DEPARTMENT of ELECTRICAL and ELECTRONIC ENGINEERING
EXAMINATIONS 2003

M.Sc and EEE/ISE PART IV: M.Eng. and ACGI

Solutions 2003
ADVANCED COMMUNICATION THEORY

— There are FOUR questions (Q1 to Q4)
— Answer Question ONE plus TWO other questions.

Comments for Question Q1:
— Question Q1 has 20 multiple choice questions numbered 1 to 20.
— Circle the answers you think are correct on the answer sheet provided.
— There is only one correct answer per question.

Distribution of marks

Question-1: 40 marks
Question-2: 30 marks
Question-3: 30 marks
Question-4: 30 marks

The following are provided:

- ***A table of Fourier Transforms***
- ***A "Gaussian Tail Function" grap***

Examiners responsible: Dr. A. Manikas

ANSWER to Q1

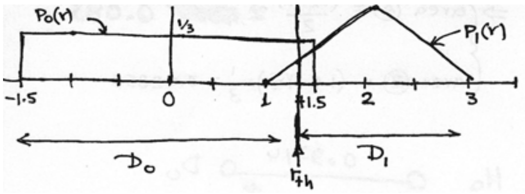
1) A B C D E
2) A B C D E
3) A B C D E
4) A B C D E
5) A B C D E

6) A B C D E
7) A B C D E
8) A B C D E
9) A B C D E
10) A B C D E

11) A B C D E
12) A B C D E
13) A B C D E
14) A B C D E
15) A B C D E

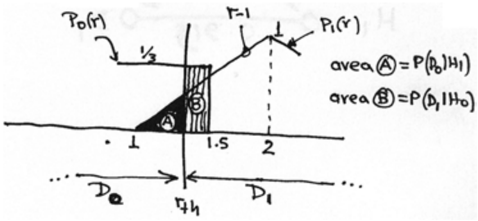
16) A B C D E
17) A B C D E
18) A B C D E
19) A B C D E
20) A B C D E

ANSWER to Q2 (aim: to examine 'decision rules')



$$C_{00}.\Pr(D_0|H_0) + C_{10}.\Pr(D_1|H_0) = C_{11}.\Pr(D_1|H_1) + C_{01}.\Pr(D_0|H_1)$$

$$\Rightarrow \qquad 3\Pr(D_1|H_0) = \Pr(D_0|H_1) \quad (1)$$



$$(1) \Rightarrow 3\underbrace{(1.5 - r_{\text{th}})\tfrac{1}{3}}_{\textbf{area}\ \boxed{\textbf{B}}} = \underbrace{\tfrac{(r_{\text{th}}-1)^2}{2}}_{\text{area}\ \boxed{\textbf{A}}}$$

$$\Rightarrow (1.5 - r_{\text{th}}) = \tfrac{r_{\text{th}}^2 - 2r_{\text{th}} + 1}{2}$$

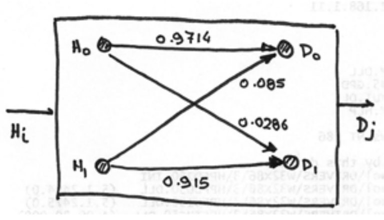
$$\Rightarrow 3 - 2r_{\text{th}} = r_{\text{th}}^2 - 2r_{\text{th}} + 1$$

$$\Rightarrow r_{\text{th}}^2 = 2$$

$$\Rightarrow r_{\text{th}} = \sqrt{2} = 1.41$$

$$\Rightarrow \begin{cases} \textbf{area}\ \boxed{\textbf{A}} = \frac{\left(\sqrt{2}-1\right)^2}{2} \simeq \mathbf{0.085} \\ \textbf{area}\ \boxed{\textbf{B}} = \left(\mathbf{1.5} - \sqrt{\mathbf{2}}\right)\frac{1}{3} \simeq \mathbf{0.02859} \end{cases}$$

$$\text{i.e. } \mathbb{F} = \begin{bmatrix} 0.9714, & 0.085 \\ 0.0286, & 0.915 \end{bmatrix}$$



ANSWER to Q3 (aim: to examine 'Spread Sectrum Theory')

- $F_g = 4\,\text{kHz}$, $Q = 256$ levels, $\gamma = 8$ bits; $P_J = 1.6\text{W}$, $N_0/2 = 0.5 \times 10^{-12}$, $A = 0.693\,V$, $p_e = 3 \times 10^{-6}$

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- $r_{cs} = 2 \times 2\gamma F_g = 2 \times 2 \times 8 \times 4 \times 10^3$ bits/sec $= 128 \times 10^3 \frac{\text{bits}}{\text{sec}}$
  $T_{cs} = \frac{1}{r_{cs}} = 7.8125\,\mu\text{sec}$
- $N_0 = 10^{-12}$
- $B_J = 0.1\,B_{ss}$
- $E_b = \frac{A^2}{2} T_{cs} = 1.876 \times 10^{-6}$

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- $p_e = \mathbf{T}\left\{\sqrt{(1-\rho)\text{EUE}_{equ}}\right\}$; BPSK $\Rightarrow \rho = -1$

  Therefore $3 \times 10^{-6} = \mathbf{T}\left\{\sqrt{2\text{EUE}_{equ}}\right\}$

  $\Rightarrow$ (using inverse tail-functions), $4.6 = \sqrt{2\text{EUE}_{equ}}$

  $\Rightarrow \; 2\frac{E_b}{N_0 + \frac{P_J}{B_J}} = 4.6^2$

  $\Rightarrow \; B_J = \frac{P_J}{-N_0 + \frac{2E_b}{4.6^2}}$

  $\Rightarrow \; B_{ss} = \frac{P_J \times 10}{-N_0 + \frac{2E_b}{4.6^2}} = 90.236 \times 10^6$
  (i.e. PN-code rate $= 90.236 \frac{\text{Mchips}}{\text{sec}}$)

  $\Rightarrow T_c = \frac{1}{B_{ss}} = 11.082 \times 10^{-9}\text{sec}$

  Therefore, $\text{PG} = \frac{T_{cs}}{T_c} = 704.9716$

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- synchronisation error of 30% etc, $\Rightarrow \tau = 0.3T_{cs} = 2.34375\,\mu\text{sec}$
  This indicates that $T_c < \tau$ which implies that the o/p code-noise has:
  mean $= 0$
  var $= A^2 T_c / T_{cs} = 0.68123\,\text{mW}$

  Furthermore, since $\tau > T_c \Rightarrow P_{\text{desired}} = 0 \Rightarrow \text{SNIR}_{out} = 0$
  $\Rightarrow p_e = \mathbf{T}\left\{\sqrt{0}\right\} = 0.5$

ANSWER to Q4 (aim: to examine 'DS-CDMA')

- No of users $= 256 \Rightarrow 1$ desired + 255 MAI
  That is $K = 255$
- AJM $= 30\text{dB} \Rightarrow 10\log_{10}(\text{EUE}_{equ}) - 10\log_{10}(\text{EUE}_{equ,pr}) = 30$
  $\Rightarrow \frac{\text{EUE}_{equ}}{\text{EUE}_{equ,pr}} = 10^3$ (Equ.1)
- 'protection' $BER = p_{e,pr} = 10^{-2}$
- $m = 21 \Rightarrow N = 2^m - 1 = 2097161$
  $P = 0.1915$
  $N_0 = 10^{-6}$

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- $\overbrace{p_{e,pr}}^{=10^{-2}} = \mathbf{T}\left\{\sqrt{2\text{EUE}_{equ,pr}}\right\}$

  $\Rightarrow \sqrt{2\text{EUE}_{equ,pr}} \simeq 2.3263$

  $\Rightarrow \text{EUE}_{equ,pr} \simeq 2.7059$

  This implies, using Equ 1, that $\text{EUE}_{equ} \simeq 2705.9$

  i.e. $\frac{E_b}{N_0 + N_J} \simeq 2705.9$

  $\Rightarrow E_b \simeq 2705.9\left(N_0 + \underbrace{\left(\tfrac{K-1}{E_b}\right)}_{=N_J}\right)$

  $\Rightarrow E_b \simeq \frac{2705.9 \times N_0 \times N}{N - 2705.9 \times K} = 0.0040$

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- $PT_{cs} = E_b \Rightarrow T_{cs} = \frac{E_b}{P} = 21.1\,\text{msec}$

  $N = \frac{T_{cs}}{T_c} \Rightarrow T_c = 10.042\,\mu\text{sec}$

  Therefore, PN-code rate $= \frac{1}{T_c} = 99.583\,\text{Mchips/sec}$

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- 1) by employment of three directional antennas each having 120° beamwidth , thereby
  diving each cell into 3 sectors (Sectorisation)
  2) by using 'voice activity' (e.g. a voice activity factor α=0.375)