IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2004

EEE PART III/IV: M.Eng., B.Eng. and ACGI

ELECTRICAL ENERGY SYSTEMS

Wednesday, 5 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Corrected Copy

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	B.C. Pal, D. Popovic
	Second Marker(s) :	D. Popovic, B.C. Pal

[3.13]

1. The 138 kV 150 km three-phase transmission line has a series impedance $z = 0.17 + j0.79$ Ω/km and shunt admittance $y = j5 \times 10^{-6}$ mho/km.

   (a) Find the characteristic impedance Z_c, the propagation constant γ, the attenuation constant α, and the phase constant β. [6]

   (b) The line is delivering 15 MW at 132 kV at a 100% power factor. Find the sending-end voltage and current, the power angle θ_{12} (i.e., $\angle V_{1a} - \angle V_{2a}$), and the transmission efficiency $\eta = -P_{21}/P_{12}$. Use the long-line model. [14]

[3.13]

2. Draw a per-unit impedance diagram for the system whose one-line diagram is shown in Figure 2.1. The three phase and line-line ratings are as follows:

   Generator G1: 50 MVA, 13.8 kV, $X = 0.15$ p.u.
   Generator G2: 20 MVA, 14.4 kV, $X = 0.15$ p.u.
   Motor M: 20 MVA, 14.4 kV, $X = 0.15$ p.u.
   Transformer T1: 60 MVA, 13.2 - 161 kV, $X = 0.1$ p.u.
   Transformer T2: 25 MVA, 13.2 - 161 kV, $X = 0.1$ p.u.
   Transformer T3: 25 MVA, 13.2 - 161 kV, $X = 0.1$ p.u.
   Line 1: $20 + j80\ \Omega$
   Line 2: $10 + j40\ \Omega$
   Line 3: $10 + j40\ \Omega$
   Load: $20 + j15$ MVA at 12.63 kV (assume parallel combination of real and reactive part of the load).

   Select a base of 100 MVA and 161 kV in the transmission line. [20]

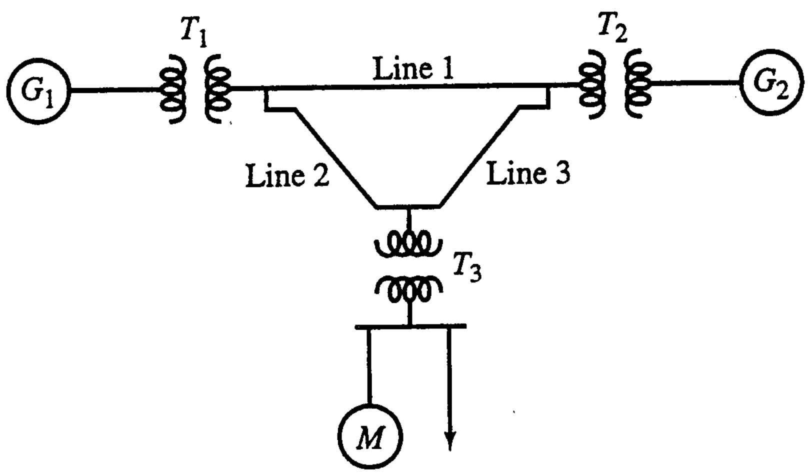


Figure 2.1

3. For the three-bus system shown in Figure 3.1, generator G1 is chosen as swing generator. Bus 2 is a generator bus and bus 3 is a load bus. The line reactances are given as: $X_{12} = 0.2$ p.u., $X_{13} = 0.25$ p.u., $X_{23} = 0.4$ p.u.

   (a) Find the bus admittance matrix Y_{BUS}. [7]

   (b) For each bus k, specify the bus type and determine which of the variables V_k, δ_k, P_k and Q_k are input data and which are unknowns. [5]

   (c) Set up the linearized system of equations that are solved at each iteration of the Newton power flow solution method. Do not solve the system of equations. [8]

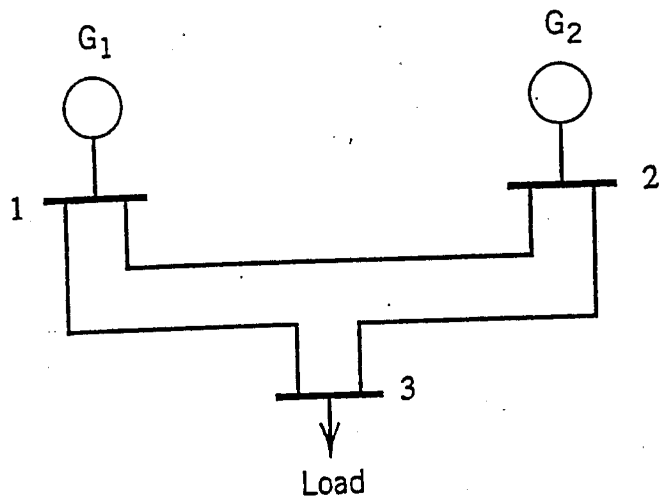


Figure 3.1

[3.13]

4. (a) What is Lagrange multiplier? What physical significance has it got in respect of unit commitment? [4]

(b) An area of an interconnected power system has three fossil-fuel units operating on economic dispatch. The operating costs of these units as function of output power are given by

$$C_1 = 612 + 8.64P_1 + 0.00174P_1^2 \quad R\$/hr \qquad 350 \leq P_1 \leq 1200 \qquad (4.1)$$
$$C_2 = 310 + 7.85P_2 + 0.00142P_2^2 \quad R\$/hr \qquad 300 \leq P_2 \leq 1000 \qquad (4.2)$$
$$C_3 = 196 + 8.77P_3 + 0.00530P_3^2 \quad R\$/hr \qquad 200 \leq P_3 \leq 800 \qquad (4.3)$$

where P_1, P_2 and P_3 are in MW and $R\$$ is symbol of an arbitrary currency. Determine the economic operating output for these three units when delivering a total of 2000 MW load. Ignore transmission losses. [16]

[3.13]

5. (a) What are the various methods of voltage control in power systems? [5]

(b) Discuss the importance of *'droop'* characteristic of turbine-governor for parallel operation? [5]

(c) Two synchronous generators A and B operate in parallel and supply a total load of 300 MW.The capacities of the generators are 150 MW and 200 MW respectively, both having 4% droop characteristic from no load to full load. Evaluate the frequency of operation and the load shared by each generator. [10]

6. (a) Why is fault current calculation so important in power system analysis? [4]

(b) Consider the one-line diagram of a simple power system shown in Figure 6.1. System data in per-unit (p.u.) on a 100 MVA base are given as follows:

Synchronous generators:

G1:	100 MVA	20 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.05$
G2:	100 MVA	20 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.05$

Transformers:

T1:	100 MVA	20/220 kV	$X_1 = X_2 = X_0 = 0.1$
T2:	100 MVA	20/220 kV	$X_1 = X_2 = X_0 = 0.1$

Transmission lines:

TL12:	100 MVA	220 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.30$
TL13:	100 MVA	220 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.30$
TL23:	100 MVA	220 kV	$X_1 = X_2 = 0.15$	$X_0 = 0.30$

The neutral of each generator is grounded through a current limiting reactor of 0.03 p.u. on 100 MVA base. All transformer neutrals are solidly grounded. The generators are operating at no-load with a voltage of 1.05 p.u. Determine fault current in p.u. for a single line to ground (SLG) fault in phase 'a' at bus location 3. [16]

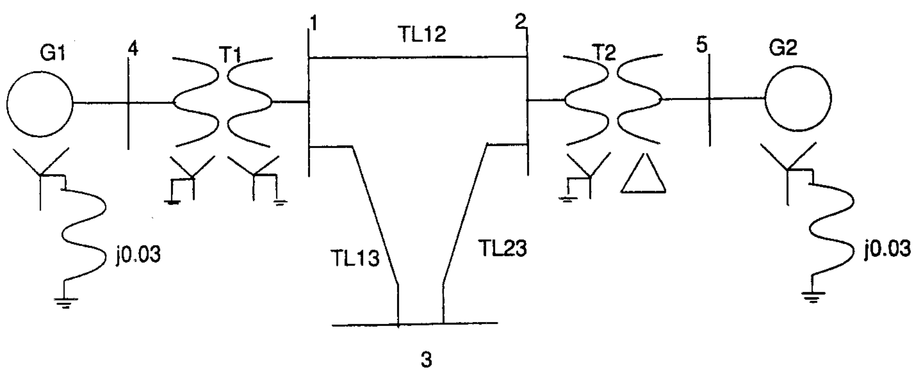


Figure 6.1: A simple power system

Question Number etc. in left margin — Mark allocation in right margin

1. (a) (bookwork / applied)

$z = 0.17 + j0.79 = 0.8081 \angle 77.86^\circ \ \Omega/km$

$y = j5.4 \cdot 10^{-6} = 5.4 \cdot 10^{-6} \angle 90^\circ$ 3

$Z_C = \sqrt{\frac{z}{y}} = 386.8 \angle -6.07^\circ$

$\gamma = 2.09 \cdot 10^{-3} \angle 83.9^\circ = \underbrace{0.221 \cdot 10^{-3}}_{\alpha} + j\underbrace{2.08 \cdot 10^{-3}}_{\beta} = \alpha + j\beta$ 3

(b) (application of theory)

$P_{load} = \frac{15 MW}{3} = 5 MW$ 2

$|V_2| = \frac{132}{\sqrt{3}} = 76.21 kV, \quad \angle V_2 = 0$ 2

$I_2 = \frac{5 \cdot 10^6}{76.21 \cdot 10^3} = 65.6 A \quad (pf = 1)$ 2

$V_1 = V_2 \cosh \gamma l + Z_C I_2 \sinh \gamma l$ 2

$I_1 = I_2 \cosh \gamma l + \frac{V_2}{Z_C} \sinh \gamma l$

$\gamma l = (0.221 \cdot 10^{-3} + j2.08 \cdot 10^{-3})\, 150 = 0.0331 + j0.3116$ 1

$\cosh \gamma l = 0.9524 \angle 0.6113^\circ$

$\sinh \gamma l = 0.3083 \angle 84.12^\circ$

$\Rightarrow$

$V_1 = 74.675 \angle 6.48^\circ \ kV$ 1

$I_1 = 87.46 \angle 44.6^\circ \ A$ 1

$\theta_{12} = 6.48^\circ - 0^\circ = 6.48^\circ$ 1

$P_{12} = Re\{V_1 I_1^*\} = 5.138 \ MW$ 1

$\eta = -\frac{P_{21}}{P_{12}} = \frac{5}{5.138} = 0.973$ 1

/20

2. (application of theory)

Base 100 MVA, 161 kV (TL)

G_1, G_2 and M all have the same base voltage = 13.2 kV 1

G_1: $X = 0.15 \frac{100}{50}\left(\frac{13.8}{13.2}\right)^2 = 0.3279$ pu 1

G_2: $X = 0.15 \frac{100}{20}\left(\frac{14.4}{13.2}\right)^2 = 0.8926$ pu 1

T_1: $X = 0.1 \frac{100}{60} = 0.1667$ pu 1

T_2: $X = 0.1 \frac{100}{25} = 0.4$ pu 1

Transmission lines:

$Z_{base} = \frac{161^2}{100} = 259.21\ \Omega$ 1

$Z_{line1} = \frac{20 + j80}{259.21} = 0.07716 + j0.3086$ pu 1

$Z_{line2} = Z_{line3} = \frac{10 + j40}{259.21} = 0.03858 + j0.1543$ pu 1

Motor M:

$X = 0.15 \frac{100}{20}\left(\frac{14.4}{13.2}\right)^2 = 0.8926$ pu 1

T_3: $X = 0.1 \frac{100}{25} = 0.4$ pu 1

Load: parallel combination of R & X

$S = V I^* = |V|^2 Y^* = |V|^2\left(\frac{1}{R} + j\frac{1}{X}\right)$

$R = \frac{|V|^2}{P} = \frac{12.63^2}{20} = 7.976$ 1

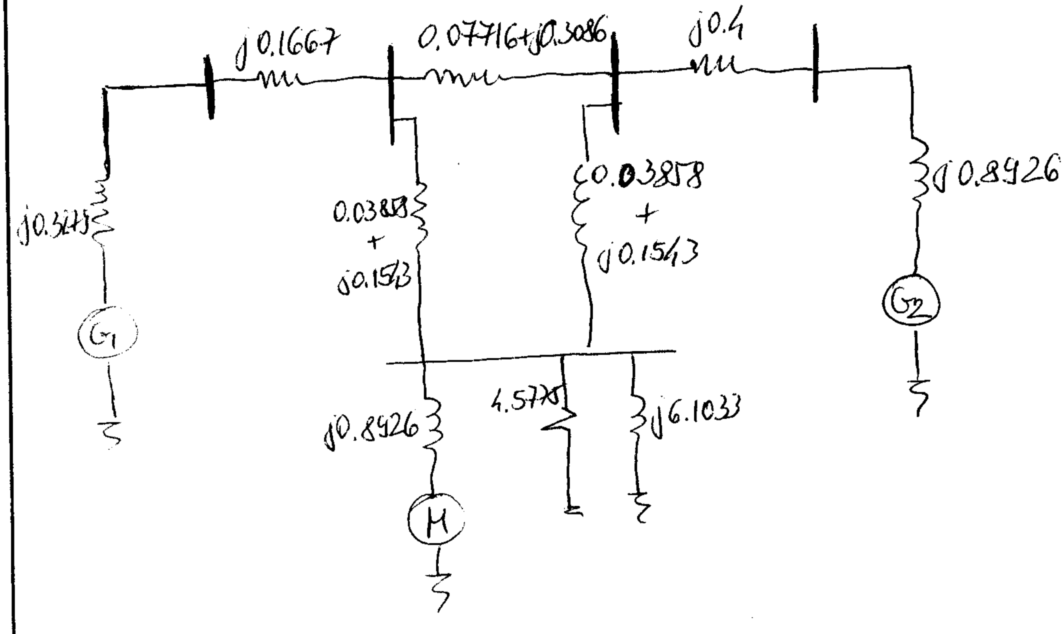
2 (cont)

$$X = \frac{|V|^2}{Q} = \frac{12.63^2}{15} = 10.635$$ 1

$$Z_{base} = \frac{V_b^2}{S_b} = \frac{(13.2 \cdot 10^3)^2}{100 \cdot 10^6} = 1.7424$$ 1

$R_{pu} = 4.5775$ 1

$X_{pu} = 6.1033$ 1



5

20

3. (a) (bookwork + application of theory) 1

$Y_{12} = Y_{21} = -\frac{1}{j0.2} = j5$ 1

$Y_{13} = Y_{31} = -\frac{1}{j0.25} = j4$ 1

$Y_{23} = Y_{32} = -\frac{1}{j0.4} = j2.5$ 1

$Y_{11} = -Y_{12} - Y_{13} = -j5 - j4 = -j9$ 1

$Y_{22} = -Y_{21} - Y_{23} = -j5 - j2.5 = -j7.5$ 1

$Y_{33} = -Y_{31} - Y_{32} = -j4 - j2.5 = -j6.5$ 1

$$Y_{bus} = \begin{bmatrix} -j9 & j5 & j4 \\ j5 & -j7.5 & j2.5 \\ j4 & j2.5 & -j6.5 \end{bmatrix}$$ 1

(b) (Bookwork)

Bus 1 : Swing bus ; input : δ_1, V_1 ; unknown: P_1, Q_1

Bus 2 : P-V (gen) ; input : P_2, V_2 ; unknown : δ_2, Q_2 5

Bus 3 : P-Q (load) ; input : P_3, Q_3 ; unknown : δ_3, V_3

(c) $\Delta P_2 = P_2^{calc} - P_2^{spec}$ 2 2

$\Delta P_3 = P_3^{calc} - P_3^{spec}$ 2

$\Delta Q_3 = Q_3^{calc} - Q_3^{spec}$

$$J \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 \end{bmatrix} = - \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}$$ 2

/20

Model Answers (Dr. B. Pal)

4 Solution

(a) (**Bookwork**) A multiplier associated with Lagrange function comprising of costs and constraints. Within the limits of operation of a unit, the Lagrange multiplier is nothing but incremental fuel cost. At maximum limit, the incremental fuel cost is less than or equal to Lagrange multiplier, at the minimum output limit, the incremental fuel cost is more than or equal to Lagrange multiplier. This indicates that at lower limit, cost of operation is not economic at upper limit it is economic but cannot be exceeded because of no over load limit available with steam turbine.
[4marks]

(b) (**New computed example**)When all of the three units are in operation, the condition of optimal operating cost is reached when incremental fuel costs for all the units are equal. In the absence of transmission loss, the total load equals total generation. i.e. at optimal point the following has to satisfy.

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \frac{dC_3}{dP_3} = \lambda \ (in \quad R\$/\text{MWhr}) \tag{4.1}$$

$$P_{load} = P_1 + P_2 + P_3 \tag{4.2}$$

[2marks]

Now,

$$\frac{dC_1}{dP_1} = 8.64 + 0.00348P_1 \tag{4.3}$$
$$\frac{dC_2}{dP_2} = 7.85 + 0.00284P_2 \tag{4.4}$$
$$\frac{dC_3}{dP_3} = 8.77 + 0.01060P_3 \tag{4.5}$$

Upon substitution of (4.3) to (4.4) into (4.1) and carrying out necessary manipulations including (4.2) the following final form is obtained

$$0.00348P_1 - \lambda = -8.64 \tag{4.6}$$
$$0.00284P_2 - \lambda = -7.85 \tag{4.7}$$
$$0.01060P_3 - \lambda = -8.77 \tag{4.8}$$
$$P_1 + P_2 + P_3 = 2000 \tag{4.9}$$

[9marks]

The solution is

$$P_1 = 687.6 \quad \text{MW} \tag{4.10}$$
$$P_2 = 1103.30 \quad \text{MW} \tag{4.11}$$
$$P_3 = 209.0 \quad \text{MW} \tag{4.12}$$
$$\lambda = 10.983 \quad \$/\text{MWhr} \tag{4.13}$$

It is seen that economic solution leads to violation of maximum limits in unit#2. Hence it has to be set to operate at its maximum limit 1000 MW and the rest of the load has to be shared economically by unit#1 and unit#3. The excercise is repeated to produce the final set of equations

$$0.00348P_1 - \lambda = -8.64 \tag{4.14}$$
$$0.01060P_3 - \lambda = -8.77 \tag{4.15}$$
$$P_1 + P_3 = 1000 \tag{4.16}$$

The solution is

$$P_1 = 765.84 \quad \text{MW} \tag{4.17}$$
$$P_3 = 234.16 \quad \text{MW} \tag{4.18}$$
$$\lambda = 11.25 \quad R\$/\text{MWhr} \tag{4.19}$$

It is interesting to note that the incremental cost of production in unit #1 and unit#3 are higher than that of unit #2 **[5marks]**

5 solution

(a) (**Bookwork**) The students are expected to write a couple of sentences on the following: Injection of reactive power (shunt capacitors/reactors, series capacitors, synchronous compensators), tap changing transformers, booster transformer, phase shifting transformer, conventional thyristor based Flexible AC transmission system (FACT) devices such as Static Var compensator (SVC) and Controllable Series Capacitor (CSC). **[5marks]**

(b) (**Bookwork**)The steady state characteristic between fuel (steam/water/gas) input to turbine power output in power-frequency plain exhibits negative slope. The closed loop control applied to governor system is such that it exhibits drop in frequency as the output power is increased. This is known as droop characteristic in a turbine governor control system. It is expressed as speed or frequency change to power output change. It has been customised in real time operation as percentage of frequency deviation to 100% change in power output.

For example, a 5% droop of an unit means, the frequency would change by 2.5 Hz in 50 Hz system when the valve/gate position of the machine causes 100% change in output i.e. from no load to full load. **[5marks]**

(c) (**New computed example**) The droop characteris of Gen A and Gen B are drawn from the data in problem 5.c and are shown in the Figure 5.1. **[4marks]**

Let the common frequency of operation is $f2$ and the power shared by

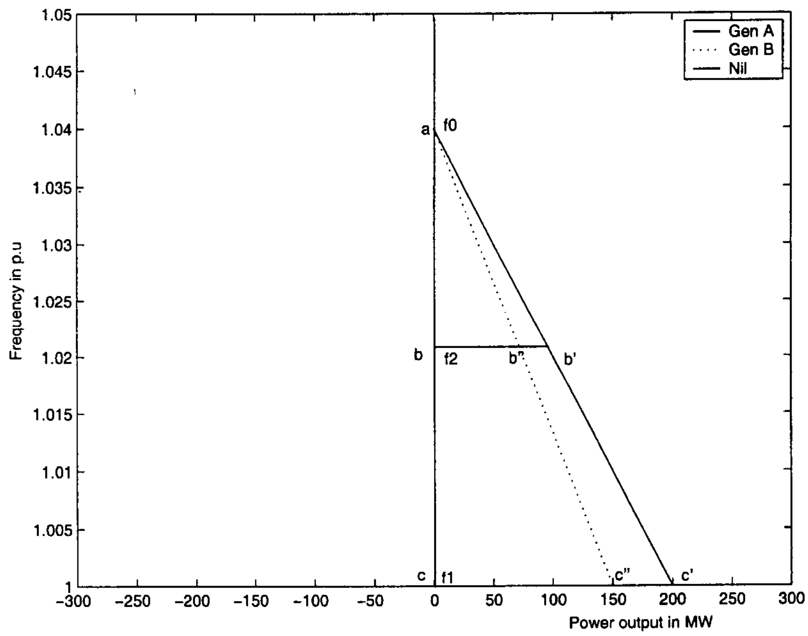


Figure 5.1: The droop characteristics of Gen A and Gen B

Gen A and Gen B are bb' and bb'' where $bb' + bb'' = 300$ when they share the total 300 MW load. Let $bb' = x$ and hence $bb'' = 300 - x$.
From the theory of identical triangles the following equations can be written:

$$\frac{x}{150} = \frac{bb''}{cc''} = \frac{f0 - f2}{f0 - f1} = \frac{1.04 - f2}{1.04 - 1.0} \tag{5.1}$$

$$\frac{300 - x}{200} = \frac{bb'}{cc'} = \frac{f0 - f2}{f0 - f1} = \frac{1.04 - f2}{1.04 - 1.0} \tag{5.2}$$

Solving (5.1)and (5.2) provides $f2 = 1.006$ pu and $x = 128.57$ MW. The share by Gen A = 171.42 MW and by Gen B = 128.58 MW. Alternatively this problem can be solved by forming two simultaneous equation with slope and intercept set from the data in the problem. **[6marks]**

6 solution

(a) (**Bookwork**)The main objectives of fault analysis are to

- (i) determine maximum and minimum three-phase short currents.
- (ii) to determine the unsymmetrical fault current for single (SLG) and double line-to-earth,(L-L-G), line to line faults (L-L) and sometimes for open circuit faults.
- (iii) investigation of the operation of protective relays
- (iv) determine the rated rapturing capacity of breakers.
- (v) determine fault-current distribution and busbar-voltage levels during fault conditions

So it is very important to have an assessment of fault current to decide rating and setting of protection systems. **[4marks]**

(b) (**New computed example**) The first task is to break this system into three sequence network. This is shown in Fig6.1 where the detail representation for each network is shown and then it is simplified with respect to fault point F.
The required star-delta transformation is adopted for network simplification. The poitive sequence network contains voltage sources of 1.05 p.u. voltage. the pre-fault voltage in the absence of load current is 1.05 p.u at fault location. The simplified total positive sequence reactance is 0.20 p.u. as shown in Figure 6.1. **[4marks]**
The negative sequence network is easy to obtain oncethe positive sequence network is obtained. This network would not have any voltage source as negative sequence voltage is not generated. since all the negative sequence reactance of generator, line and transformer are equal to respective positive sequence reactances as seen in the problem, the negative sequence network in its simplest form is similar to positive sequence network without pre-fault voltage at fault location F. **[2marks]**
It is interesting to see that the path for zero sequence current is open between bus #5 and #2 because of delta-star transformer T2. Hence the zero sequence network would be different. There would be no zero sequence component current coming from generator G2 because of this broken path. The simplified equivalent is a reactance of 0.226 p.u. with out any prefualt voltage because of obvious reason. **[5marks]**
For a single line to ground fault the three sequence networks needs to be connected in series through fault point. The positive sequence current is calculated from the following

$$I_{a1} = I_{a2} = I_{a0} = \frac{1.05\angle 0^0}{j0.20 + j0.20 + j0.226} = 1.677\angle{-90^0}\ p.u. \qquad (6.1)$$

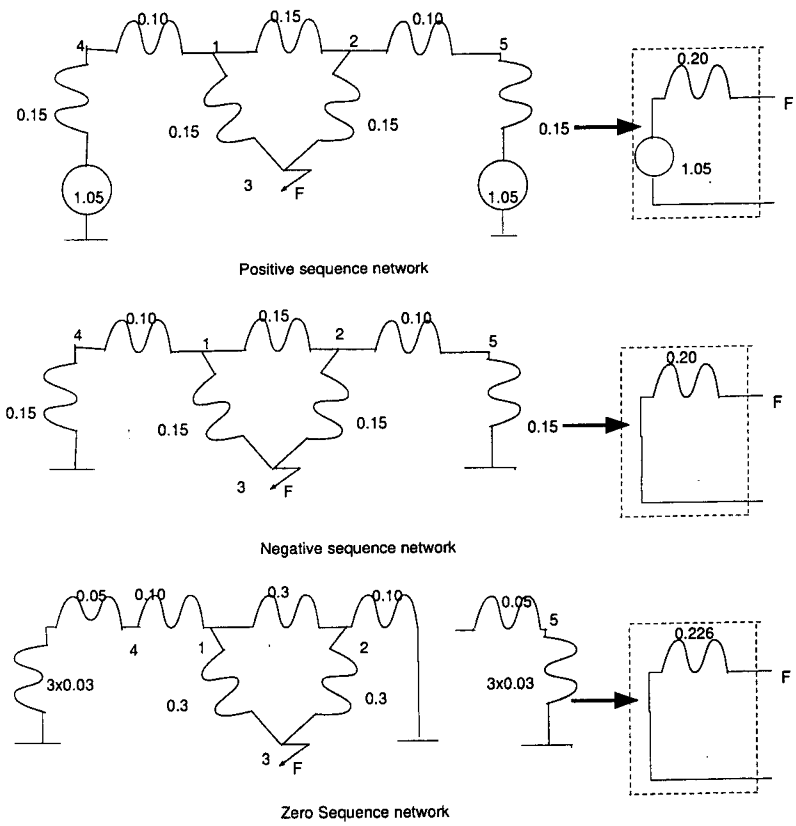


Figure 6.1: The sequence networks representation of the system

The total current I_a is sum of three component currents i.e. $I_{a1} + I_{a2} + I_{a3} = 3 * 1.677 = 5.0319\ p.u.$. The current lags the voltage by 90 degree as all the elements in the circuits are inductive. **[5marks]**