41} \leq 1$
(symmetry & power conservation).
Passive LAN application:



Fabrication: integrated or fused fibre.

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5 c) Using the upper path:

$$f_A = \frac{P_R}{P_T} = P_{21} e^{-\alpha L_A} P_{21} = P_{21}^2 e^{-\alpha L_A}$$

lower path:

$$f_B = \frac{P_R}{P_T} = P_{41} e^{-\alpha L_B} P_{23}$$

but $P_{23} = P_{41} = 1 - P_{21}$

$\therefore \quad f_B = (1 - P_{21})^2 e^{-\alpha L_B}$

need $f_B = f_A$

$$P_{21}^2 e^{-\alpha L_A} = (1-P_{21})^2 e^{-\alpha L_B}$$

$$P_{21}^2 = (1-P_{21})^2 e^{-\alpha \Delta L}$$

$$P_{21}^2 (e^{\alpha \Delta L} - 1) + 2 P_{21} - 1 = 0$$

$$P_{21} = \frac{-2 \pm \sqrt{4 + 4(e^{\alpha \Delta L} - 1)}}{2(e^{\alpha \Delta L} - 1)} = \frac{\sqrt{e^{\alpha \Delta L}} - 1}{e^{\alpha \Delta L} - 1}$$

Also, need $\Delta t = n \Delta L / c$ to be $\ll$ bit length $1/B$

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6 a) Should include diagram of structure, Er^{3+} energy level diagram, information and values for gain, spectral width, noise figure, pump power and λ, ASE fibre length. Extras could include discussion of quenching effects, gain saturation, cross-talk, gain flatness and methods to correct it.

b) Without the amplifier, the received optical power is S_o. If we assume a detector of perfect (unity) quantum efficiency, then the responsivity:

$$R = e\lambda/hc$$

and so the photocurrent $I_{ph} = S_o e\lambda/hc$
and the shot noise:

$$(I_{sn}^*)^2 = 2eI_{ph}$$

So in terms of photocurrent

$$SNR = \frac{I_{ph}}{\sqrt{(I_{sn}^*)^2}\sqrt{\Delta f}} = \frac{S_o e\lambda/hc}{\sqrt{2eS_o e\lambda/hc}\sqrt{\Delta f}}$$

$$= \sqrt{\frac{S_o\lambda/hc}{2\Delta f}}$$

With the amplifier, taking $G \gg 1$

$$(I_{ase}^*)^2 \simeq 4e^2G^2S_o\lambda/hc$$

$$I_{ph} = GS_o e\lambda/hc$$

ASE will now be the dominant noise source:

$$SNR = \frac{GS_o e\lambda/hc}{\sqrt{4e^2G^2S_o\lambda/hc}\sqrt{\Delta f}} = \sqrt{\frac{S_o\lambda/hc}{4\Delta f}}$$

which is reduced by $1/\sqrt{2}$

In terms of electrical power, SNR degrades by $3dB \times n$ where n is the no. of amplifiers. The amplifiers are useful, ie improve the SNR, where thermal or preamplifier (electrical) noise dominate.