IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

EEE PART III/IV: MEng, BEng and ACGI

Corrected Copy

MICROWAVE TECHNOLOGY

Monday, 9 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : S. Lucyszyn
Second Marker(s) : E. Rodriguez-Villegas

1 Consider a balanced transmission line feeding a simple dipole antenna.

(a) If the dielectric material between, and completely surrounding, the current carrying metal conductors is non-magnetic and has a dielectric constant of 2.3, calculate the phase velocity of a TEM-wave as it propagates down the transmission line, inside the dielectric.

[4]

(b) Valence electrons within the current carrying metal conductors are free to travel. What is the name commonly given to the mean velocity at which these electrons travel and give the order of magnitude for its speed in high conductivity metals?

[2]

(c) Within the current carrying metal conductors an evanescent wave propagates. Calculate the phase velocity of this wave if the metal is made from copper (having a bulk DC conductivity of 5.8×10^7 S/m) at 10 GHz.

[4]

(d) Within the metal conductors, the current density is given by, $Jc = N\ e\ v_d$, where electron charge is $e = 1.6 \times 10^{-19}$ C. For copper at room temperature, the number of valence electrons is $N = 8.47 \times 10^{22}$ /cm^3. If the metal conductor has a circular cross-sectional diameter of 2.25 mm and the current flowing is 10 A, calculate the time average drift velocity, v_d.

[4]

(e) For a half-wave dipole in free-space, calculate the distance to the boundary of the radiating far field at 10 GHz. In relation to the speed of light, comment on the phase velocities of the electric field within both the near-field and radiating far field.

[6]

2 (a) Describe, with the aid of a diagram, a branch-line coupler having a distributed-element implementation, indicating lengths and impedances. Indicate its main characteristics.

[5]

(b) Replace the distributed-element implementation in 2(a) with an equivalent lumped-element version. How does the performance compare with that in 2(a)? With a coupler having an impedance of $Zo = 75\ \Omega$, calculate the components values for a design frequency of 3.0 GHz.

[5]

(c) Replace the lumped-element implementation in 2(b) with an equivalent lumped-distributed version. With a coupler having an impedance of $Zo = 75\ \Omega$ and line sections of $\phi = 30°$, calculate the components values for a design frequency of 3.0 GHz.

[5]

(d) Draw the topology of a balanced amplifier employing the coupler given in 2(a).

[5]

3 (a) Describe the various mechanisms associated with power loss in guided-wave structures.

[3]

(b) Describe the various modes that can be supported by CPW, GCPW, CBCPW and FGC transmission lines and, where possible, indicate how unwanted modes can be suppressed. Calculate the cut-off frequency for the TM_1 and TM_2 modes, given an alumina substrate having a height of 635 μm and dielectric constant of 9.8.

[5]

(c) Define the quality factor of a dielectric, in terms of both energy and also propagation constant. From this latter definition, derive from first principles the relationship between the quality factor of a low loss dielectric in terms of both frequency and in loss tangent. The alumina substrate in 3(b) has a loss tangent of 0.0002, calculate its quality factor.

[5]

(d) State the general equation for power flux density and also the equation for surface power density on a conducting surface. From first principles, prove that power flux density at the surface of a metal is the same as the surface power density on a conducting surface.

[7]

4 A short transmission line is used to transform an arbitrary termination impedance, $Z = R + jX$, to the reference impedance, Zo. The impedance looking into the transformer is given by the following expression:

$$z_{IN} = \frac{z + jz_{TX} \tan \vartheta}{z_{TX} + jz \tan \theta} \qquad (4.1)$$

where the normalised variables used have their usual meanings

(a) Using this equation, derive the equations for the characteristic impedance of the transmission line, Z_{TX}, and the corresponding electrical length, ϑ.

[7]

(b) From the expressions derived in 4(a), what are the mathematical limits for the resistive and reactive values of the termination impedance that can be mapped into the input impedance of the short transmission line transformer?

[3]

(c) A load termination consisting of a 5 pF capacitance in series with a 8 Ω resistance must be matched at 1800 MHz to a 50 Ω reference impedance using a short transmission line transformer. Using expressions derived in 4(a) and 4(b), calculate Z_{TX} and ϑ for the short transmission line transformer.

[7]

(d) Comment on the suitability, or otherwise, of implementing the short transmission line transformer calculated in 4(c) using conventional microstrip and thin-film microstrip technologies.

[3]

5 (a) State the boundary conditions for electric and magnetic fields at the interface between air and a perfect conductor.

[2]

(b) Explain why a rectangular waveguide of cross-sectional dimensions *a* and *b* has a lowest frequency at which electromagnetic waves will not propagate. Derive an expression for the lowest mode cut-off frequency.

[4]

(c) For a rectangular waveguide of cross-sectional dimensions 2.0 mm x 1.0 mm, determine the cut-off frequencies of the following modes: TE_{10}, TE_{01}, TE_{11}, TM_{11}, TE_{20}.

[4]

(d) List which of these modes are degenerate and discuss mode conversion between degenerate modes.

[2]

(e) Define the terms phase velocity and group velocity. Also derive formulas for them in terms of the guide mode cut-off frequency and signal frequency.

[4]

(f) Identify the centre of the band of frequencies in the waveguide where only one mode will propagate and calculate the group and phase velocity of the propagating mode at this band centre. Show that the product of your group and phase velocities is 9×10^{16} in SI units.

[4]

6 (a) A microwave oven operates at 2.45 GHz. If the internal dimensions of the oven are 36.0 cm x 34.5 cm x 24.5 cm, illustrate the E-field energy distribution inside the oven when it is empty. What happens to this distribution when food is placed inside the oven and what is the role of the turntable?

[5]

(b) With a peak E-filled intensity of 200 V/m, estimate the average power dissipated per unit volume in a portion of meat having a dielectric constant of 38 and loss tangent of 0.4.

[5]

(c) If the portion of meat in 6(b) has a mass of 1.0 kg and dimensions of 20 cm x 20 cm x 20 cm and is assumed to have a specific heat of 5 kJ/kg K, estimate the approximate cooking time to raise the temperature of the meat from 0 °C to 100 °C.

[5]

(d) Given the complex wavenumber, calculate skin depth from first principles, for the portion of meat in 6(b). If the portion of meat is thicker than a few skin depths, what action can be taken to ensure that the meat is cooked more uniformly?

[5]

Model answer to Q 1(a): Computed Example

With a transmission line supporting TEM-mode propagation, the E- & H-field waves propagate outside the conductors at the phase velocity inside the dielectric, v_{pd}:

$$Vpd = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \rightarrow \frac{3.0x10^8 \ m/s}{\sqrt{2.3}} = 2.0x10^8 \ m/s$$

where, c = speed of light in a vacuum; μ_r = relative permeability of surrounding dielectric = 1; ε_r = relative permittivity of the surrounding dielectric = 2.3

[4]

Model answer to Q 1(b): Computed Example

The free electrons inside the conductors travel at the Fermi speed, v_f:

$$v_f = \sqrt{\frac{2E_f}{m}} \cong 1.6x10^6 \ m/s \quad \textit{for copper}$$

where, E_f = Fermi Energy; m = rest mass of the electron The student does not need to know this detailed calculation, but should appreciate the order of magnitude.

[2]

Model answer to Q 1(c): Computed Example

The E- & H-field waves propagate inside the conductors at the phase velocity, v_{pc}:

$$v_{pc} = \frac{\omega}{\beta} = \omega\delta = \sqrt{\frac{2\omega}{\mu_o \sigma_o}} \cong 4.1x10^4 \ m/s \quad \textit{for copper at } 10 \ GHz$$

[4]

Model answer to Q 1(d): Computed Example

Conduction current, $Jc = N e v_d$, flows when (Ne) electrons drift at a time average drift velocity, v_d:

$$v_d = \frac{1}{N \cdot e} \frac{I}{Area} \cong 1.86x10^{-4} \ m/s \quad \textit{with } 10A \textit{ flowing in } 2.25 \textit{ mm dimeter copper wire}$$

[4]

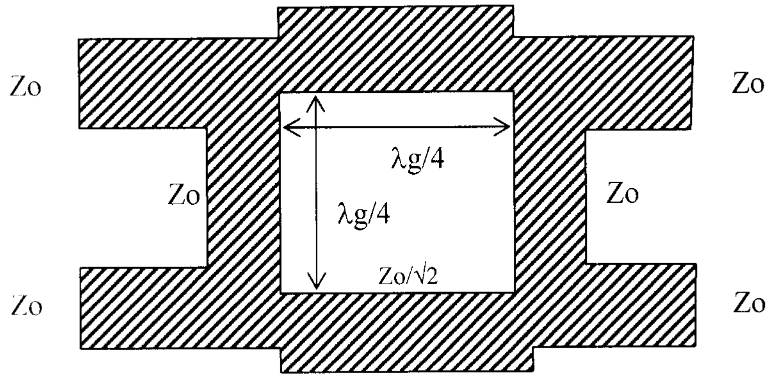
Model answer to Q 1(e): Computed Example

As the radius of the detached electric field loop increases, its curvature straightens out towards a line. With antenna radiation, this spherical wave approximates a plane wave, whereby the E- & H-field and direction of propagation are all mutually orthogonal (i.e. TEM propagation in free space). At a distance greater than $2L^2/\lambda o$ (where L = largest spatial dimension of the antenna's aperture = $\lambda o/2$ in this case), the radiating far field (or Fraunhofer) region exists. What is important is to note that, within this region, the radiation pattern of the antenna is not a function of distance from the antennas. Therefore, the boundary for the radiating far field is $\lambda o/2 = 3 \times 10^8/(2 \times 10 \times 10^9) = 15$ mm. Beyond this boundary, the electric field will be propagating at the speed of light (3×10^8 m/s). However, close to the antenna and well within this boundary the velocity will be greater than the speed of light.

[6]

Model answer to Q 2(a): Bookwork

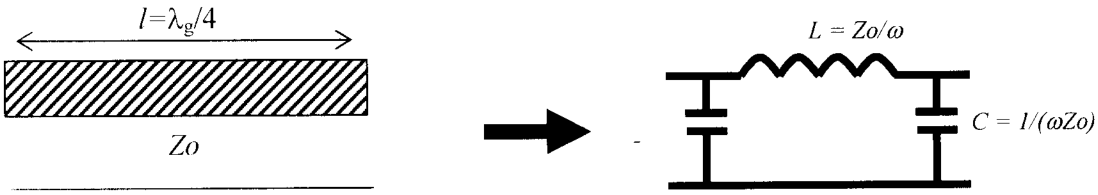
90° 3dB Directional Coupler (Branch-line Coupler)



- Works on the interference principle, therefore, narrow fractional bandwidth (15% maximum)
- No bond-wires or isolation resistors required
- Wider tracks make it easier to fabricate and is, therefore, good for lower loss and higher power applications
- Simple design but large
- Meandered lines are possible for lower frequency applications

Model answer to Q 2(b): Bookwork and Computed Example

The lumped-element equivalent of a $\lambda_g/4$ transmission line is shown below.

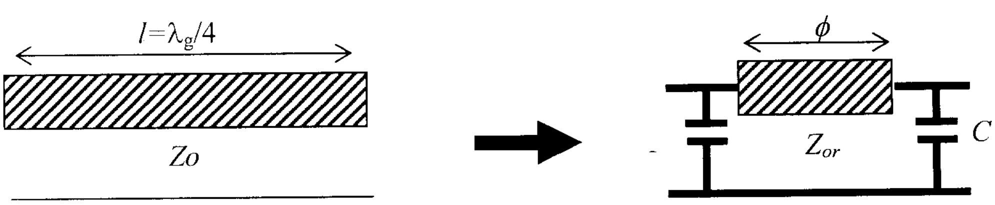


All the previous distributed-element couplers can be transformed into equivalent lumped-element couplers by simply replacing all the $\lambda_g/4$ lengths of transmission lines with the above π-network. Since lumped-element components have a lower Q-factor, when compared to distributed-element components, there is an insertion loss penalty. Also, because this π-network is clearly a low-pass filter, having a cut-off frequency, $f_c = \frac{1}{2\pi\sqrt{LC}}$, there is also a bandwidth penalty.

L = 4.0 nH and C = 1.6 pF for the Zo = 75 Ω sections of line
L = 2.8 nH and C = 1.0 pF for the Zo = 53 Ω sections of line

Model answer to Q 2(c): Bookwork and Computed Example

- Lumped-Distributed Couplers

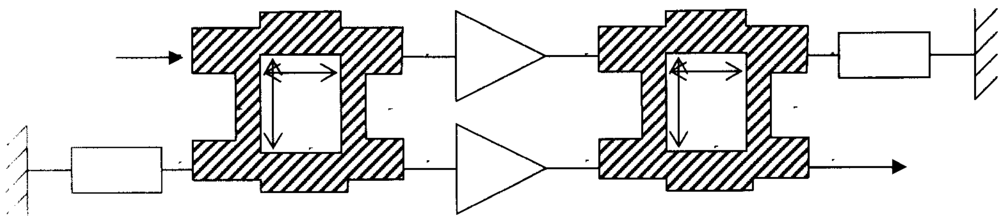


In this 'reduced-size' technique, each $\lambda_g/4$ line is replaced with the above π-network.

$$Z_{or} = \frac{Zo}{Sin\phi} \quad and \quad C = \frac{Cos\phi}{\omega Zo}$$

With ϕ = 30°,
Z_{or} = 150 Ω and C = 0.61 pF for the Zo = 75 Ω sections of line
Z_{or} = 106 Ω and C = 0.87 pF for the Zo = 53 Ω sections of line

Model answer to Q 2(d): Solution given in class



Model answer to Q 3(a): Bookwork

Power Loss in Guided-Wave Structures

Transmission lines are generally realised using both conductor and dielectric materials, both of which should be chosen to have low loss characteristics. Energy is lost by joule's heating, multi-modeing and leakage. The former is attributed to ohmic losses, associated with both the conductor and dielectric materials. The second is attributed to the generation of additional unwanted modes that propagate with the desired mode. The latter is attributed to leakage waves that either radiate within the substrate (e.g. dielectric modes and surface wave modes) or out of the substrate (e.g. free space radiation and box modes).

Model answer to Q 3(b): Bookwork and Computed Example

Multi-Modeing in Conductor-Backed CPW Lines

With an ideal CPW line, only the pure-CPW (quasi-TEM) mode is considered to propagate. In the case of a grounded-CPW (GCPW) line, where the backside metallization is at the same potential as the two upper-ground planes (through the use of through-substrate vias), a microstrip like mode can also co-exist with the pure-CPW mode. With the conductor-backed CPW (CBCPW) line, where the backside metallization has a floating potential, parallel-plate line (PPL) modes can also co-exist. The significant PPL modes that are associated with CBCPW lines include the fundamental TEM mode (designated TM_0) found at frequencies from DC to infinity and the higher order TM_n modes that can only be supported above their cut-off frequency, $f_{cn} = nc/(2h\sqrt{\varepsilon_r})$. By inserting a relatively thick dielectric layer (having a lower dielectric constant than that of the substrate), between the substrate and the lower ground plane, the pure CPW mode can be preserved. This is because the capacitance between the upper and lower conductors will be significantly reduced and, therefore, there will be less energy associated with the parasitic modes. Alternatively, the parallel-plate line modes can also be suppressed by reducing the width of the upper-ground planes, resulting in finite ground CPW (or FGC). Finally, in addition to all the modes mentioned so far, the slot-line mode can also propagate if there is insufficient use of air-bridges/underpasses to equalise the potentials at both the upper-ground planes.

PURE-CPW + SLOT-LINE + MICROSTRIP + PARALLEL-PLATE (TEM + TM_n)
|------------CPW------------|
|-----------------GCPW and FGC---------------|
|---CBCPW|--|

For TM1 and TM2, given a substrate having a height of 365 μm and dielectric constant of 9.8. the cut-off frequencies will be 75.5 and 150.9 GHz, respectively.

Model answer to Q 3(c): Bookwork and Computed Example

Lossy Dielectric, Lossless Conductor

Here, electromagnetic energy flows through the dielectric and not inside the conductor (which merely acts as a guide).

$$Q = \omega \cdot \frac{Peak\ Energy\ Stored\ in\ either\ the\ E-field\ or\ H-field}{Total\ Energy\ Lost\ per\ Second\ (Average\ Power\ Loss)}$$

$$Q = \frac{\beta}{2\alpha}, \quad where \quad \beta = \frac{2\pi}{\lambda g} \quad \therefore Q = \frac{\pi}{\alpha\lambda g}$$

$$where,\ attenuation\ per\ unit\ guided-wavelength = \alpha\lambda g \quad [Np/\lambda g]$$

$$\{Note\ that,\ Power\ Attenuation = 10\log_{10}\left(e^{-2\alpha\lambda g}\right) = -20\alpha\lambda g\log_{10}(e) = -8.686\,\alpha\lambda g \quad [dB/\lambda g]\}$$

$$For\ a\ low\ loss\ dispersionless\ dielectric: \quad \alpha = \frac{\sigma'}{2}\sqrt{\frac{\mu}{\varepsilon'}} = f(\omega) \quad and \quad \beta = \omega\sqrt{\mu\varepsilon'} = f(\omega)$$

$$therefore,\ \alpha\lambda g = \frac{\pi\sigma'}{\omega\varepsilon'} \neq f(\omega) \quad since\ \sigma' = \omega\varepsilon''\ and\ tan\delta = \frac{\varepsilon''}{\varepsilon'} \quad \therefore Q = \frac{\omega\varepsilon'}{\sigma'} \equiv \frac{1}{tan\delta} \neq f(\omega)$$

Q-factor = 1/tanδ = 5,000

Model answer to Q 3(d): Bookwork

Lossless Dielectric, Lossy Conductor

The losses in the conductor result from a flow of electromagnetic energy, from the dielectric into the conductor.

Surface Power Density, $P_S = |J_S|^2\ Rs \quad [W/m^2]$

where, J_S = surface current density [A/m]

$$now,\ Power\ Flux\ Density,\ P_D(z) = \text{Re}\{E_X(z)\ H_Y(z)^*\}$$

$$where\ \eta = \frac{E_X}{H_Y} \equiv Zs \quad \text{within a conductor}$$

$$\therefore P_D(z) = \text{Re}\left\{E_X(z)\left(\frac{E_X(z)}{Zs}\right)^*\right\} = \text{Re}\left\{\frac{|E_X(z)|^2}{Zs^*}\right\}$$

$$but,\ E_X(0) = ZsJs$$

$$\therefore P_D(0) = \text{Re}\left\{\frac{|ZsJs|^2}{Zs^*}\right\} = \text{Re}\left\{\frac{ZsZs^*|Js|^2}{Zs^*}\right\} = |J_S|^2\ Rs \equiv Ps \quad \text{Q.E.D.}$$

Model answer to Q4(a): New Derivation

$$z_{IN} = \frac{z + jz_{TX} \tan\vartheta}{z_{TX} + jz \tan\theta} \equiv z_o$$

$$\therefore Z_{TX}\left(Z + jZ_{TX} \tan\vartheta\right) = Zo(Z_{TX} + jZ \tan\vartheta)$$

$$\mathrm{Re}\{LHS\} \equiv \mathrm{Re}\{RHS\}$$

$$\therefore \theta = \tan^{-1}\left\{\frac{Z_{TX}(Zo - R)}{XZo}\right\}$$

$$\mathrm{Im}\{LHS\} \equiv \mathrm{Im}\{RHS\}$$

$$\tan\theta = \frac{Z_{TX}X}{ZoR - Z_{TX}^2} \equiv \frac{Z_{TX}(Zo - R)}{XZo}$$

$$\therefore Z_{TX} = \sqrt{ZoR - \frac{X^2 Zo}{Zo - R}}$$

Model answer to Q4(b): New Derivation

From the last expression in 4(a), the limits are:

$$R \neq Zo \quad and \quad X < \sqrt{R(Zo - R)}$$

Model answer to Q4(c): Computed Example

For a 5 pF capacitance in series with a 8 Ω resistance at 1800 MHz, the termination load impedance is Z = 8 – j17.684 Ω.

Using the expressions from 4(b), R is not equal to 50 Ω and X < 18.33 Ω, so both values are within the acceptable mathematical limits.

Using the expressions from 4(a), Z_{TX} = 5.26 Ω and ϑ = 14°.

Model answer to Q4(d): Bookwork

The value of Z_{TX} calculated in 4(c) would be considered very low in general. In practice, a conventional microstrip line could not be used to implement such a low impedance because the width of the signal line would be too wide. However, thin-film microstrip technology may be suitable as the widths of the lines are much narrower.

Model answer to Q 5(a): bookwork

E-fields can only meet a perfect conductor at normal incidence, while the H-field must be perfectly parallel.

Model answer to Q 5(b): bookwork

There must be m, n half wavelengths of fields across the a, b dimensions, respectively, so that the E-field components vanish at the guide walls.

The cut-off wavelength is given by:

$$k_c = \sqrt{k_x^2 + k_y^2} = \sqrt{\left(m\frac{\pi}{a}\right)^2 + \left(n\frac{\pi}{b}\right)^2} \equiv \frac{2\pi}{\lambda_c}$$

$$\therefore \lambda_c = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}}$$

For example, with the TE_{10} mode, $m = 1$ and $n = 0$:

Model answer to Q 5(c): computed example

For $a = 0.002$ m and $b = 0.001$ m:

$$f_c|_{TE10} = \frac{c}{2a} = 75\ GHz$$

$$f_c|_{TE01} = \frac{c}{2b} = 150\ GHz$$

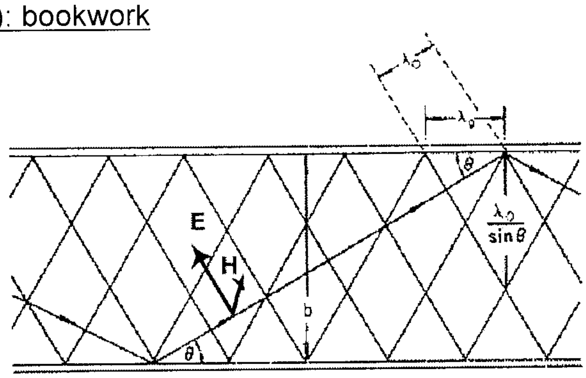
$$f_c|_{TE11} = f_c|_{TM11} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 167.7\ GHz$$

$$f_c|_{TE20} = 2f_c|_{TE10} = 150\ GHz$$

Model answer to Q 5(d): bookwork

TE_{11} and TM_{11} are the degenerate modes as they have different field patterns but share the same guided wavelength at any frequency (i.e. they must, therefore, share the same cut-off frequency). They are 'phase matched' over long distances and so mode conversion happens easily at discontinuities.

Model answer to Q 5(e): bookwork



The above figure shows the side-view of a generic parallel-plate waveguide, supporting two plane-waves propagating between and reflecting off the wall plates. Propagation within the waveguide can only be maintained as long as $\theta < \pi/2$:

$$\sin\theta = \frac{n\lambda_o/2}{b} = \frac{\lambda_o}{\lambda_c} < 1 \quad since \quad \lambda_c = \frac{2b}{n}$$ i.e. the n^{th} mode cut-off wavelength (e.g. $n = 3$ above)

with λ_0 = free space wavelength and n = the mode index.

The guided wavelength is shown in the above figure, and is given by:

$$\cos\theta = \frac{n\lambda_o/2}{n\lambda_g/2} = \frac{\lambda_o}{\lambda_g} < 1$$

$$\lambda_g = \frac{\lambda_o}{\cos\theta}$$

$$but.\ (\sin^2\theta + \cos^2\theta) \equiv 1$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{\lambda_o}{2b/n}\right)^2} = \sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}$$

$$\lambda_g = \frac{\lambda_o}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}} > \lambda_o$$

The guided-wave pattern travels at the phase velocity, $v_p = \frac{\omega}{\beta} = \frac{c}{\cos\theta} > c$, while the energy travels at the group velocity $v_g = \frac{\partial\omega}{\partial\beta} = c\cos\theta < c$. The product of these two velocities is $v_p v_g = c^2$.

Model answer to Q 5(f): computed example

$$f_c\big|_{TE10} = 75\,GHz \quad and \quad f_c\big|_{TE20} = 150\,GHz$$

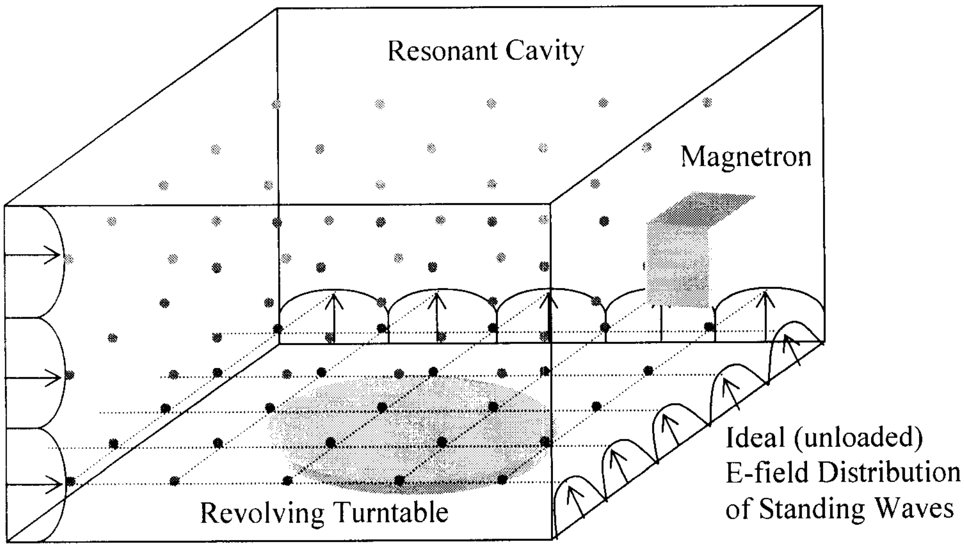
$$f\big|_{midband} = \frac{f_c\big|_{TE10} + f_c\big|_{TE20}}{2} = 112.5\,GHz$$

$$v_p\left(f\big|_{midband}\right) = 4.02x10^8\ m/s$$

$$v_g\left(f\big|_{midband}\right) = 2.24x10^8\ m/s$$

$$v_p\left(f\big|_{midband}\right)v_g\left(f\big|_{midband}\right) = 9x10^{16}\ m^2/s^2$$

Model answer to Q 6(a): Bookwork and Computed Example



At 2.45 GHz the wavelength in free space is 122.5 mm. Therefore, there are approximately 6 x 6 x 6 = 144 points of E-field maxima (not shown above).

The E-field distribution illustrated above will be completely altered at all points other than at the cavity walls. Because there are both E-field maxima and minima distributed within the cavity, there will be localised hot-spots and cold-spots in any food. With time, thermal conduction will help to even out these hot and cold spots. In an attempt to avoid hot and sold spots, a turntable below the food or metallic stirring fan above the food "sees" a time and space variant E-field distribution.

Model answer to Q 6(b): Computed Example

The peak E-field intensity is E_{pk} = 200 V/m. For meat having ε_r' = 38 and tanδ = 0.4.

$\sigma = \omega\varepsilon_o\varepsilon_r' \tan\delta = 2.07$ S/m

Power absorbed per unit volume, $P_v = \sigma E_{RMS}^2|_{dielectric\ interface} \sim 41.4$ kW/m^3

Model answer to Q 6(c): Computed Example

Power dissipated (i.e. absorbed) within the food, $P = \dfrac{mS_p\Delta T}{t}\ [W]$

m = mass [g]
S_p = specific heat capacity [kJ/kg K]
ΔT = raise in temperature [K or °C]
t = time [s]

- Power dissipated per unit volume, $P_V = \dfrac{\rho S_p \Delta T}{t}\left[W/m^3\right]$

ρ = mass/volume [g/m^3]

If the meat has dimensions of 20 x 20 x 20 cm^3 then using the result from 6(b):

P_v = 41.4 kW/m^3 and volume = 8 x 10^{-3} m^3, $\therefore P = P_v \times volume = 331.2$ W

With m = 1.0 kg, S_p = 5 kJ/kg K, ΔT = 100°C

$$\therefore\ t = \frac{mS_p\Delta T}{P} = \frac{1000 \times 5 \times 100}{331.2} = 1510\,s \cong 25 \text{ minutes}$$

Model answer to Q 6(d): Computed Example

Propagation constant, $\gamma = \alpha + j\beta \equiv jk$ where $k = \omega\sqrt{\mu\varepsilon}$

$$\therefore\ \gamma = j\omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} \text{ where } \varepsilon'' = \frac{\sigma}{\omega}$$

$$\gamma = j\omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\sigma}{\omega\varepsilon'}\right)^{\frac{1}{2}}$$

$$\gamma \approx j\omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\sigma}{2\omega\varepsilon'}\right)$$

$$\alpha \approx \omega\sqrt{\mu\varepsilon'}\frac{\sigma}{2\omega\varepsilon'} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon'}}$$

$$\therefore\ \alpha \approx \frac{\sigma}{2}\left(\frac{120\pi}{\sqrt{\varepsilon_r'}}\right)$$

But, skin depth, $\delta = \frac{1}{\alpha}$

With σ = 2.07 S/m and ε_r' = 38: δ = 1.58 cm. At this depth the power will have fallen to $1/e^2$ or 14% of its surface value. Since the thickness of the meat is approximately 10 skin depths thick, the meat should be cubed and separated so that it can cook more uniformly (e.g. 1.5 inches width/length/height dimensions almost correspond to 2 skin depths).