E4.03
AS5
SO10
ISE4.3

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

MSc and EEE/ISE PART IV: MEng and ACGI

MOBILE RADIO COMMUNICATION

Corrected Copy

Q1

Wednesday, 3 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : M.K. Gurcan

Second Marker(s) : K.K. Leung

Special Instructions for Invigilators: None

Information for candidates :

1) a) Determine the critical distance for the two-ray model in [3]

i) an urban microcell with a transmitter antenna height $h_t = 10\,m$ and a receiver antenna height $h_r = 3m$,

ii) an Indoor microcell having a transmitter antenna height $h_t = 3\,m$ and a receiver antenna height $h_r = 2m$, [3]

given that the radio transmission frequency is $f_c = 2\,GHz$. Comment on the results. [4]

b) For a radio system operating at 900 MHz, Table 1 gives the set of empirical measurements of the logarithmic power ratios, $P_{r,\text{dBm}} - P_{t,\text{dBm}}$, of the received to the transmitted signals at varying distances. Note the measurements include the effects of log-normal shadowing.

Distance, d_i, from transmitter	$P_{r,\text{dBm}} - P_{t,\text{dBm}}$
10 m	-70 dB
20 m	-75 dB
~~30 m~~ 50 m	-90 dB
100 m	-100 dB
300 m	-125 dB

Announced at start

Table 1 Path loss measurements

At distance d_i, the simplified path loss model estimates the received signal power in dB from

$$P_{r,\text{dBm}} = P_{t,\text{dBm}} + K - 10 \cdot \gamma \cdot \log_{10}(d_i)$$

where $K = 20\log_{10}(\lambda/4\pi)$ is the free-space-path loss at unit distance $d_0 = 1\,m$, and $\gamma = 3.71$ is the path loss exponent. [10]

Find $\sigma^2_{\Psi_{db}}$ the variance of the log-normal shadowing about the mean path loss based on these empirical measurements.

2) a) Consider a wireless LAN operating in a factory. The transmitter and receiver have a Line-of-Sight path between them with gain α_0, phase ϕ_0, and delay τ_0. Operating machines create an additional reflected signal path every T_0 seconds. The reflected signal has gain α_1, phase ϕ_1, and delay τ_1. Find the time-varying impulse response $c(\tau,t)$ for the link between the transmitter and receiver pair. [4]

b) The root-mean-square (rms) delay spreads are measured to be $\sigma_{T_m} \approx 50ns$ and $\sigma_{T_m} \approx 30\mu s$ for indoor channels and outdoor microcells, respectively. Find the maximum symbol rate $R_s = 1/T_s$ for these environments if a linearly modulated signal transmitted through the channel can be received with negligible Intersymbol Interference (ISI). Comment on the data rates achievable over indoor channels and outdoor microcells. [3] [2] [3]

c) For a channel with Doppler spread $B_D = 80Hz$, find the time difference between two received signal samples in order for the samples to be approximately independent.

d) Consider the time-varying multipath channel in the frequency domain by taking the Fourier transform of the time-varying impulse response $c(\tau,t)$. Using this Fourier transform description explain the meaning of [3]

i) The ***coherence bandwidth*** of the channel, [3]

ii) ***flat fading***, [2]

iii) ***frequency selective*** fading.

3) a) Consider a wireless channel where the signal power attenuation with distance d follows the formula $P_r(d) = P_t \frac{d_0^3}{d^3}$ for $d_0 = 10m$ where P_t and P_r are the transmitted and received signal powers respectively. Assume that the channel has a bandwidth $B = 30\ kHz$ and it is subjected to AWGN having a noise power spectral density of $N_0/2$, where $N_0 = 10^{-9} W/Hz$. For a transmitter power of 1 *W*, find the capacity of this channel for a transmitter-to-receiver distance of [6]

ii) 100 *m* and

iii) 1 *km*.

b) Consider a flat-fading channel with independent-identical-distributed channel gain $\sqrt{g}$ which can take on three possible values: $\sqrt{g_1} = 0.05$ with probability $p_1 = 0.1$, $\sqrt{g_2} = 0.5$ with probability $p_2 = 0.5$, and $\sqrt{g_3} = 1$ with probability $p_3 = 0.4$. The transmitted power is $P_t = 10\ mW$, and the noise power spectral density is $N_0/2$ where $N_0 = 10^{-9}\ W/Hz$, and the channel bandwidth is 30 *kHz*. Assume that the receiver has knowledge of the instantaneous value of g but the transmitter does not. Find the Shannon capacity of this channel and compare this with the capacity of an AWGN channel with the same average signal-to-noise ratio. [7]

b) Assume the same channel as in part (b), with a bandwidth of 30 *kHz* and three possible received SNRs: $\gamma_1 = 0.8333$ with probability $p(\gamma_1) = 0.1$, $\gamma_2 = 83.33$ with probability $p(\gamma_2) = 0.5$, and $\gamma_3 = 333.33$ with probability $p(\gamma_3) = 0.4$. Find the ergodic capacity of this channel assuming that both transmitter and receiver have instantaneous channel side information. [7]

4) Consider the downlink of a direct sequence spread spectrum (DSSS) radio system where at the output of the j^{th} chip matched filter, the received discrete time signal corresponding to the i^{th} information data bit, b_j, is given by

$$\mathbf{r}[i] = \sqrt{p_j h_j}\, b_j[i]\mathbf{s}_j + \sum_{\substack{k=1 \\ k \neq j}}^{K} \sqrt{p_k h_j}\, b_k[i]\mathbf{s}_k + \mathbf{n}$$

where $\mathbf{s}_j = \begin{bmatrix} s_{j,1} & s_{j,2} & \cdots & \cdots & s_{j,N-1} & s_{j,N} \end{bmatrix}^T$ is the spreading sequence with the property that $\mathbf{s}_j^T\mathbf{s}_j = 1$. The term $\mathbf{n}$ is the noise vector having dimension N with corresponding variance σ^2. For $k = 1 \cdots\cdots K$, p_k is the transmission power for code k. The term $\sqrt{h_j}$ is the amplitude of the channel impulse response $c(\tau) = \sqrt{h_j}\,\delta(\tau)$ and K is the total number of codes.

a) Given that the system is overloaded, i.e $K>N$, and each information data bit $b_j[i]$ can be estimated using $\hat{b}_j = sign(\mathbf{c}_j^T\mathbf{r})$, produce expressions for the detection filter coefficients, $\mathbf{c}_j$, for [5]

   i) the matched-filter detection, and

   ii) the minimum-mean-square-error (mmse) detection.

b) Produce an expression for the signal-to-noise ratio at the output of the mmse detection filter. [5]

c) Given that the channel side information is known both at the transmitter and receiver, explain how the transmission power can be iteratively adjusted to maintain a fixed signal-to-noise ratio at the output of the receiver detection filter. [5]

d) Produce an expression for the sum-capacity per chip for the dowlink if the system described in part (c) uses all K parallel codes to transmit to a single user. [5]

5) Consider the third generation wideband UTRA/FDD radio system, and answer the following questions.

a) Describe how the OVSF channelization and scrambling codes are used to spread the information data bit to realize a physical channel. [5]

b) Describe how the scrambling codes for the downlink are organized to reduce the search time for the identification of the cell-specific scrambling codes. [5]

c) Describe how [5]

i) the Primary Synchronization transport channel, and

ii) the Secondary Synchronization transport channel

is organised to use the cell-specific scrambling codes to establish the frame timing synchronization.

d) Describe how the Random Access Channel and the Acquisition Indicator channels are used to provide access control. [5]

6. a) Consider a direct sequence spread spectrum wideband CDMA system, where a total of K spreading signature waveforms are used to spread the information data bits over the downlink. Assume that both transmitter and receiver have knowledge of the channel gain h_k and the channel signal-to-noise ratio (SNR) $g_k \triangleq \frac{h_k}{\sigma^2}$ for each code k where σ^2 is the noise variance. Given that γ_k^* is the minimum required signal-to-noise ratio at the output of the detector and that the transmitter adjusts the transmission power P_k for each code k while maintaining a SNR $\gamma_k \geq \gamma_k^*$, derive an expression for the power P_k as a function of the inverse-channel-SNR (in accordance with the Perron-Frobenius theorem). [10]

b) Assume that the inverse-channel-SNR power allocation method is to be replaced with the iterative water filling power allocation method. Describe how the iterative-water filling algorithm calculates the power for each spreading code in order to maximize the sum-capacity under the constraint that the total transmission power is limited. [10]

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MODEL ANSWERs and MARKING SCHEME	
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Second Examiner: Leung, K.K..	**Question** 1 **Page** 1 *out of* 13
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1-a

$h_t = 10$ m $\quad f_c = 2$ GHz $\Rightarrow \lambda = \frac{3 \cdot 10^8}{2 \cdot 10^9} = 0.15$ m

$h_r = 3$ m

$d_c = \frac{4 h_t h_r}{\lambda} = \frac{4 \times 10 \times 3}{0.15} = 800$ m for urban microcell.

—///—

$h_t = 3$ m

$h_r = 2$ m

$d_c = \frac{3 \times 2 \times 4}{0.15} = 160$ m for indoor system

—///—

A cell radius of 800 m in an urban microcell system is a bit large. Usually micro cells are on the order of 100m However if we use a cell size of 800 m with the specified system parameters, then the desired signal power would fall off as d^2 inside the cell while interference from neighbouring cells would fall off as d^4 and thus would be greatly reduced.

—///—

Similarly 160 m is quite large for the cell radius of an indoor system. As there are many walls, hence the signal is attenuated quite rapidly, this enables us to use smaller cell radius.

—///—

1-b

The sample variance relative to the simplified path-loss model with $\gamma = 3.71$ is

$$\sigma^2_{\psi_{dB}} = \frac{1}{5} \sum_{i=1}^{5} \left[M_{measured}(d_i) - M_{model}(d_i) \right]^2$$

$M_{measured}(d_i)$ is the path loss measurement in table 1 at distance d_i and $M_{model}(d_i) = K - 37.1 \log_{10}(d)$

This yields $\quad K = 20 \log_{10}(\lambda/4\pi) = -31.54$ dB

$$\sigma^2_{\psi_{dB}} = \frac{1}{5}\left[(-70-31.54+37.1)^2 + (-75-31.54+48.27)^2 + (-90-31.54+63.03)^2 + (-110-31.54+74.2)^2 + (-125-31.54+91.90)^2\right] = 13.29$$

Thus the standard deviation of shadow fading on this path is $\sigma_{\psi_{dB}} = 3.65$ dB.

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2.a For $t \neq nT_0$ $(n = 1, 2, \ldots)$ the channel impulse response simply corresponds to the line of sight path.
For $t = nT_0$ the channel impulse response includes both the LOS and reflected paths. Thus $c(\tau, t)$ is given by

$$c(\tau,t) = \begin{cases} \alpha_0 \exp(j\phi_0)\,\delta(\tau-\tau_0) & t \neq nT_0 \\ \alpha_0 \exp(j\phi_0)\,\delta(\tau-\tau_0) + \alpha_1 \exp(j\phi_1)\,\delta(\tau-\tau_1) & \text{for } t = nT_0 \end{cases}$$

—///—

2.b We assume that negligle ISI requires that
$T_S \gg \sigma_{T_m}$ (ie $T_S \geq 10\,\sigma_{T_m}$)
This gives us $R_S = \frac{1}{T_S} \leq \frac{0.1}{\sigma_{T_m}}$
for $\sigma_{T_m} \approx 50$ ns this yields $R_S \leq 2$ Mbps
for $\sigma_{T_m} \approx 30\ \mu s$ this yields $R_S \leq 3.33$ kbps

The indoor systems currently support upto 50 Mbps and outdoor systems upto 2.4 Mbps. To maintain these data rates for a linearly modulated signal without severe performance degradation by ISI some form ISI mitigation is needed. ISI is also less severe in indoor systems.

—///—

2.c $B_D = 80$ Hz,
coherence time $T_c \approx \frac{1}{B_D} = \frac{1}{80} = 12.5$ ms
so samples spaced by 12.5 ms are approximately uncorrelated.

2.d

$$C(f;t) = \int_{-\infty}^{\infty} C(\tau,t)\exp(-j2\pi f\tau)\,d\tau$$

since $C(\tau,t)$ is WSS, its integral $C(f;t)$ is also.

Thus autocorrelation

$$A_c(f_1,f_2,\Delta t) = E\left(C^*(f_1;t)\,C(f_2;t+\Delta t)\right)$$

$$A_c(f_1,f_2,\Delta t) = E\left[\int_{-\infty}^{\infty} C^*(\tau_1,t)\exp(j2\pi f_1\tau_1)d\tau_1 \int_{-\infty}^{\infty} C(\tau_2;t+\Delta t)\exp(-j2\pi f_2\tau_2)\,d\tau_2\right]$$

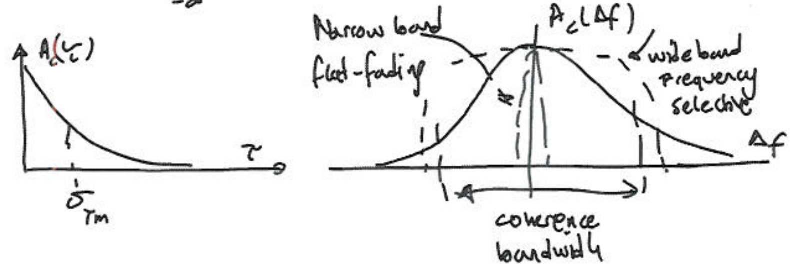
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left(C^*(\tau_1;t)\,C(\tau_2;t+\Delta t)\right)\exp(j2\pi f_1\tau_1)\exp(-j2\pi f_2\tau_2)\,d\tau_1\,d\tau_2$$

$$= \int_{-\infty}^{\infty} A_c(\tau,\Delta t)\exp(-j2\pi(f_2-f_1)\tau)\,d\tau$$

$$= A_c(\Delta f;\Delta t)$$

$\Delta f = f_2 - f_1$ define $A_c(\Delta f) \triangleq A_c(\Delta f;0)$

$$A_c(\Delta f) = \int_{-\infty}^{\infty} A_c(\tau)\exp(-j2\pi\Delta f\tau)\,d\tau$$



2.d.i The frequency at which $A_c(\Delta f) = 0$ for $\Delta f > B$ is the coherence bandwidth

2d.ii Narrow band signal with bandwidth $B \ll B_c$, is refered to as flat fading

2d.iii If $B \gg B_c$, in this case the fading is called frequency selective.

3.a The received SNR is
for $d = 100$ m

$$\gamma = \frac{P_r(d)}{N_0 B} = \frac{(0.1)^3}{10^{-9}\, 30\times10^3} = 33 = 15\text{dB}$$

for $d = 1000$ m

$$\gamma = \frac{(0.01)^3}{10^{-9}\times 30\times10^3} = 0.033 = -15\text{dB}.$$

The corresponding capacities are
for $d = 100$ m

$$C = B\log_2(1+\gamma) = 3000\log_2(1+33) = 152.6 \text{ kbps}$$

for $d = 1000$ m

$$C = B\log(1+\gamma) = 3000\log_2(1+0.033) = 1.4 \text{ kbps}$$

The signifant decrease in capacity at greater distances is due to the path loss exponent of 3.

—///—

3.b The channel has three possible received SNRs

$$\gamma_1 = \frac{P_t\, g_1}{N_0 B} = \frac{10\times10^{-3}\,(5\times10^{-2})^2}{10^{-9}\times 30\times10^3} = \frac{25\times10^{-6}}{3\times10^{-5}} = 0.833 = -0.79 \text{ dB}$$

$$\gamma_2 = \frac{P_t \cdot g_2}{N_0 B} = \frac{10\times10^{-3}\times(5\times10^{-1})^2}{10^{-9}\times 3\times10^3} = 83.33 = 19.2 \text{ dB}$$

$$\gamma_3 = \frac{P_t \cdot g_3}{N_0 B}\ \frac{10\times10^{-3}}{10^{-9}\times 3\times10^3} = 333.33 = 25 \text{ dB}$$

$p(\gamma_1) = 0.1 \quad p(\gamma_2) = 0.5 \quad p(\gamma_3) = 0.4$

$$C = \sum_{i=1} B\log_2(1+\gamma_i)\, p(\gamma_i)$$

$$= 3000\left[0.1\log_2(1.833) + 0.5\log_2(84.33) + 0.4\log_2(334.33)\right]$$

$$= 199.26 \text{ kbps}$$

—///—

The average SNR $\bar{\gamma} = 0.1\times(0.8333) + 0.5\times(83.33) + 0.4\times(334.33) = 175.08 = 22.43$ dB
with this SNR, the capacity is $C = 30\log_2(1+175.08) = 223.8$ kbps
This is approximately 25 kbps higher than flat fading channel with receiver CSI and the same average SNR

3-c $\gamma_1 = 0.833$ $p(\gamma_1) = 0.1$

$\gamma_2 = 83.33$ $p(\gamma_2) = 0.5$

$\gamma_3 = 333.33$ $p(\gamma_2) = 0.4$

Need to find the cut-off value

$$\sum_{\gamma_i \geq \gamma_0} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) p(\gamma_i) = 1$$

we first assume that all channel states are used to obtain γ_0 (assume that $\gamma_0 < \min \gamma_i$)

$$\sum_{i=1}^{3} \frac{p(\gamma_i)}{\gamma_0} - \sum_{i=1}^{3} \frac{p(\gamma_i)}{\gamma_i} = 1$$

$$\Rightarrow \frac{1}{\gamma_0} = 1 + \sum_{i=1}^{3} \frac{p(\gamma_i)}{\gamma_i} = 1 + \left(\frac{0.1}{0.8333} + \frac{0.5}{83.33} + \frac{0.4}{333.33}\right) = 1.13$$

solving for $\gamma_0 = \frac{1}{1.13} = 0.89 > 0.833 = \gamma_1$

since this value is greater than the weakest channel we modify the water filling optimization

$$\sum_{i=2}^{3} \frac{p(\gamma_i)}{\gamma_0} - \sum_{i=2}^{3} \frac{p(\gamma_i)}{\gamma_i} = 1 \Rightarrow \frac{0.9}{\gamma_0} = 1 + \sum_{i=2}^{3} \frac{p(\gamma_i)}{\gamma_i} = 1 + \frac{0.5}{83.33} + \frac{0.4}{333.3} = 1.13$$

we get

$$\gamma_0 = \frac{0.9}{1.0072} = 0.89$$

we now have $\gamma_1 < \gamma_0 < \gamma_2 < \gamma_3$

The sum capacity is

$$C = \sum_{i=2}^{3} B \log_2\left(\frac{\gamma_i}{\gamma_0}\right) p(\gamma_i) = 30000\left(0.5 \log_2 \frac{83.33}{0.89} + 0.4 \log_2 \frac{333.33}{0.89}\right) = 200.82$$

This rate is slightly higher than for the case of receiver CSI only. It is significantly below that of an AWGN channel.

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MODEL ANSWERs and MARKING SCHEME

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4a-i For matched filter detection

$c_j = s_j$

4a-ii For the mmse receiver

$c_j = \alpha_j (h \underline{S} \underline{P} \underline{S}^T + \sigma I)^{-1} s_j$

where

$\underline{S} = [s_1 \; \ldots \; s_K]$

$P = \mathrm{diag}(P_1 \; \ldots \; P_K)$

4b

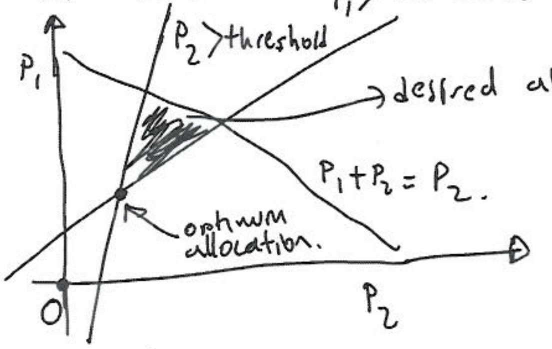
$$SNIR = \frac{P_j h}{h \sum_{k \neq j} P_k (c_j s_k)^2 + \sigma^2 c_j^T c_j}$$

4c

$$P_j \geq \gamma_j^* \sum_{k \neq j} P_k (c_j s_k)^2 + \frac{\gamma_j^* \sigma^2}{h} c_j^T c_j$$

where γ_j^* desired SNIR

let K = 2



4d Sum capacity per chip is given by

$$C = \frac{1}{2N} \log_2 \left(\det \left(h \, S P S^T + \sigma^2 I \right) \right)$$

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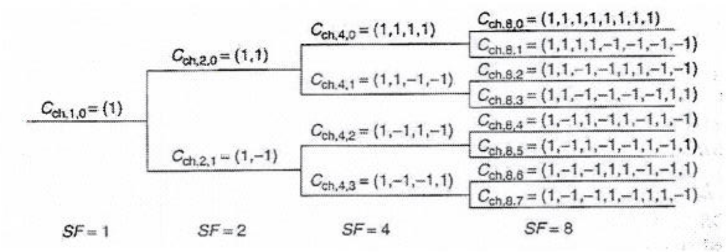
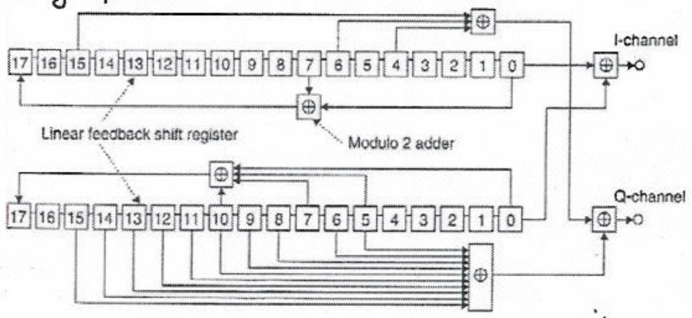
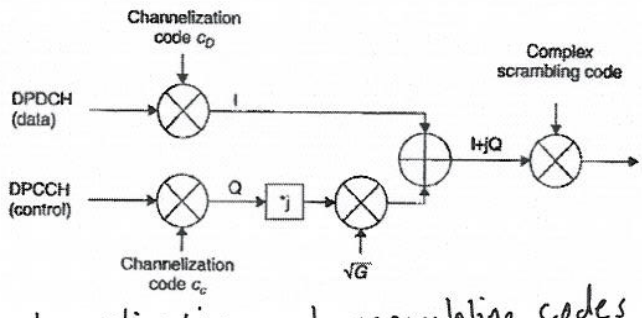
5.a



The OVSF codes are generated using Hadamar codes
They provide the channelization codes



The Gold sequences are used to provide complex scrambling codes.



Both channelization and scrambling codes are used to to spread the information data bits.

5b The following synchronization code is used to identify the starting point for each time slot.

$C_{PSC} = (1+j)x < a, a, a, -a, -a, a, -a, -a, a, a, a, -a, a, -a, a, a >,$

$a = \{x_1, x_2, x_3, \ldots, x_{16}\}$

$= \{1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1, 1, -1, -1, 1\}$

Secondary scrambing codes are generated as follows

$n = 16 \times (k-1)$

$\mathbf{H}_8$ Hadamar codes

$C_{SSC,k} = (1+j) \times < h_n(0) \times z(0), h_n(1) \times z(1), h_n(2)$

$\times z(2) \cdots h_n(255) \times z(255), >,$

$Z = \{b, b, b, -b, b, b, -b, -b, b, -b, b, -b, -b, -b, -b, -b\}$

$b = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, -x_9,$

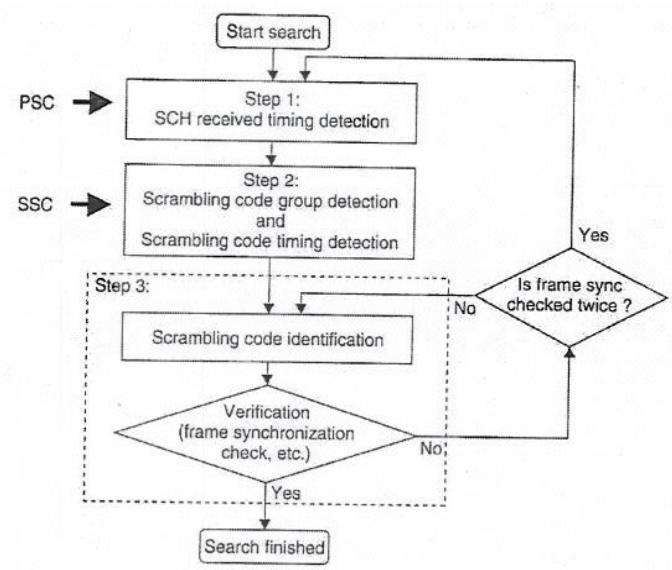
$-x_{10}, -x_{11}, -x_{12}, -x_{13}, -x_{14}, -x_{15}, -x_{16}\}$

To produce a total of 16 distinctive codewords.

64 different combinations of these codes are generated as outlined in the following table.

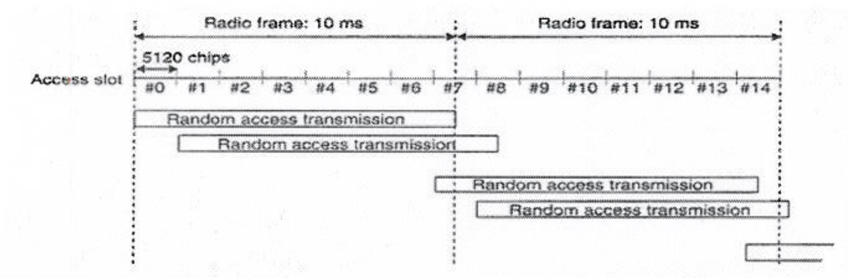
Scrambling Code Group	slot number														
	#0	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14
Group 0	1	1	2	8	9	10	15	8	10	16	2	7	15	7	16
Group 1	1	1	5	16	7	3	14	16	3	10	5	12	14	12	10
Group 2	1	2	1	15	5	5	12	16	6	11	2	16	11	15	12
Group 3	1	2	3	1	8	6	5	2	5	8	4	4	6	3	7
Group 4	1	2	16	6	6	11	15	5	12	1	15	12	16	11	2
Group 5	1	3	4	7	4	1	5	5	3	6	2	8	7	6	8

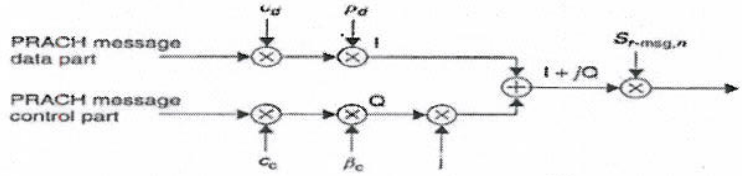
5-c A total of 512 different scrambling codes are used and They are grouped into groups of 8 codes giving us a a total of 64 groups. using the above grouping arrangement a three step search algorithm is used as follows to identify which group of codes is used in a cell.


Start search
PSC
Step 1:
SCH received timing detection
SSC
Step 2:
Scrambling code group detection
and
Scrambling code timing detection
Yes
Step 3:
Is frame sync
checked twice ?
No
Scrambling code identification
Verification
(frame synchronization
check, etc.)
No
Yes
Search finished

—//—

5d


Radio frame: 10 ms
Radio frame: 10 ms
5120 chips
Access slot
#0 #1 #2 #3 #4 #5 #6 #7 #8 #9 #10 #11 #12 #13 #14
Random access transmission
Random access transmission
Random access transmission
Random access transmission


PRACH message
data part
PRACH message
control part
I
Q
I + jQ
$S_{r\text{-}msg,n}$
c_c
β_c
β_d

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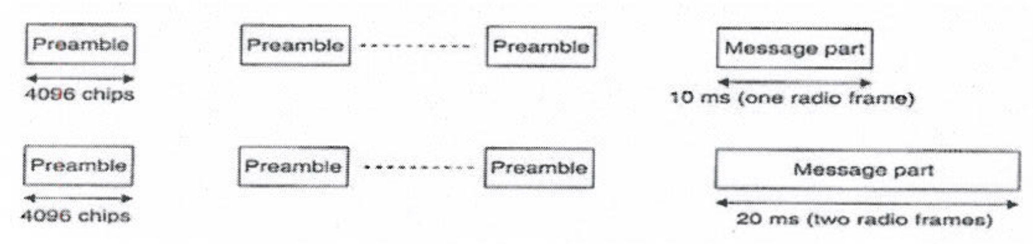
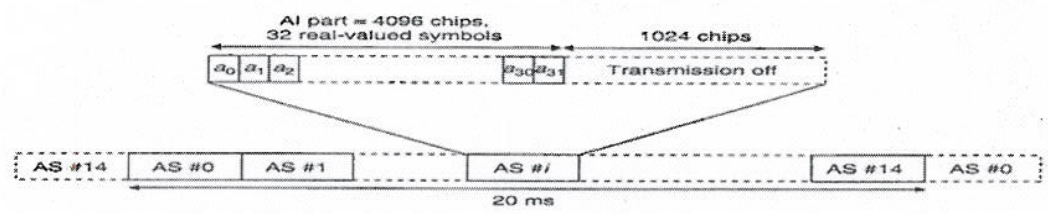
Question labels in left margin **Marks allocations in right margin**





6 a Assume that the channel impulse response is

$h(t) = h_j \, \delta(t)$ where h_j is channel gain.

$$SNIR_j = \frac{P_j h_j (c_i^T s_i)^2}{\sum_{k \neq j} P_k h_j (c_j s_k)^2 + \sigma^2 c_j^T c_j}$$

assume $c_j = s_j$ and $s_j^T s_j = 1$ $s_k^T s_j = \rho_{ij}$

if Gold sequences are used with code length hence processing gain N

$$|\rho_{ij}| = \frac{1}{\sqrt{N}} \Rightarrow \rho_{ij}^2 = \frac{1}{N} = PG = \rho \quad , \text{ let } n = \sigma^2 c_j^T c_j$$

$$SNIR_j = \frac{P_j g_j}{\rho \sum_{J \neq k} h_j P_k + n} \qquad J = 1 \ldots K$$

in the matrix form

$$(I - F) P \geq u \quad \text{with } P > 0$$

where

$P = (P_1 \ldots P_K)^T$ is the vector transmitter powers

$$u = \left(\frac{n \gamma_1^*}{h_1}, \frac{n \gamma_2^*}{h_2}, \ldots \frac{n \gamma_K^*}{h_K} \right)^T$$

$$F_{jk} = \begin{cases} 0 & J = K \\ \dfrac{\gamma_j^* h_K \rho}{h_j} & j \neq K \quad \text{for } J = 1, \ldots K. \end{cases}$$

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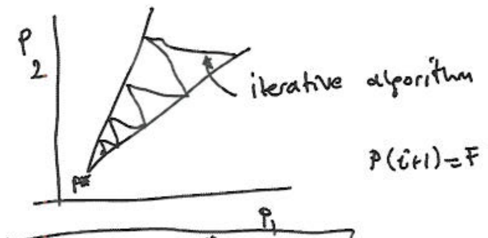
The matrix F has non-negative elements and is irreducible

Let ρ_F Perron-Frobenious eigen value of F.

ρ_F is the maximum eigen value of F.

There exists a vector P such that $(I-F)P \geq u$

$(I-F)^{-1}$ exists and is positive component wise



$P(i+1) = F\,P(i) + u$

$$P_j(i+1) = \frac{\gamma_j^*}{\gamma_j(i)} P_j(i)$$

—//—

6.b

Iterative water filling for bits/chip

$$C_{sum} = \frac{1}{2N} \sum_{k=1}^{K} \log_2(1+\gamma_k)$$

$\sigma^2 = n$.
capacity

$\bar{s}_j = [s_{j,1} \; --- \; s_{j,N}]^T$ for all $j = 1 \ldots k$

$S_{s,j} = [\bar{s}_1 \; --- \; \bar{s}_K]^T$ for all i , $i \neq j$

$P_{s,j}$ diag $(P_1 \; --- \; P_K)$ for all i , $i \neq j$

$$A_j = \sigma^2 I + h_j \bar{S}_{I,j} P_{I,K} S^T_{I,K}$$

SIR simplifies to

$$\gamma_j = SIR_j = P_j h_j s_j^T A_j^{-1} s_j$$

$$\text{Channel\_SNR}_j = \frac{SNR_j}{P_j} = h_j s_j^T A_j^{-1} s_j$$

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I WF algorithm

maximize $C_{sum} = \frac{1}{N} \sum_{k=1}^{K} \log_2 (1 + \gamma_k)$

s.t. $\sum_{k=1}^{K} P_k \leq P_T$

<u>Algorithm</u>

① Allocate $P_k = \frac{P_T}{K}$ for $k = 1 \ldots K$

② Calculate A_j

③ Calculate channel gain

$$g_j = \frac{\gamma_j}{P_j} = h_j s_j^T A_j^{-1} s_j \quad \text{for } j = 1 \ldots K$$

④ Order g_j so that g_1 is largest g_j
g_K is smallest g_j

⑤ Calculate Lagrange multiplier

$$K_\lambda = \frac{1}{K} \left[P_T + \sum_{j=1}^{K} \frac{1}{g_j} \right]$$

$$= \frac{1}{K} \left[P_T + \sum_{j=1}^{K} \frac{1}{h_j s_j^T A_j^{-1} s_j} \right]$$

⑥ Calculate new power values

$$P_{j,new} = \max\left(0, K_\lambda - \frac{1}{h_j s_j^T A_j^{-1} s_j}\right)$$

⑦ if $K_\lambda - \frac{1}{h_K s_K^T A_K^{-1} s_K} < 0$

Then $K = K - 1$ go to ②

⑧ $|P_{j,new} - P_j| > \text{Threshold}$ Stop

else go to ②.