E3.12
SO13

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

MSc and EEE PART III/IV: MEng, BEng.and ACGI

Corrected Copy

OPTOELECTRONICS

Tuesday, 1 May 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	R.R.A. Syms
	Second Marker(s) :	W.T. Pike

Fundamental constants

$e = 1.6 \times 10^{-19}\ C$

$m_e = 9.1 \times 10^{-31}\ kg$

$\varepsilon_0 = 8.9 \times 10^{-12}\ F/m$

$\mu_0 = 4\pi \times 10^{-7}\ m\ kg/C^2$

$c = 3.0 \times 10^{8}\ m/s$

$h = 6.6 \times 10^{-34}\ Js$

1. Figure 1 shows a three-layer asymmetric slab waveguide of height h, formed from materials of refractive indices n_1, n_2 and n_3.

a) Sketch the transverse variation of the refractive index, explaining any limitation on the values of n_1, n_2 and n_3. Sketch ray paths for i) a guided mode, and ii) a substrate mode.

[7]

b) For single frequency fields in dielectric media, Maxwell's equations may be assumed in the form:

$\mathrm{div}(\varepsilon \underline{E}) = 0 \quad \mathrm{div}(\mu_0 \underline{H}) = 0 \quad \mathrm{curl}(\underline{E}) = -j\omega\mu_0 \underline{H} \quad \mathrm{curl}(\underline{H}) = j\omega\varepsilon_0\varepsilon_r \underline{E}$

What are all the boundary conditions that are satisfied by the electric field, for TE guided modes?

[6]

c) Describe and sketch the full range of modal solutions that can be found for this structure.

[7]

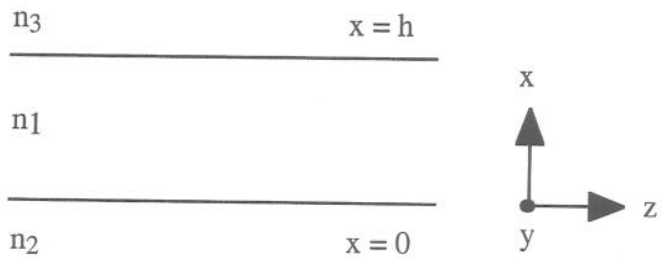


Figure 1.

2a) Explain briefly how graded index optical fibre is constructed.

[4]

b) A parabolic index fibre has the radial variation in refractive index shown in Figure 2. Propose and sketch a suitable approximation that may be used to model this variation. Write down the corresponding scalar waveguide equation in Cartesian co-ordinates, assuming that the fibre is oriented parallel to the z-direction. Show how the equation may be transformed into polar co-ordinates in the case of a radially symmetric transverse field. Show that the modal field $E = E_0 \exp(-r^2/a^2) \exp(-j\beta z)$ satisfies the equation, where r is the distance from the centre of the fibre, and find how a and β relate to the parameters of the fibre and wavelength.

[12]

c) A particular fibre has $n_0 = 1.46$ and $r_0 = 25\ \mu m$. Find the mode field diameter and the propagation constant when $\lambda = 1.55\ \mu m$.

[4]

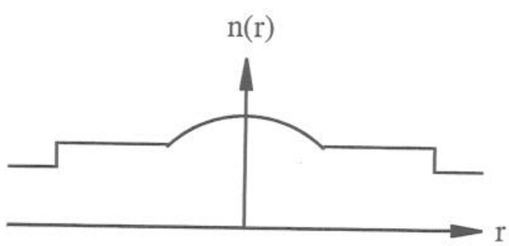


Figure 2.

3a) Sketch the transverse variation of refractive index in the coupling region of a step-index directional coupler. On the same scale, sketch the transverse variation of the modal fields. Explain the advantage of the directional coupler over a Y-junction.

[8]

b) The coupled mode equations for a synchronous coupler can be written as:

$$dA_1/dz + j\kappa A_2 = 0$$
$$dA_2/dz + j\kappa A_1 = 0$$

Here, A_1 and A_2 are the amplitudes of the modes in Guides 1 and 2, κ is the coupling coefficient, and z is the distance in the direction of propagation. Solve the equations, assuming that light is input to Guide 1. Show how the powers in the two guides may be obtained, and prove that power is conserved.

[8]

c) The coupling coefficient in a particular waveguide system is $\kappa = 0.157\ mm^{-1}$. How long should the coupler be, to achieve a 50 : 50 power split?

[4]

4a) Explain how channel waveguides are fabricated in $LiNbO_3$. What is the electro-optic effect, and why is it important?

[6]

b) Sketch two alternative arrangements of electrodes and waveguides used in a $LiNbO_3$ electro-optic phase modulator, and describe their application. Explain the limitations of lumped element electro-optic phase modulators, and suggest how these may be overcome.

[6]

c) Explain how phase modulators may be constructed in InP (which is electro-optic) and in Si (which is not).

[8]

5a) Explain in detail the operation of a thermo-optic Y-junction based Mach-Zehnder interferometer. Include in your answer a sketch of the device and its response characteristic.

[10]

b) Figure 2 below shows the layout of a fibre-pigtailed 2 x 2 silica-on-silicon matrix switch based on thermo-optic Mach-Zehnder interferometric gates.

A crosspoint connection is desired between Port 1 and Port 4. How should the MZI gates be configured? Neglecting fibre coupling and propagation losses, what is the insertion loss? If the transmission is less than 100%, where and how has the power been lost?

[6]

c) Repeat the calculation for a broadcast connection between Port 1 and both Ports 3 and 4 together.

[4]

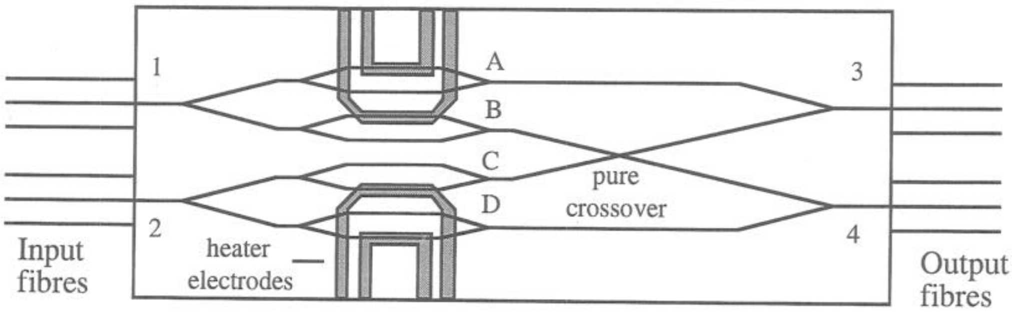


Figure 2.

6a) Explain the difference between the mechanism of light generation in light emitting diodes (LEDs) and in semiconductor lasers. Hence explain why laser diodes are the preferred source for high bit-rate optical communications, rather than LEDs.

[10]

b) The rate equations for a LED can be expressed as:

$$dn/dt = I/ev - n/\tau_e \qquad d\phi/dt = n/\tau_{rr} - \phi/\tau_p$$

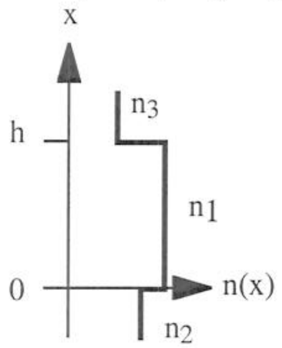
Use these equations to find the frequency dependence of the light modulation when the drive current is sinusoidally varied, as $I = I' + I'' \exp(j\omega t)$.

[10]

E3.12

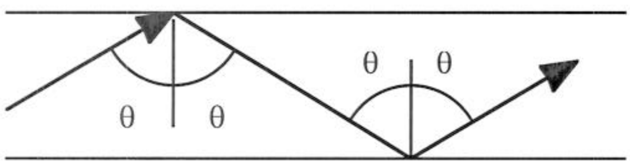
Optoelectronics 2007 - Solutions

1a) For total internal reflection at both interfaces of an asymmetric slab dielectric waveguide we require $n_1 > n_2 > n_3$. The refractive index variation must therefore be as shown below:



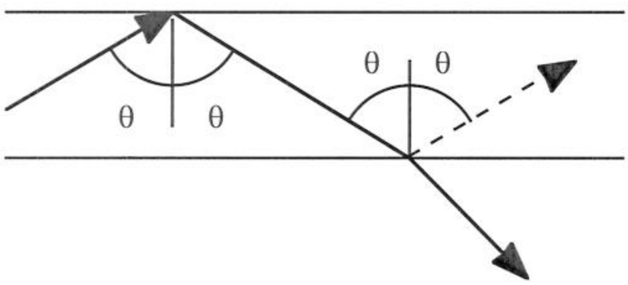
[3]

i) Guided mode rays are totally reflected at both the upper and the lower interface:



[2]

ii) Substrate modes have ray components that are transmitted into the substrate:



[2]

b) The boundary conditions that must be satisfied are continuity of the tangential components of the electric and magnetic fields at the two interfaces.

For TE incidence, the electric field is y-polarized for the geometry given, so $\underline{E} = E_y \underline{j}$. This field is wholly tangential, so E_y must match on x = 0 and x = h.

[2]

The magnetic field can be found from $\underline{H} = (j/\omega\mu_0)\,\text{curl}\,(\underline{E})$.
Evaluating curl ($\underline{E}$) we get:

$$\text{curl}\,(\underline{E}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}$$

Or curl ($\underline{E}$) = $-\partial E_y/\partial z\ \underline{i} + \partial E_y/\partial x\ \underline{k}$

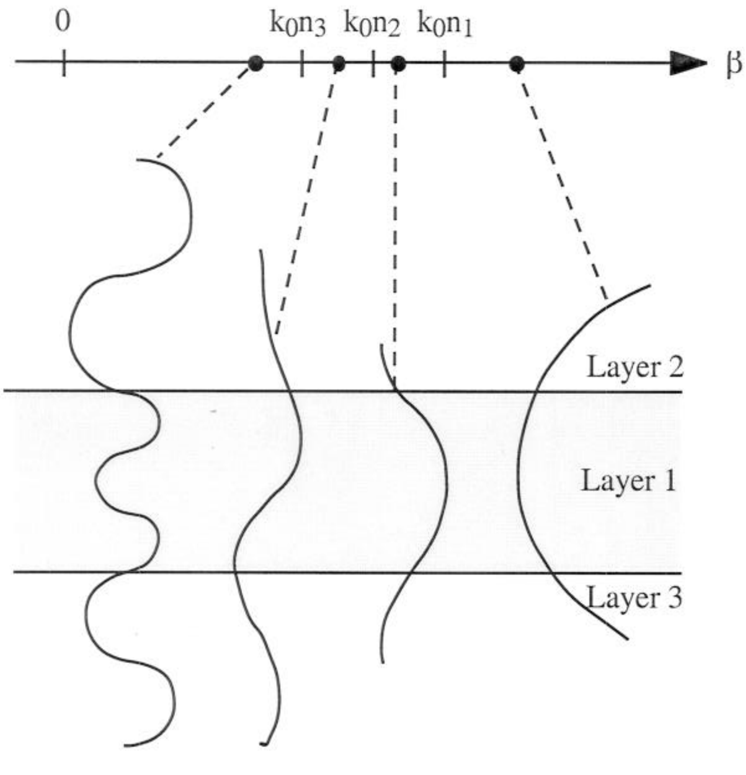
The k-component is tangential, so $\partial E_y/\partial x$ must match on x = 0 and x = h.

[4]

c) Referring to the figure below:

- For $\beta > k_0n_1$, the solutions are exponential in all three layers. This implies infinite field amplitudes at large distances from the guide, so these solutions are physically unrealistic.
- For $k_0n_1 > \beta > k_0n_2$, there are a discrete number of bound or guided modes. These vary cosinusoidally inside the guide core, and decay exponentially outside the guide.
- For $k_0n_2 > \beta > k_0n_3$, the solutions vary exponentially in the cover (layer 3), and cosinusoidally in both the guide (layer 1) and substrate (layer 2). Since these fully penetrate the substrate region, they are called substrate modes. Any value of β is allowed, between the two limits given above, so the set forms a continuum.
- For $k_0n_3 > \beta$, solutions vary cosinusoidally in all three layers. These particular field patterns are known as radiation modes. Once again, any value of β is allowed in the range above, so the set forms another continuum.

[4]

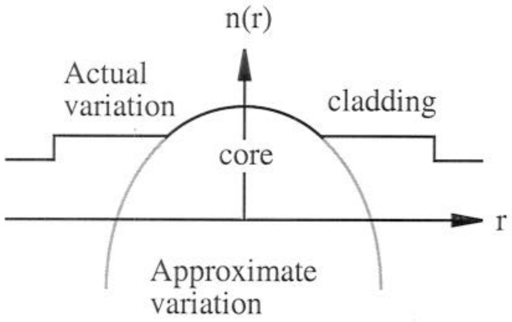


[3]

2a) All optical fibre is constructed using a two-step process, involving the fabrication of a preform followed by a pull down to the final diameter. Telecoms fibre is based on silica, with index differences needed for wave guidance supplied by doping with germania. The preform is typically formed by chemical vapour deposition, with the gas flow rates being varied to obtain the required radial variation in refractive index. The glass is normally deposited either on the outside of a silica rod or on the inside of a silica tube.

[4]

b) For the central region of the refractive index variation shown, a suitable approximation is $n^2 = n_0^2[1 - (r/r_0)^2]$.



[2]

The scalar wave equation is $\nabla^2 E + n^2 k_0^2 E = 0$, where n is the refractive index and $k_0 = 2\pi/\lambda$ is the propagation constant of free space.
If the refractive index is a function of x and y alone, and hence describes a guide oriented in the z-direction, we may assume a solution in the form $E(x, y, z) = E_T(x, y) \exp(-j\beta z)$.
Here $E_T(x, y)$ is the transverse field and β is the propagation constant.
By substituting into the wave equation, we obtain the waveguide equation:
$\partial^2 E_T/\partial x^2 + \partial^2 E_T/\partial y^2 + \{n^2 k_0^2 - \beta^2\} E_T = 0$

[2]

Hence the waveguide equation reduces to:
$\partial^2 E_T/\partial x^2 + \partial^2 E_T/\partial y^2 + \{n_0^2 k_0^2 [1 - (r/r_0)^2] - \beta^2\} E_T = 0$

[2]

For circularly symmetric fields, $\partial E_T/\partial x = dE_T/dr\ \partial r/\partial x = (x/r)\ dE_T/dr$.
Similarly, $\partial^2 E_T/\partial x^2 = (x^2/r^2)\ d^2E_T/dr^2 + [1/r - x^2/r^3]\ dE_T/dr$
$\partial^2 E_T/\partial y^2$ may be evaluated in the same way, so we obtain
$\nabla^2_{xy} E_T = d^2E/dr^2 + (1/r)\ dE_T/dr$
The waveguide equation is then:
$d^2E_T/dr^2 + (1/r)\ dE_T/dr + \{n_0^2 k_0^2 [1 - (r/r_0)^2] - \beta^2\} E_T = 0$.

[2]

Assuming $E_T(r) = E_0 \exp(-r^2/a^2)$, and substituting into the above we get:
$r^2 \{4/a^4 - n_0^2 k_0^2/r_0^2\} + \{n_0^2 k_0^2 - 4/a^2 - \beta^2\} = 0$.
Here the terms are grouped into two blocks, one dependent on r^2, the other a constant. Both must be zero independently.

[2]

The former condition leads to $a = \sqrt{(2r_0/n_0 k_0)}$
The latter requires $\beta^2 = n_0^2 k_0^2 - 4/a^2$ or $\beta = n_0 k_0 \sqrt{\{1 - 2/(n_0 k_0 r_0)}$

[2]

Since the substitution leads to conditions on a and β that can be satisfied, the solution is a mode of the fibre.

c) At $\lambda = 1.55\ \mu m$, $k_0 = 2\pi/1.55 \times 10^{-6} = 4.054 \times 10^6\ m^{-1}$.
For $n_0 = 1.46$ and $r_0 = 25\ \mu m$, we then get

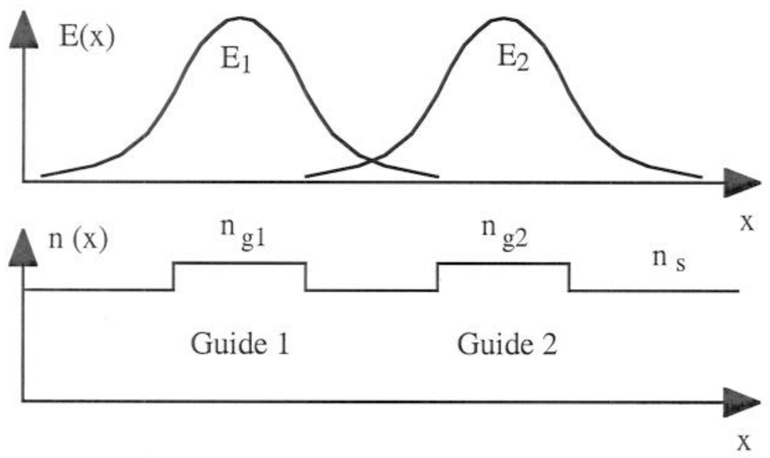
$a = \sqrt{(2 \times 25 \times 10^{-6}/1.46 \times 4.054 \times 10^{6})} = 2.91 \times 10^{-6}$ m, or 3 μm.
The mode field diameter is then 6 μm.

[2]

The propagation constant is then:
$\beta = 1.46 \times 4.054 \times 10^{6} \sqrt{\{1 - 2/(1.46 \times 4.054 \times 10^{6} \times 25 \times 10^{-6})\}} = 5.88 \times 10^{6}\ \mathrm{m}^{-1}$.

[2]

3a) Transverse fields and refractive index profiles in a step-index directional coupler are:



[4]

Y-junctions are 4 port devices, in which the fourth port is connected to radiation. For some input conditions, power can therefore be lost to radiation. The directional coupler has four guided output ports, and can therefore be used as a lossless bidirectional switch.

[4]

b) The governing first order coupled mode differential equations are:

$dA_1/dz + j\kappa A_2 = 0$

$dA_2/dz + j\kappa A_1 = 0$

Differentiating the upper equation, we get:

$d^2A_1/dz^2 + j\kappa dA_2/dz = 0$

Substituting using the lower equation, we then get:

$d^2A_1/dz^2 + \kappa^2 A_1 = 0$

This second order equation has the general solution:

$A_1 = C_1 \cos(\kappa z) + C_2 \sin(\kappa z)$

For an input of unity amplitude into (say) guide 1, the boundary conditions are that $A_1 = 1$ and $A_2 = 0$ on $z = 0$. From the upper coupled mode equation, the second condition is equivalent to $dA_1/dz = 0$. The solution must then be:

$A_1 = \cos(\kappa z)$

$A_2 = -j \sin(\kappa z)$

[4]

The power carried by a guided mode is proportional to the modulus squared of the field amplitude. The powers in the two guided modes are therefore:

$P_1 = |A_1|^2 = \cos^2(\kappa z)$

$P_2 = |A_2|^2 = \sin^2(\kappa z)$

Clearly, power is conserved, since $P_1 + P_2 = 1$ throughout.

[4]

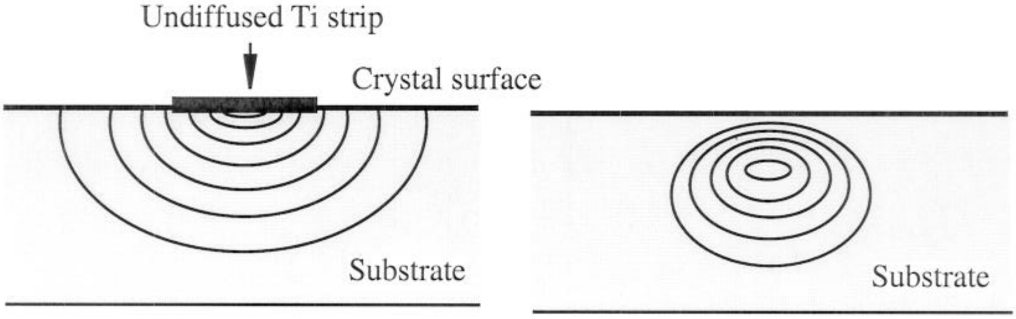
c) For a 50 : 50 power split, we require $\kappa L = \pi/4$.

Hence, $L = \pi/4\kappa = 3.14/(4 \times 0.157) = 5$ mm.

[4]

R.R.A. Syms

4a) Channel waveguides are fabricated in $LiNbO_3$ by in-diffusion. Titanium metal is first deposited in patterned strips of ≈ 1000 Å thickness and then in-diffused at a high temperature (≈ 1000°C) for several hours. The additional impurities cause a change in refractive index that is proportional to their concentration. Typical maximum $\Delta n \approx 0.01$.



Contours of constant refractive index

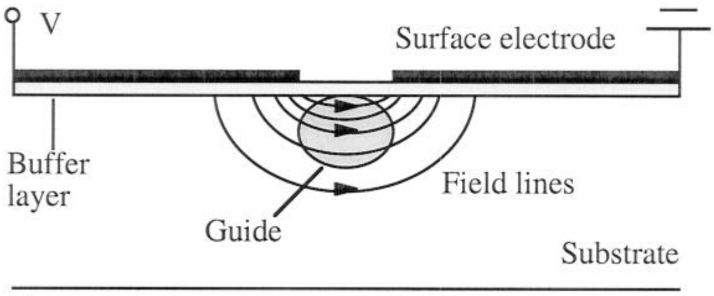
Contours of constant light intensity

[3]

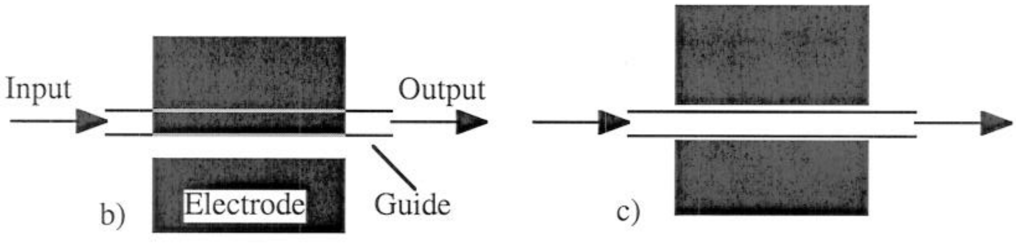
The electro-optic effect is a phenomenon that occurs in non-centro-symmetric crystals, where by application of an electric field E is used to change the refractive index n by an amount: $\Delta n = -n^3 r_{ij} E/2$, where r_{ij} is the electro-optic coefficient. The electro-optic effect is important because it allows the construction of high-speed phase modulators, which in turn can be used to construct high-speed amplitude modulators in interferometric optical waveguide circuits. The acheivable bandwidth is higher than that possible by other means, e.g. direct modulation of a semiconductor laser.

[3]

b) Ti: $LiNbO_3$ electro-optic phase modulators are constructed using surface electrodes, which are conventionally spaced away from the substrate by a thin layer of dielectric (a). The waveguide is arranged to run underneath, in the vicinity of the electrode gap. Two geoemetries are important: asymmetric placement, which exploits the vertical component of the field (b), and symmetric placement (c), which exploits the horizontal component.



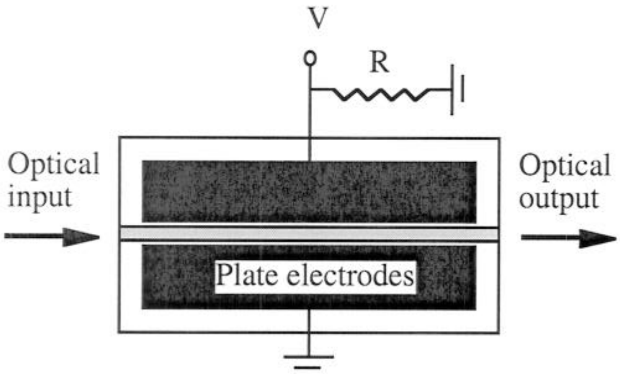
a)

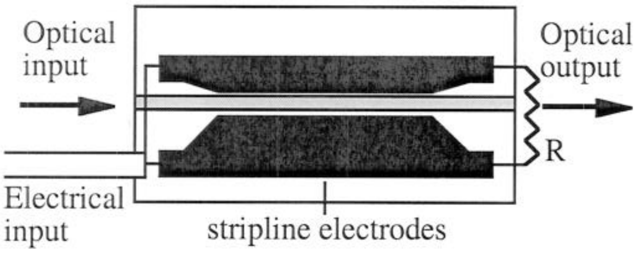


b) c)

[3]

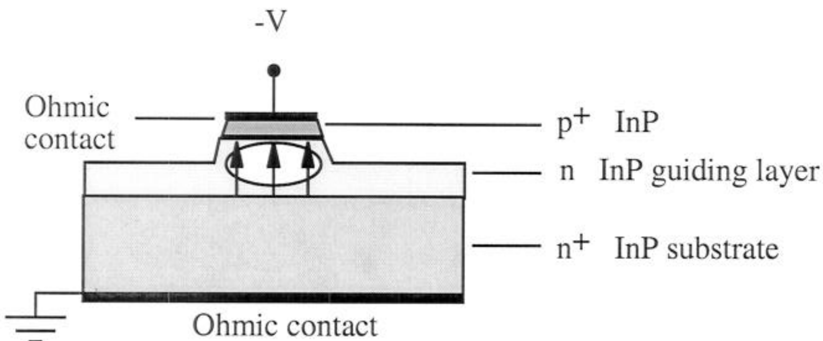
The electrode structures may act as a lumped element parallel plate capacitor. In this case, the upper frequency is determined by a RC time constant in which C is the electrode capacitance and R is the source impedance. Alternatively they may be arranged as a stripline waveguide, whose impedance is real. In this case, the upper frequency is detemined by velocity mismatch between the electrical and optical waves.





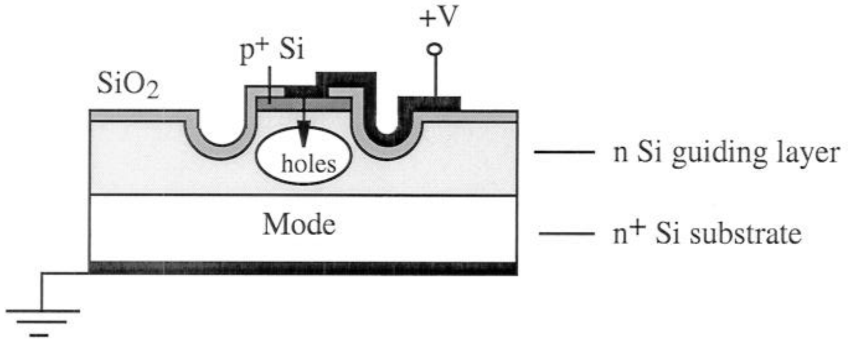
[3]

c) In a semiconductor, carriers normally prevent the development of the high electric field needed in an electro-optic device. The p-n junction provides a solution. The figure below shows a cross-section of a phase modulator based on an n-type InP homostructure strip-loaded guide on a n^+ substrate. The guide is capped with a p^+ layer, thus forming a p^+-n homojunction, with the bulk of the depletion region inside the guide itself. One ohmic contact is placed on the p^+ layer, while the other is connected to the substrate. When the junction is reverse-biased, the resistive depletion region grows downwards, so that the applied voltage is eventually dropped across the guide. The field is then sufficient to obtain an index change through the relatively weak r_{41} electro-optic coefficient.



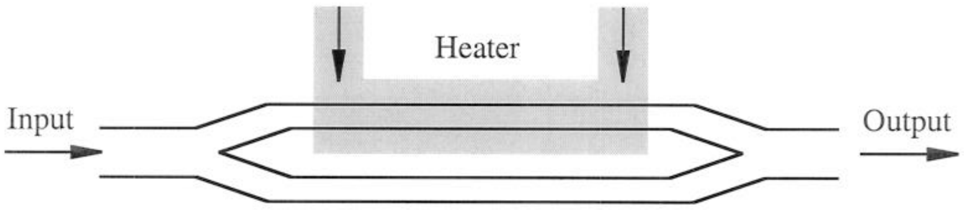
[4]

A refractive index change can be created in materials such as Si that are not electro-optic, by the electrically- controlled injection of minority carriers, this time using a forward-biassed p-n junction. The figure below shows the cross-section of a silicon-based phase modulator. Again, a p^+-n junction is constructed at the top of the guide. When the junction is forward-biassed, holes are injected into the guiding region, thus generating a refractive index change through the change in free carrier concentration.



[4]

5a) The thermo-optic Mach-Zehnder interferometric gate consists of two back-to-back Y-junctions, separated by a region containing straight waveguides, one of which carries a thermo-optic phase modulator based on a Ti thin film heater.



[3]

The gate operates as follows. The input wave is first split into two components by the left-hand Y-junction. The transverse field across the device just before the phase shifter region can be written as:
$E(x, y) = (a_{in}/\sqrt{2})\,[E_U(x, y) + E_L(x, y)]$

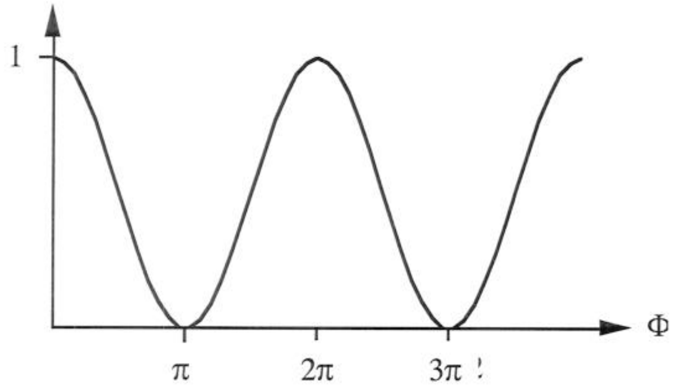
Here a_{in} is the modal amplitude of the input and $E_U(x, y)$ and $E_L(x, y)$ are the transverse fields in the upper and lower guides, respectively. Each beam then travels through the central section, and a phase shift of ϕ is applied to the upper one. The transverse field just beyond the phase-shifter can then be written as:
$E'(x, y) = (a_{in}/\sqrt{2})\,[E_U(x, y)\exp(-j\Phi) + E_L(x, y)]$
$= (a_{in}/\sqrt{2})\,[E_U(x, y)\exp(-j\phi/2) + E_L(x, y)\exp(j\phi/2)]\exp(-j\phi/2)$

This field can be expressed in terms of the characteristic modes of the two-guide system forming the central region of the device, as:
$E(x, y) = (a_{in}/\sqrt{2})\,[E_S(x, y)\cos(\phi/2) - j\,E_A(x, y)\sin(\phi/2)]\exp(-j\phi/2)$

where E_S and E_A are the transverse fields of the symmetric and antisymmetric supermodes in the central region. This distribution now passes into the right-hand Y-junction. The component carried by the symmetric supermode emerges from the single guide output, while the anti- symmetric component is radiated.

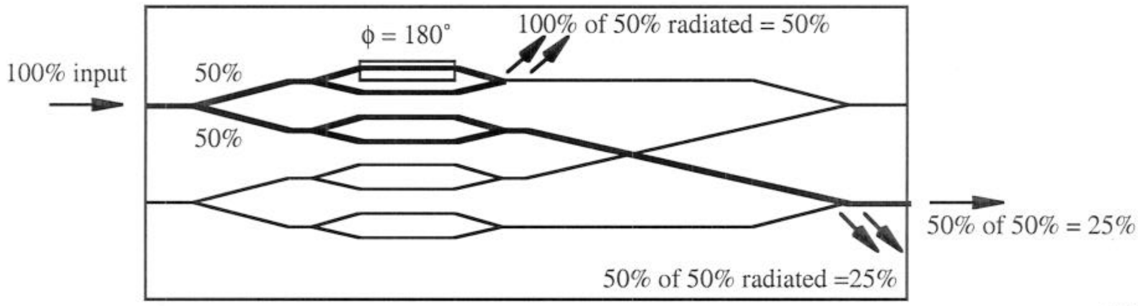
The amplitude of the guided output is thus $a_{out} = a_{in}\cos(\phi/2)$ so that the normalised output power $P = P_{out}/P_{in}$ is $P = \cos^2(\phi/2)$. The normalised output therefore varies cosinusoidally with ϕ, as shown in below. The output power is unity when no phase shift is applied, falling to zero when $\phi = \pi, 3\pi$ etc.

[4]



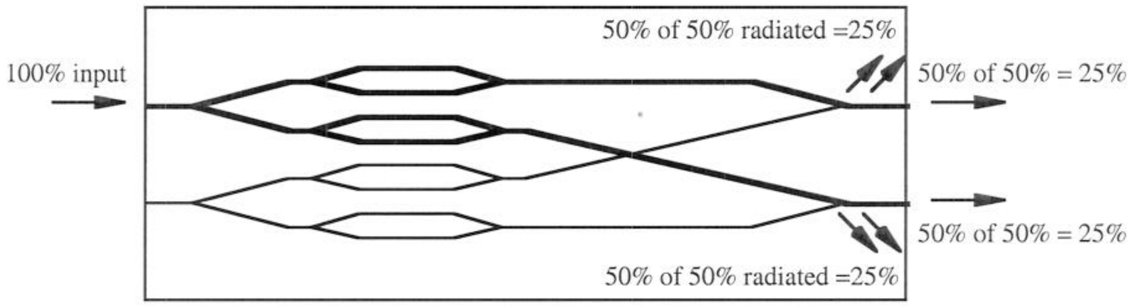
[3]

b) For the point to point connection:



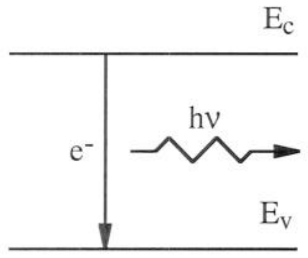
[6]

c) For the broadcast connection:



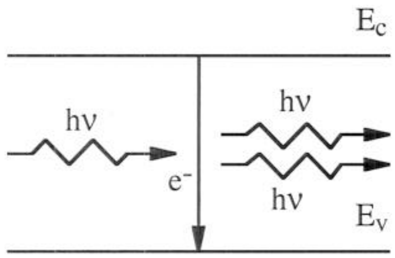
[4]

6a) LEDs generate light by spontaneous emission. This process involves a conduction band electron recombining with a valence band hole. The energy thus released gives rise to a photon of energy $h\nu \approx E_c - E_v$, and random phase, direction and polarization.



[2]

Lasers generate light by stimulated emission. This process involves a conduction band electron recombining with a valence band hole, when triggered by a photon of appropriate energy. The energy thus released gives rise to a second photon of identical energy (and hence identical frequency), phase, direction and polarization.



[2]

The laser diodes is therefore the preferred source for optical communications because:

LEDs generate light isotropically, and hence have:
Low external efficiency due to total internal reflection at the semiconductor-air interface
Low coupling efficiency to optical fibres with low numerical aperture
LED light is very broad band, and hence unsuitable as a single frequency carrier
LEDs cannot be modulated at high speed, due to their first-order response

[3]

In contrast, laser light is generated with preferred emission directions, and hence has
High external efficiency
High coupling efficiency to fibres
Laser light is inherently narrow band
Lasers can be directly modulated at rates up to a few GHz

[3]

b) The electron rate equations is:
$dn/dt = I/ev - n/\tau_e$ or $\{n + \tau_e\, dn/dt\} = I\tau_e/ev$

Assume $I = I' + I'' \exp(j\omega t)$ and $n = n' + n'' \exp(j\omega t)$
Differentiating and substituting into the electron rate equation, we get:
$n' + n''(1 + j\omega\tau_e) \exp(j\omega t) = \{I' + I'' \exp(j\omega t)\}\, \tau_e/ev$
Hence $n' = I'\tau_e/ev$ and $n'' = (I''/ev)\, \{\tau_e / (1 + j\omega\tau_e)\}$

The photon rate equation is:
$d\phi/dt = n/\tau_{rr} - \phi/\tau_p$

R.R.A. [signature]

Since τ_p is so small, we can neglect $d\phi/dt$, so that $\phi/\tau_p = n/\tau_{rr}$

Now the light flux out is $\Phi = v\phi/\tau_p$.
Since each photon carries energy $hc/\lambda_g = eE_g$, the optical power P emitted is:
$P = hc\Phi/\lambda_g = (hc/\lambda_g)\ vn/\tau_{rr}$

Assume that $P = P' + P''\ \exp(j\omega t)$; hence:
$P' + P''\ \exp(j\omega t) = (hc/e\lambda_g)\ [(I'\tau_e/\tau_{rr}) + (I''/\tau_{rr})\ \{\tau_e/(1 + j\omega\tau_e)\}\ \exp(j\omega t)]$

Equating DC and AC terms separately, we get:
$P' = \eta' hcI'/e\lambda_g$ and $P'' = \eta'' hcI''/e\lambda_g$

Where the DC and AC internal efficiencies η' and η'' are:
$\eta' = \tau_e/\tau_{rr}$ and $\eta'' = \tau_e/\{\tau_{rr}(1 + j\omega\tau_e)\}$

[10]