IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2006

EEE PART III/IV: MEng, BEng and ACGI

ELECTRICAL ENERGY SYSTEMS

Wednesday, 3 May 10:00 am

Time allowed: 3:00 hours

Corrected Copy
Q3

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Please use separate answer books for Sections A and B.

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	C.A. Hernandez-Aramburo, B.C. Pal
	Second Marker(s) :	G. Strbac, G. Strbac

Section A

1. A three-phase transmission line is connected to an 115kV source. It has a length of 200km and the following characteristics per unit length:

$y_C = j0.333\times10^{-5}$ [S/km]

$X_L = 0.5$ [Ω/km]

$R = 0.11$ [Ω/km]

Use the appropriate model (short, medium-length or long line model) to solve the following problems.

a) Calculate the total series impedance and the total shunt admittance per phase for this line

[3]

b) If there is no load connected to the receiving end of the line, calculate the following:

i) the line voltage at the receiving end

[4]

ii) the current delivered by the source per phase

[4]

iii) the total reactive power at the sending end

[5]

iv) the total losses (I^2R) in the line

[4]

2. Solve the following problems regarding synchronous machines and power transformers

a) For a synchronous generator:

i) explain what power factor control is and what it is useful for

[3]

ii) with the aid of a phasor diagram, explain how power factor control can be achieved.

[6]

b) A 50Hz, three-phase, star-star, transformer is rated at 1300MVA, 24.5kV/345kV. Its equivalent impedance is exclusively reactive with a value of 11.5%. If the high-voltage side of this transformer delivers 810MVA at 370kV with a 0.9 lagging power factor, calculate the following:

i) the per-unit phase current at the high voltage side

[6]

ii) the line voltage across the low-voltage side (express your results in volts and degrees)

[5]

3. The one-line diagram of a three-phase system is shown below. In this system the following parameters are known:

Bus 1 has a voltage of $(400\angle 0^{\circ})$ kV.

Bus 2 has a voltage of $(480\angle -16^{\circ})$ kV.

The load at bus 3 is S_3=110+j20 MVA.

The impedances of the two transmission lines are shown in the diagram.

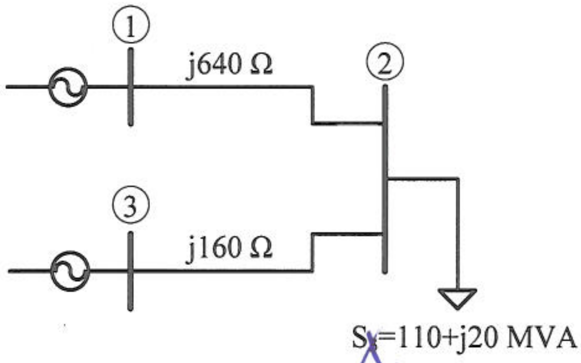


Figure 3.1

Using a per-unit system with a voltage base of 400kV and a power base of 100MVA, calculate the following:

a) the per-unit impedances of the lines [4]

b) the current through each line (in per-unit values) [6]

c) the voltage at bus 3 (in per-unit value) [4]

d) the power at buses 1 and 3 (in MVA) [6]

Section B

4. (a) Why do fuel cost characteristics in fossil-fuel units have minimum and maximum limits? [4]

(b) A power company has a mixed portfolio of hydro, nuclear, coal and gas based power generation units. Which units in your opinion would be committed for base load operation and why? [4]

(c) A power company has two units that can be engaged on economic dispatch. The operating costs of these units are given by

$$C_1 = 350 + 8.5P_1 + 0.004P_1^2 \quad \$/hr \qquad 300\ MW \leq P_1 \leq 1200\ MW \qquad (4.1)$$
$$C_2 = 400 + 7.5P_2 + 0.008P_2^2 \quad \$/hr \qquad 400\ MW \leq P_2 \leq 1000\ MW \qquad (4.2)$$

where P_1 and P_2 are in MW.

Determine the power output of each unit and the incremental operating cost(s), if the system is to operate at minimum cost to meet a load demand of 1000 MW. Transmission losses are neglected. [12]

5. (a) What is usually meant by *system, service and utilization voltage*? [5]

(b) Why is the low voltage problem at higher loading is more severe in an uncompensated over-head transmission system than in an underground cable system of same voltage rating? [3]

(c) Figure 5.1 shows an 11 kV power generating station feeds a load of 50 MW at 0.9 pf lagging through a radial transmission system. The line is connected at two ends by two 50 MVA transformers 11kV/132 kV and 132kV/33 kV. The percentage leakage reactance of each the transformers is 12.5%. The line has resistance and reactance of 5 Ω and 40 Ω respectively. Find the required generation voltage and angle when the load is served at 30 kV. [12]

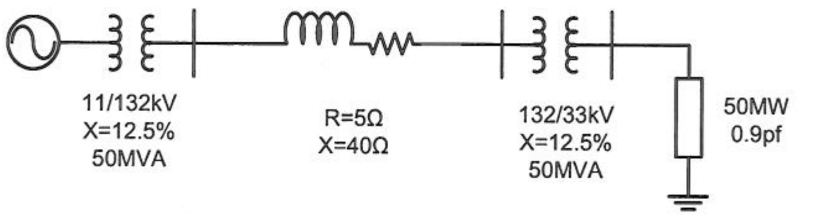


Figure 5.1: A simple power system

6. (a) Explain why zero sequence currents do not flow for faults not involving ground. [3]

(b) A synchronous generator usually has three reactances; *steady state, transient* and *subtransient*. Which one should be considered for fault calculation and why? [5]

(c) Figure 6.1 shows a schematic representation of a faulted power system. A single line to ground fault through a fault impedance of Z_F occurs at *phase 'a'*. Apply the method of *symmetrical components* to show that the three sequence networks need to be connected in series at the fault point to compute the fault current. [6]

(d) Find an expression for the fault current in all three phases. Neglect the pre-fault load current and assume the Thevenin's equivalent of the positive sequence generation voltage at the fault point V_F and Z_1 Z_2 and Z_0 are respectively the positive, negative and zero sequence impedance of the network. [6]

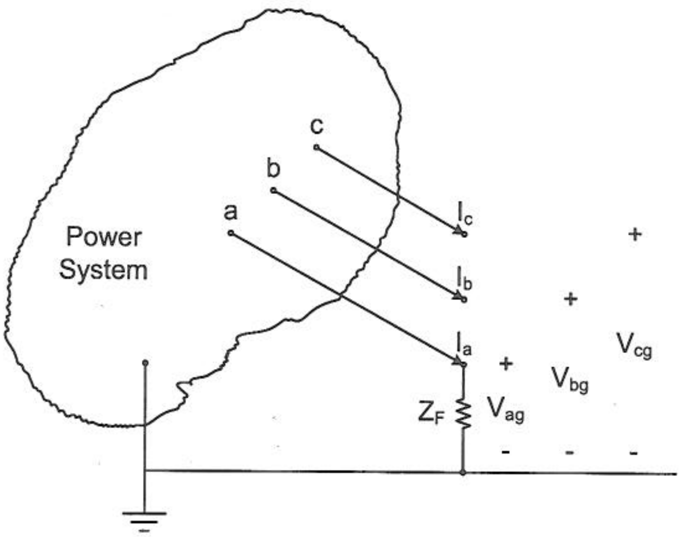


Figure 6.1: Schematic representation of a faulted system

E3.13

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

Department of Electrical and Electronic Engineering

Examinations 2006

MODEL ANSWERS

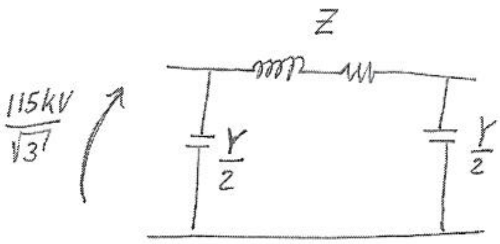
E 3.13 Electric Energy Systems:

Author: Dr C A Hernandez and Dr B. Pal

E3.13 Part A - Answers

1. **A three-phase transmission line is connected to a 115kV source. It has a length of 200km and the following characteristics per unit length: y_C=j0.333×10^{-5} [S/km], X_L=0.5 [Ω/km] and R=0.11 [Ω/km]. Use the appropriate model (short, medium-length or long line model) to solve the following problems.**

   a) **Calculate the total series impedance and the total shunt admittance per phase for this line**



$$Z = (R + jX_L) \times length$$
$$= \left(0.11 \frac{\Omega}{Km} + j0.5 \frac{\Omega}{Km}\right) 200\,km$$
$$= 22 + j100\ [\Omega]$$

$$Y = Y_c \times length$$
$$= j\,0.333 \times 10^{-5} \frac{S}{km} \times 200\,km$$
$$= j\,0.666 \times 10^{-3}\ [S]$$

[Simple numerical problem to get the student started]
[3 marks]

   b) **If there is no load connected to the receiving end of the line, calculate the following:**

      i) **The line voltage at the receiving end**

From a medium-length transmission line, we have the following model:

$$V_s = \left[1 + \frac{YZ}{2}\right] V_R + Z I_R \quad (I_R = 0 \text{ for an open-end transmission line})$$

then

$$V_R = \frac{V_s}{1 + \frac{YZ}{2}} = \frac{\frac{115\,kV}{\sqrt{3}}}{1 + \frac{(j0.666\times10^{-3})(22+j100)}{2}} = \frac{66.395\,kV}{0.9667 + j0.0073} =$$

$$V_R = 68.68 \angle -0.4342^\circ \, kV$$

$$V_{R\,line} = \sqrt{3}\, V_R = 118.96 \angle -0.4342^\circ$$

[Variation of a classroom problem]
[4 marks]

ii) The current delivered by the source per phase

Again, from the medium-length model

$$I_s = V_R \left[Y\left(1 + \frac{YZ}{4}\right)\right]$$

$$= (68.68 \angle -0.4342^\circ)\left[-2.4396\times10^{-6} + j6.549\times10^{-4}\right]$$

$$= 0.1733 + j44.9796$$

$$= 44.98 \angle 89.77^\circ$$

[Variation of a classroom problem]
[4 marks]

iii) The total reactive power at the sending end

Reactive power at the sending end. Calculate **S** first:

$S = 3\, V_s I_s^*$

$= 3\,(66.395\ kV)(0.1733 - j\,44.9796)$

$= 34.522\,[kW] - j\,8.959\,[MVar]$

$Q = Im\{S\} = \underline{8.959\ MVar}$

[Variation of classroom problem]
[5 marks]

iv) The total joule losses (I^2R) in the line

Losses in the line

In this open-ended transmission line the losses of the line correspond to the active power delivered by the source; thus: $P = Re\{S\} = 34.522\ kW$

[Variation of classroom problem]
[4 marks]

2. **Solve the following problems regarding synchronous machines and power transformers**

a) **For a synchronous generator:**

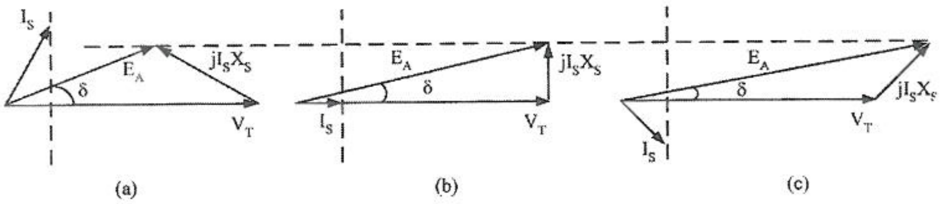
i) **Explain what power factor control is and what it is useful for.**

Power factor control

Power factor control means, in essence, that a synchronous generator can absorb or deliver reactive power independently of the real power being exported. This is a very useful characteristic for providing reactive power support to power systems, which is an essential ancillary service for the transmission and distribution of electrical power. Synchronous machines installed in an industrial site can also provide local compensation for a low power factor (this application is not so common nowadays).

[Book reading, basic understanding]
[3 marks]

ii) **With the aid of a phasor diagram, explain how power factor control can be achieved.**



Power factor control in a synchronous generator:
(a) Under-excited, (b) normally excited and (c) over-excited conditions

Assume (just to facilitate the reading of the phasor diagrams) that the machine is connected to an infinite bus (V_T = constant) and that the stator winding resistance is negligible (R_S=0).

From the three figures above, it may be graphically established that by increasing the real component of E_A, the imaginary component of the current I_S is changed from positive to negative. This means that the current I_S (and consequently the power factor) is controlled from leading V_T to lagging V_T.

Mathematically, the power factor control can be explained with the following expression:

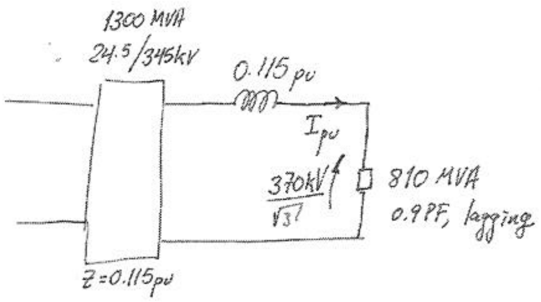
$$I_S = \frac{E_A - V_T}{jX_S} = \frac{1}{X_S}\left[E_A \sin(\delta) - j\left(E_A \cos(\delta) - V_T\right)\right]$$

The complex component of I_S can be made positive or negative according to the value of $E_A \cos(d)$ (recall that V_T was assumed constant).

[Book reading, intermediate level]
[6 marks]

b) **A 50Hz, three-phase, star-star, transformer is rated at 1300MVA, 24.5kV/345kV. Its equivalent impedance is exclusively reactive with a value of 11.5%. If the high-voltage side of this transformer delivers 810MVA at 370kV with a 0.9 lagging power factor, calculate the following:**

i) **the per-unit phase current at the high voltage side**



Solving the problem on a per-phase basis.

$$S_\varphi = \frac{S_{3\varphi}}{3} = \frac{810}{3}\left(\cos\left(\cos^{-1}PF\right) + j\sin\left(\cos^{-1}PF\right)\right)$$

$$= \frac{810}{3}\left[0.9 + j\,0.4359\right] = 270\left(0.9 + j0.4359\right)$$

$$= 243 + j\,117.69 \text{ MVA}$$

$$V_\varphi = \frac{V_L}{\sqrt{3}} = 213.6196 \text{ kV}$$

$$I_\varphi = \frac{S_\varphi^*}{V_\varphi^*} = \frac{243 - j117.69 \text{ MVA}}{213.6196 \text{ KV}} = 1137.5 - 550.93\ [A]$$

$$I_{base} = \frac{S_{\varphi\,base}}{V_{\varphi\,base}} = \frac{\frac{1300}{3}\times10^6}{\frac{345}{\sqrt{3}}\times10^3} = \frac{433.333\times10^6}{199.1858\times10^3} = 2175.5\ [A]$$

$$I_{\varphi_{pu}} = \frac{I_\varphi}{I_{base}} = \frac{1137.5 - j550.93}{2175.5} = 0.5229 - j0.2532 = 0.581\angle -25.8425^\circ$$

[Numerical application of classroom theory]
[6 marks]

ii) **the line voltage across the low-voltage side (express your results in volts and degrees)**

$$V_{\phi_{pu}} = \frac{V_\phi}{V_{base}} = \frac{213.6196\,kV}{\frac{345}{\sqrt{3}}\,kV} = 1.0725$$

$$V_{sec_{pu}} = V_{\phi_{pu}} + I_{\phi_{pu}} Z_{pu} = 1.0725 + (0.5229 - j0.2532)(j0.115)$$

$$= 1.0725 + (0.0291 + j0.0601)$$

$$= 1.1016 + j0.0601 = 1.1033 \angle 3.1243^\circ$$

$$V_{prim_{pu}} = V_{sec_{pu}}$$

$$V_{prim} = V_{prim_{pu}} \cdot V_{base_{prim}} = (1.1033 \angle 3.1243^\circ)(24.5\,kV)$$

$$= 27.03 \angle 3.1243^\circ$$

[Numerical example of classroom theory]
[5 marks]

3. **The one-line diagram of a three-phase system is shown below. In this system the following parameters are known:**

Bus 1 has a voltage of (400∟0°) kV

Bus 2 has a voltage of (480∟-16°) kV

The load at bus 3 is S_3=110+j20 MVA

The impedances of the two transmission lines shown in the diagram are given in ohms

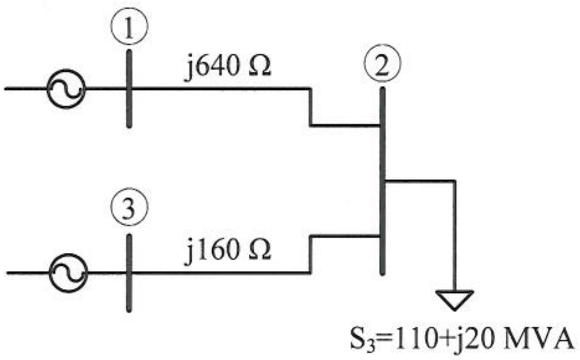


Figure 3.1

Using a per-unit system with a voltage base of 400kV and a power base of 100MVA, calculate the following:

a) The per-unit impedances of the lines

$S_{base} = 100\ MVA$

$V_{base} = 400\ kV$

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(400\ kV)^2}{(100\ MVA)} = 1600\Omega$$

$$Z_{12pu} = \frac{Z_{12}}{Z_{base}} = \frac{j640\Omega}{1600\Omega} = j0.4$$

$$Z_{23pu} = \frac{Z_{23}}{Z_{base}} = \frac{j160\Omega}{1600\Omega} = j0.1$$

[Simple numerical problem to get the student started. Basic knowledge of per-unit systems]
[4 marks]

b) The current through each line (in per-unit values)

Converting given data into pu. values:

$S_{2pu} = 1.1 + j0.2 = 1.118 \angle 10.3048^\circ$

$V_{2pu} = 1.2 \angle -16^\circ = 1.1535 - j0.3308$

$$I_{2pu} = \frac{S_2^*}{V_{2pu}^*} = \frac{1.118 \angle -10.3048^\circ}{1.2 \angle 16^\circ} = 0.9317 \angle -26.3048^\circ = 0.8352 - j0.4129$$

$$I_{12pu} = \frac{V_{1pu} - V_{2pu}}{Z_{12pu}} = \frac{1 - (1.1535 - j0.3308)}{j0.4} = 0.8269 + j0.3838$$

$I_{21pu} = -0.8269 - j0.3838$

$I_{23pu} = -I_{2pu} - I_{21pu} = -(0.8352 - j0.4129) + (0.8269 + j0.3838) = -0.0083 + j0.7967$

$I_{32pu} = -I_{23pu} = 0.0083 - j0.7967$

[Use of per-unit systems, numerical problem]
[6 marks]

c) The voltage at bus 3 (in per-unit value)

$$V_{3pu} = +I_{32pu} Z_{23pu} + V_{2pu} = (0.0083 - j0.7967)(j0.1) + (1.1535 - j0.3308)$$
$$= 1.2332 - j0.3299$$
$$= 1.2766 \angle -14.97$$

[Use of per-unit systems, numerical problem]
[4 marks]

d) The power at buses 1 and 3 (in MVA)

$$S_{12_{pu}} = V_{1pu} I_{12_{pu}}^* = (1)(0.8269 + j0.3838)^*$$
$$= 0.8269 - j0.3838 \quad S_1 = 82.69 - j38.38 \text{ MVA}$$

$$S_{32_{pu}} = V_{3pu} I_{32_{pu}}^*$$
$$S_{32_{pu}} = (1.2332 - j0.3299)(0.0083 - j0.7967)^*$$
$$= 0.2731 + j0.9797 \quad S_3 = 27.31 + j97.97 \text{ MVA}$$

[Fundamentals of power the "more-complex" flow problem]
[6 marks]

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Answers - Part II

E3.13

Problem 4.

(a) (**Bookwork**) The minimum load or output limitation of a thermal unit is generally caused by fuel combustion stability and inherent steam generator design constraint. For example most supercritical units cannot operate below 30 % of design capability. A minimum flow of 30 % is required to cool the tubes in the furnace of the steam generator adequately. The maximum limit is for obvious reason that the units are designed to their rated capacities with slight overloading margin. **[4marks]**

(b) (**Bookwork** The idea behind choosing base load units is influenced by two factors: economy and reliability. When enough water head is available (rainy season) hydro units must be put for base load if they can cover the total base load requirement. The next option is nuclear as it is always available given enough fuel stock for forseeable future. The fuel is costly though but from reliability point of view any short fall in base load allocation because of inadequate hydro capacity, it is an option. Coal and gas based power generation should always be used for meeting peak power requirements. The operating cost varies depending upon the fuel cost in the international market. They have some advantage to be used as peak units because of their fast start up and response time. However the best startegy for base load allocation must reflect the business interest and technical constraints of the company. **[4marks]**

(c) (**New computed example**) When both units are in operation, the condition of optimal operating cost is reached when incremental fuel costs for both of them are equal. In the absence of transmission loss, the total load equals total generation. i.e. at optimal point the following has to satisfy.

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \lambda \; (in \quad R\$/\text{MWhr}) \tag{0.1}$$

$$P_{load} = P_1 + P_2 \tag{0.2}$$

Now,

$$\frac{dC_1}{dP_1} = 8.5 + 0.008P_1 \tag{0.3}$$

$$\frac{dC_2}{dP_2} = 7.5 + 0.016P_2 \tag{0.4}$$

Upon substitution of (0.3) to (0.4) into (0.1) and carrying out necessary manipulations including (0.2) the following final form is obtained

$$0.008P_1 - \lambda = -8.5 \quad (0.5)$$
$$0.016P_2 - \lambda = -7.5 \quad (0.6)$$
$$P_1 + P_2 = 1000 \quad (0.7)$$

[7marks]

The solution is

$$P_1 = 625 \quad \text{MW} \quad (0.8)$$
$$P_2 = 375 \quad \text{MW} \quad \text{MW} \quad (0.9)$$
$$\lambda = 13.5 \quad \$/\text{MWhr} \quad (0.10)$$

[5marks]

Problem 5.

(a) (**Bookwork**) **System voltage**: The RMS phase-to-phase voltage of a portion of an AC electric system. It is widely referred in two values; nominal system voltage and maximum system voltage.
System voltage is further classified based on ANSI C84.1-1995)as: *Low voltage* (distribution or utilization level), ($< 1kV$) *Medium voltage* (distribution or utilization level) ($> 1kV but < 100kV$), *High voltage* (sub transmission or feeder level utilization level, usually not owned by transmission company) ($> 100kV but < 230kV$), *Extra high voltage* (grid voltage or transmission voltage) ($> 230kV < 765kV$) and *Ultra high voltage* (grid or transmission voltage) ($> 765kV, e.g. 1100kV$) **[3marks]**
Service voltage: The voltage at the point where the electrical system of the supplier and the electrical system of the user are connected. **[1marks]**
Utilization voltage The voltage at the terminals of utilization equipment. **[1marks]**

(b) (**Bookwork**) The reactive power generated by 132kV cable system is larger in amount than that by overhead line of same rating. The primary reason is higher cable conductor to sheath capacitance. As a result reactive requirement of the load and series line inductance are appreciably met by the cable system and it need not be transported from sending end. This obviously improves the voltage profile. In overhead case reactive power consumed by line series inductance is far more higher than that genearted by shunt capacitance; particularly at higher loading. The consequence is a poor voltage in the line. **[3marks]**

(c) (**New computed example**) Solution steps:
Step 1: Select a base MVA: 100
Step 2: Base currents at different voltage levels:
437.4 A, 1749.6 A Step 3: Base impedance
174 ohms at 132 kV **[4marks]**
Step 4: Calculate impedance in pu.
Line: (0.0287 +j0.2296) p.u Tx: j0.25 and j0.25 pu

Step 5: Express load currents for in pu. 0.6111

[3marks]

Step 6: Apply KVL with load voltage at reference and find source voltage (pu) pu

Step 7: Find voltage: 13.05 kV at 19.37 degree. **[5marks]**

Problem 6.

(a) (**Bookwork**) The zero sequence voltage and current are co-phasor quantities. They can only exit in a three phase line if they are provided a return path. Usually various components are earthed. Any fault not involving ground does not offer any return path to the fault current. **[3marks]**

(b) (**Bookwork**) Depending on operating situation (steady-state, transient or sub-transient) the magnetic effects of the rotor winding forces (constant flux linkage theorem) the stator flux to follow different paths. Immediately after the fault inception, the field and damper winding action of the rotor do not allow stator flux to flow through them. This leave the stator flux to flow largely through air gap. The magnetic reluctance offered is very high. The effective inductance is low and known as subtransient reactance. The high fault current initially is limited by subtransient reactance. Eventually rotor windings allow stator flux to penetrate through rotor. This increases the reactance, (to transient and finally steady-state). The magnitude of fault current decreases gradually as a result. Circuit breaker has to carry fault current right from the inception of the fault. Since the maximum fault current has the impact on circuit breaker rating, it is the subtransient reactance that has to be considered for fault calculation.

[5marks]

(c) (**Theory**) The transformation matrix T relating phase variables with sequence component variables is given by

$$T = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{0.11}$$

where $a = 1\angle 120^0$. Referring to the figurein the problem, under the assumption of no pre-fault load currents $I_b = I_c = 0$ In view of this one can write the following

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \tag{0.12}$$

On simplification the values of $I_0 = I_1 = I_2$. This probably suggests that three sequence networks must carry same current (a clever student at this stage must think of series connection before going further) **[6marks]**
Voltage in phase 'a' at faulted point: $V_{ag} = I_a * Z_F = (I_0 + I_1 + I_2) * Z_F =$

$3 * I_1 * Z_F$ $V_0 + V_1 + V_2 = V_a = 3 * Z_F * I_F$ If the pre-fault positive sequence voltage is V_F it can be related to positive voltage at the faulted point as $V_F = V_1 + I_1 * Z_1$ The fault current $V_{ag} = (V_1 + V_2 + V_0) = V_F - I_1 * Z_1 + I_2 * Z_2 + I_0 * Z_0 = V_F - 1/3I_F(Z_0 + Z_1 + Z_2) = I_F * Z_F$. It can be simplified to $I_F = 3V_F/(Z_0 + Z_1 + Z_2 + 3Z_F)$. **[4marks]**
Since all the impedances are in series, the sequence network need to connect in series and close through three times the fault impedance as shown in Figure 6.1. (A very cleaver student is expected to use symmetrical component of phase 'a' current to mathematically show currents in other phases are zeros.) **[2marks]**

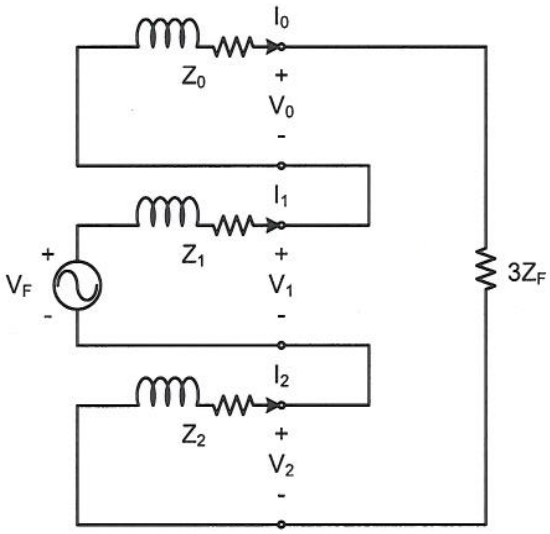

Figure 6.1: Interconnection of sequence network for single line to ground fault simulation