Paper Number(s): **E3.12**
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IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2000

MSc and EEE PART III/IV: M.Eng., B.Eng. and ACGI

OPTOELECTRONICS

Monday, 22 May 2000, 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks.

Corrected Copy

Time allowed: 3:00 hours

Examiners: Prof R.R.A. Syms, Dr A.S. Holmes

Special instructions for invigilators: None

Information for candidates:

Fundamental constants

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

$c = 3 \times 10^{8}$ m/s

$h = 6.62 \times 10^{-34}$ Js

1. a) Explain what is meant by the following terms:
Alhazen's law, Snell's law, critical angle, total internal reflection.

b) The amplitude reflection coefficient for a TE polarised electromagnetic wave incident on a boundary between two dielectric media as shown in Figure 1 is:

$$\Gamma_E = \{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)\} / \{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)\}$$

Show how the power reflection coefficient can become unity.

c) The time-independent electric fields in media 1 and 2 of Figure 1 can be written as:

$$E_{y1} = E_I \exp\{-jk_0 n_1\{z \sin(\theta_1) - x \cos(\theta_1)\}\} + E_R \exp\{-jk_0 n_1\{z \sin(\theta_1) + x \cos(\theta_1)\}\}$$
$$E_{y2} = E_T \exp\{-jk_0 n_2\{z \sin(\theta_2) - x \cos(\theta_2)\}\}$$

Assuming that total internal reflection has occurred, rewrite these expressions in modal form and sketch the transverse field variation across the interface.

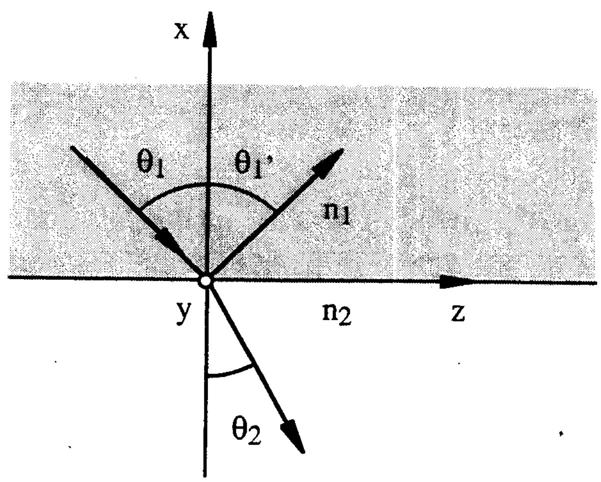

Figure 1.

2. The behaviour of a y-polarized scalar wave propagating in the (x, z) plane and defined by the time-independent electric field variation $E_y(x, z)$ may be described by the Helmholtz equation:

$$\nabla^2 E_y(x, z) + n^2(x, z)\, k_0^2\, E_y(x, z) = 0$$

a) What do the terms $\nabla^2 E_y(x, z)$, $n(x, z)$ and k_0 represent? Assuming that n is a function of x alone, derive a scalar waveguide equation for modes travelling in the z direction. Briefly describe the different solutions to this equation that may be obtained for constant n.

b) A three-layer slab optical waveguide of width h and with $n_1 > n_2 > n_3$ is constructed as shown in Figure 2. Explain how the waveguide equation is solved for the entire structure. What boundary conditions must be satisfied at the interfaces between the layers?

c) Sketch all of the substantially different variations in the transverse field distribution that may be found. For each variation, indicate the approximate range of propagation constant involved, and describe the physical significance of the solution.

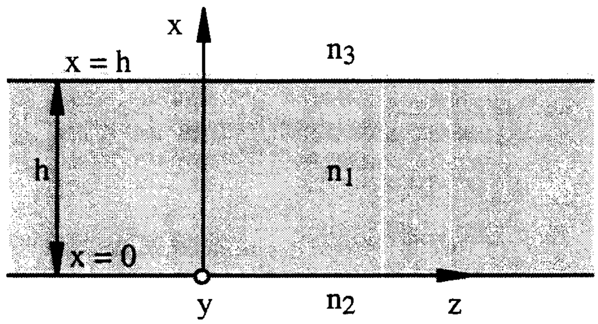

Figure 2.

3. a) Explain why titanium-diffused lithium niobate electro-optic modulators are used in ultra-high-speed optical communications links, in preference both to direct modulation of a laser, and to external modulators based either on carrier injection or the thermo-optic effect.

b) Figure 3a shows a Ti : $LiNbO_3$ Mach-Zehnder interferometric modulator. Explain its operation.

c) The two phase shifters in a Ti : $LiNbO_3$ interferometric modulator are each 5 mm in length, and have a voltage-length product for π radians phase-shift of $(VL)_\pi = 50$ V mm. Assuming that $V_1(t) = V(t) = -V_2(t)$, calculate and sketch the drive voltage variation V(t) needed to obtain the variation of guided output power with time shown in Figure 3b. Assume a constant optical input power.

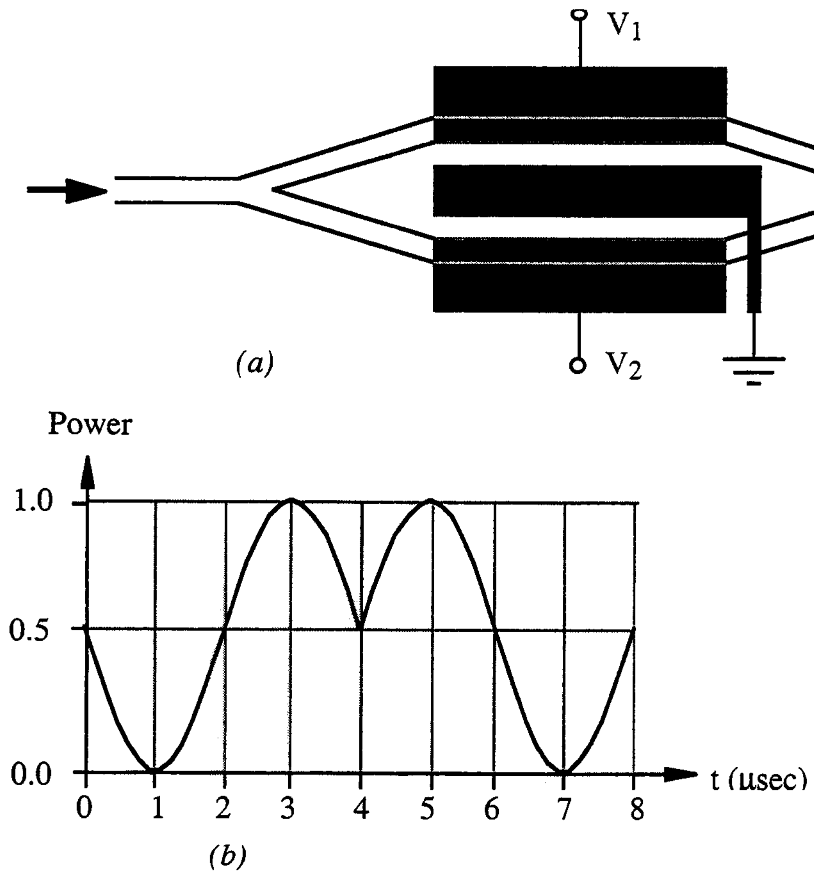

Figure 3.

4. a) Explain in detail how the photocurrent is generated in a p-n junction photodiode.

b) Table 1 below shows the energy gaps of a number of different semiconductors. On a single graph, plot the theoretical variation of responsivity with wavelength for all five materials, explaining your reasoning and giving accurate numerical values.

c) Explain briefly the principle of a p-i-n photodiode. A substrate-entry heterostructure p-i-n photodiode is to be constructed from two of the materials in Table 1, for use in an optical communications link operating at 1.5 μm wavelength. Which two materials would be suitable, and why?

Material	E_g *(eV)*
Ge	0.66
$In_{0.53}Ga_{0.47}As$	0.74
Si	1.14
InP	1.35
GaAs	1.42

Table 1.

5. a) Derive the gain and phase conditions that must be satisfied for lasing in a Fabry-Perot laser operating at a nominal wavelength λ, assuming an effective index n and a gain coefficient g. Sketch the variation of the electric field amplitude as the wave travels up and down the cavity.

b) Derive an expression for the longitudinal mode separation Δλ in the laser above. Sketch the mode spectra of a semiconductor Fabry-Perot laser operating i) below threshold, and ii) substantially above threshold.

c) An indium phosphide laser diode designed to operate at 1.5 μm wavelength has a longitudinal mode separation of 1.28 nm. If the effective index of the gain block is 3.5, how long is the laser cavity? Assuming that the end mirrors are constructed simply by cleaving, calculate the gain coefficient required for lasing.

6. a) Explain the process of light emission by electroluminescence in an LED. What factors limit the internal efficiency, the external efficiency, and the modulation bandwidth?

b) In a two-state, lumped-element model, the rate equations governing the operation of an LED are:

$dn/dt = I/ev - n/\tau_e$ and $d\phi/dt = n/\tau_{rr} - \phi/\tau_p$

where n and ϕ are the electron and photon densities, I is the current, v is the active volume, τ_e is the electron lifetime, τ_{rr} is the radiative recombination lifetime and τ_p is the photon lifetime. Using these equations, derive an expression for the frequency dependence of the internal efficiency.

c) Table 2 shows data for the internal efficiency of a GaAs LED at different frequencies, predicted from measurements of the external efficiency. Use these to estimate values for τ_e and τ_{rr}.

Frequency	Efficiency
DC	0.5
10 kHz	0.5
100 MHz	0.25

Table 2.

[END]

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1. a) *Alhazen's law* The angle of incidence equals the angle of reflection, so $\theta_1 = \theta_1'$. **1**

Snell's law The relation between the angles of incidence and transmission is $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ **1**

Critical angle The angle of the transmitted wave is $\theta_2 = \sin^{-1}\{(n_1/n_2)\sin(\theta_1)\}$. θ_2 ceases to be real if $(n_1/n_2)\sin(\theta_1) > 1$. This can occur when $\theta_1 > \theta_c = \sin^{-1}(n_2/n_1)$, where θ_c is the critical angle, if $n_1 > n_2$. **2**

Total internal reflection For $\theta_1 > \theta_c$, there is no propagating transmitted wave; instead the field in layer 2 is an evanescent wave. All the incident power is reflected at the interface between the two media. **2**

b) The transmitted wave angle is defined by $\sin(\theta_2) = (n_1/n_2)\sin(\theta_1)$
However, for $\theta_1 > \theta_c$, $\sin(\theta_2) > 1$, so there is no real solution for θ_2.
However, $\cos(\theta_2)$ may still be defined as $\cos(\theta_2) = \sqrt{1 - \sin^2(\theta_2)} = \sqrt{1 - (n_1/n_2)^2 \sin^2(\theta_1)}$
This can be written as $\cos(\theta_2) = \pm j\sqrt{(n_1/n_2)^2 \sin^2(\theta_1) - 1}$, i.e. as a purely imaginary number.
For a decaying exponential field in layer 2, the negative sign of the root is appropriate. **2**

The reflection coefficient is $\Gamma_E = \{n_1\cos(\theta_1) - n_2\cos(\theta_2)\} / \{n_1\cos(\theta_1) + n_2\cos(\theta_2)\}$.
Thus the amplitude reflection coefficient has the form: $\Gamma_E = \{\kappa + j\gamma\} / \{\kappa - j\gamma\} = z / z^*$
The power reflection coefficient is then $|\Gamma_E|^2 = \Gamma_E \Gamma_E^* = (z / z^*)(z^* / z) = 1$, i.e. unity. **2**

c) In modal form, the entire time-independent electric field is written as $E_y = E_T(x)\exp(-j\beta z)$, where $E_T(x)$ is the transverse field variation and β is the propagation constant.

The fields in media 1 and 2 are:

$$E_{y1} = E_I \exp\{-jk_0n_1\{z\sin(\theta_1) - x\cos(\theta_1)\}\} + E_R \exp\{-jk_0n_1\{z\sin(\theta_1) + x\cos(\theta_1)\}\}$$
$$E_{y2} = E_T \exp\{-jk_0n_2\{z\sin(\theta_2) - x\cos(\theta_2)\}\}$$

Assuming that the reflection and transmission coefficients are defined as $\Gamma_E = E_R/E_I$ and $T_E = E_T/E_I$, these expressions can be written as:

$$E_{y1} = E_I\,[\exp\{+jk_0n_1x\cos(\theta_1)\} + \Gamma_E \exp\{-jk_0n_1x\cos(\theta_1)\}]\exp\{-jk_0n_1z\sin(\theta_1)\}$$
$$E_{y2} = E_I\,T_E \exp\{+jk_0n_2x\cos(\theta_2)\}\exp\{-jk_0n_2z\sin(\theta_2)\}$$ **1**

Defining the propagation constant as $\beta = k_0n_1\sin(\theta_1)$, we can write:
$k_0n_1\cos(\theta_1) = \sqrt{k_0^2n_1^2 - k_0^2n_1^2\sin^2(\theta_1)} = \sqrt{k_0^2n_1^2 - \beta^2} = \kappa$ (say), and
$k_0n_2\cos(\theta_2) = \sqrt{k_0^2n_2^2 - k_0^2n_2^2\sin^2(\theta_2)} = \sqrt{k_0^2n_2^2 - k_0^2n_1^2\sin^2(\theta_1)}$. However, $\theta > \theta_c$,
so $k_0n_2\cos(\theta_2) = -j\sqrt{k_0^2n_1^2\sin^2(\theta_1) - k_0^2n_2^2} = -j\sqrt{\beta^2 - k_0^2n_2^2} = -j\gamma$ (say) **2**

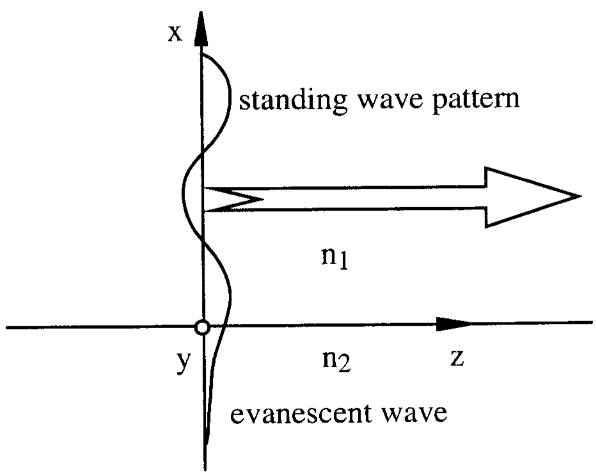
Using these definitions, the two field distributions can be written as:

$E_{y1} = E_I\,[\exp\{+j\kappa x\} + \Gamma_E \exp\{-j\kappa x\}]\exp\{-j\beta z\}$
$E_{y2} = E_I\, T_E \exp\{+\gamma x\}\ \exp\{-j\beta z\}$

If total internal reflection has occurred, $|\Gamma_E| = 1$, so we may write the reflection coefficient as a complex exponential of the form $\Gamma_E = \exp(j2\phi)$. The field in medium 1 is then:
$E_{y1} = E_I \exp(j\phi)[\exp\{+j(\kappa x - \phi)\} + \exp\{-j(\kappa x - \phi)\}]\exp\{-j\beta z\} = E_0 \cos\{\kappa x - \phi\}\exp\{-j\beta z\}$ **2**

Without needing to know T_E explicitly (although this gives the same answer), the field in medium 2 can then be found by assuming appropriate boundary matching as:
$E_{y2} = E_0 \cos(\phi) \exp\{+\gamma x\}\ \exp\{-j\beta z\}$ **2**

These two fields represent a standing wave pattern in medium 1, and an evanescent wave in medium 2, as shown below.



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2. a) $\nabla^2 E_y(x, z) = \partial^2 E_y/\partial x^2 + \partial^2 E_y/\partial z^2$ is the Laplacian of the electric field. **1**
$n(x, z)$ is the refractive index variation of the medium in which the wave is propagating. **1**
$k_0 = 2\pi/\lambda$ is the free space propagation constant, where λ is the optical wavelength. **1**

If n is a function of x alone, we must solve the scalar wave equation:
$\partial^2 E_y(x, z)/\partial x^2 + \partial^2 E_y(x, z)/\partial z^2 + n^2(x)\, k_0^2\, E_y(x, z) = 0.$

Assuming the solution $E_y(x, z) = E_T(x) \exp\{-j\beta z\}$, where E_T is a transverse field variation and β is a propagation constant, we obtain:
$\partial^2 E_y/\partial x^2 = d^2 E_T/dx^2 \exp\{-j\beta z\}$ and $\partial^2 E_y/\partial z^2 = -\beta^2 E_T(x) \exp\{-j\beta z\}$

Substituting into the wave equation and removing common exponential terms, we then obtain the scalar waveguide equation $d^2 E_T/dx^2 + \{n^2(x) k_0^2 - \beta^2\} E_T = 0$ **2**

For constant n, the waveguide equation has the general form $d^2 E_T/dx^2 + \alpha^2 E_T = 0$. For $\alpha^2 > 0$, solutions have the form $E_T(x) = E_0 \sin\{(n^2 k_0^2 - \beta^2)x\}$ or $E_0 \cos\{(n^2 k_0^2 - \beta^2)x\}$. For $\alpha^2 < 0$, the alternative solutions $E_T(x) = E_0 \exp\{\pm(\beta^2 - n^2 k_0^2)x\}$ are obtained. **2**

b) To solve the slab guide problem, use the following procedure in each region i = 1, 2 and 3:
Start with the wave equation: $\nabla^2 E_{yi} + n_i^2(x, z)\, k_0^2\, E_{yi} = 0$
Assume the solution: $E_{yi}(x, z) = E_{Ti}(x) \exp\{-j\beta z\}$
Leading to the waveguide equation: $d^2 E_{Ti}/dx^2 + \{n_i^2 k_0^2 - \beta^2\} E_{Ti} = 0$ **2**

The conditions that must be satisfied are continuity of the tangential components of the electric and magnetic field across at each interface. Since E_y is tangential throughout, we must have $E_{y1} = E_{y2}$, and hence $E_{T1} = E_{T2}$, on x = 0; similarly, we require $E_{T1} = E_{T3}$ on x = h. **2**

To perform the matching for the magnetic field, we use the Maxwell relation $\nabla \times \underline{E} = -j\omega\mu\underline{H}$. Since $E_x = E_z = 0$, we obtain $H_x = (-j/\omega\mu)\, \partial E_y/\partial z$, $H_y = 0$ and $H_z = (+j/\omega\mu)\, \partial E_y/\partial x$. Here the only tangential component is H_z, so we require continuity in $\partial E_y/\partial x$. We must therefore have $\partial E_{y1}/\partial x = \partial E_{y2}/\partial x$ or $dE_{T1}/dx = dE_{T2}/dx$ on x = 0; similarly, $dE_{T1}/dx = dE_{T3}/dx$ on x = h. **2**

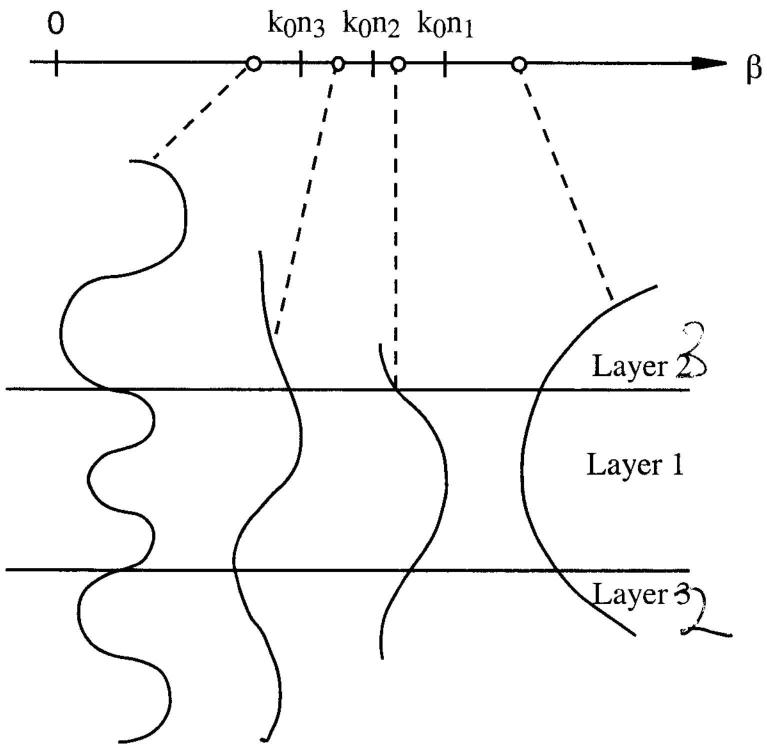
c) Referring to the figure below:

i) For $\beta > k_0 n_1$, the solutions are exponential in all three layers. This implies infinite field amplitudes at large distances from the waveguide, so these solutions are physically unrealistic, except under special circumstances. **1**

ii) For $k_0 n_1 > \beta > k_0 n_2$, there are a discrete number of bound or guided modes. These vary cosinusoidally inside the waveguide core, and decay exponentially outside the guide. **1**

iii) For $k_0n_2 > \beta > k_0n_3$, the solutions vary exponentially in the cover (layer 3), and cosinusoidally in both the guide (layer 1) and substrate (layer 2). Since these fully penetrate the substrate region, they are called substrate modes. Any value of β is allowed, between the two limits given above, so the set forms a continuum. **1**

iv) For $k_0n_3 > \beta$, the solutions vary cosinusoidally in all three layers. These particular field patterns are known as radiation modes. Once again, any value of β is allowed in the range above, so the set forms another continuum. **1**



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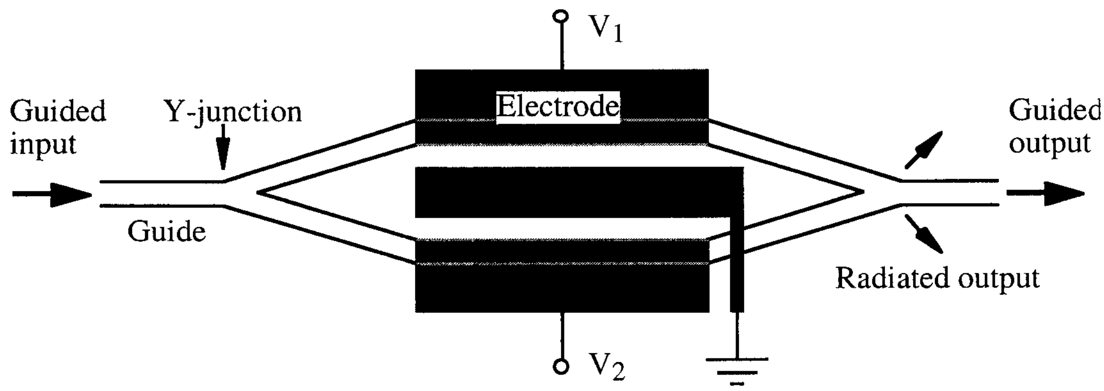
3. a) $Ti:LiNbO_3$ electro-optic modulators are used in ultra-high-speed optical communications links, because devices fitted with travelling wave electrodes allow modulation bandwidths in excess of 20 GHz. In contrast, direct modulation of a laser by varying the injection current is limited to a few GHz, and causes simultaneous modulation of the output spectrum. External modulators based on carrier injection are limited to ≈ 100 MHz by recombination lifetimes. Similarly, modulators based on the thermo-optic effect are limited to ≈ 1KHz by thermal time constants. **4**

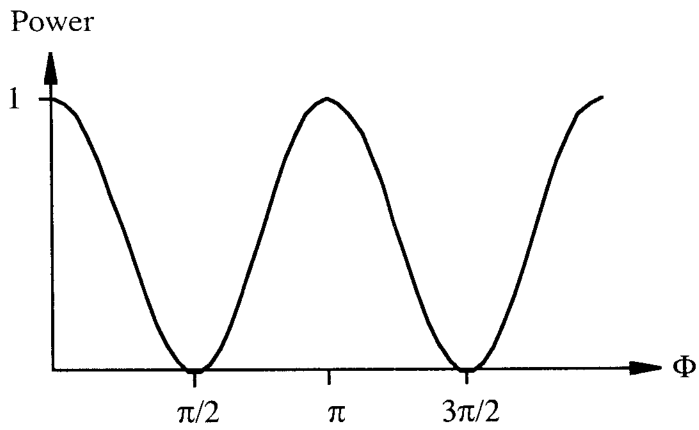
b) The Mach-Zehnder interferometer consists of two back-to-back Y-junctions, separated by a region containing a pair of identical electro-optic phase modulators. It operates as follows. The input wave is first split into two components by the left-hand Y-junction. The transverse field across the device just before the phase shifters can be written as: $E(x, y) = (a_{in}/\sqrt{2})\ [E_U(x, y) + E_L(x, y)]$



where a_{in} is the modal amplitude of the input, and $E_U(x, y)$ and $E_L(x, y)$ are the transverse fields in the upper and lower guides, respectively. Each beam then travels through a phase-shifter, and opposite shifts of Φ are applied to each, to get a push-pull effect. The transverse field just beyond the phase-shifters is then: $E(x, y) = (a_{in}/\sqrt{2})\ [E_U(x, y) \exp(-j\Phi) + E_L(x, y) \exp(+j\Phi)]$

This field can also be expressed in terms of the characteristic modes of the two-guide system at the centre of the device, as: $E(x, y) = (a_{in}/\sqrt{2})\ [E_S(x, y) \cos(\Phi) - j\ E_A(x, y) \sin(\Phi)]$

where E_S and E_A are the transverse fields of the symmetric and antisymmetric supermodes in the central region. This distribution now passes into the right-hand Y-junction. The symmetric supermode emerges from the single guide output, while the anti-symmetric component is radiated. The amplitude of the guided output is thus $a_{out} = a_{in} \cos(\Phi)$ so that the normalised output power is $P = \cos^2(\Phi)$. The normalised output therefore varies cosinusoidally with Φ, as below. The output power is unity without an applied phase shift, falling to zero when $\Phi = \pi/2, 3\pi/2$ etc. **6**



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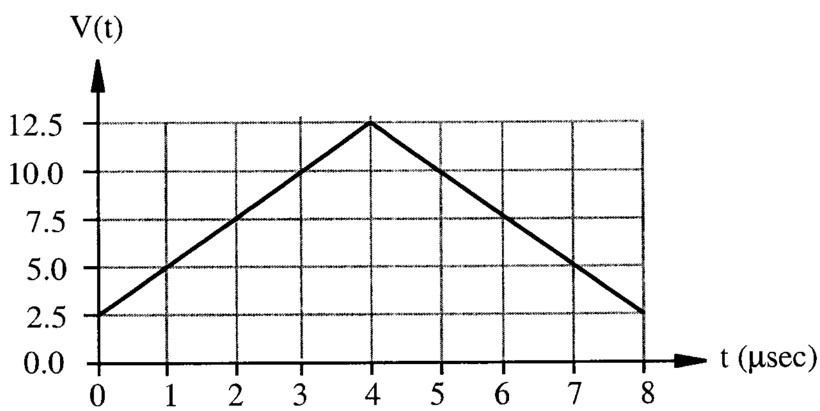
c) The normalised output from a dual-modulator Mach-Zehnder interferometer is $P = \cos^2(\Phi)$, where Φ is the phase shift applied by one of the modulators. The variation in output power required contains cosinusoidal variations as above, corresponding to a linear ramp in Φ from $\pi/4$ up to $5\pi/4$, followed by a linear ramp back down to $\pi/4$. **2**

If $(VL)_\pi = 50$ V mm, and L = 5 mm, then $V_\pi = 10$ V. Consequently, $V_{\pi/4} = 2.5$ V and $V_{5\pi/4} =$ 12.5 V. The drive voltage variation is therefore as shown below. Other solutions are possible, based on the periodic nature of the response characteristic. **2**



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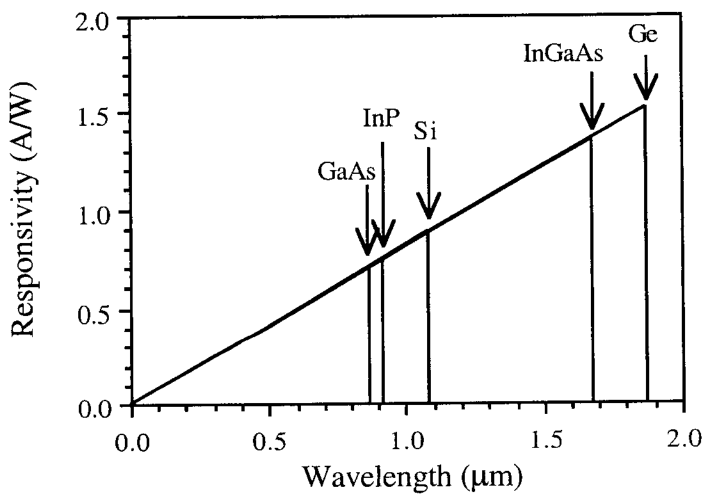
4. a) Carrier pairs are generated in a semiconductor as a result of the absorption of a photon causing the promotion of an electron from the valence band to the conduction band. A minimum photon energy of $h\nu \geq eE_g$ is required to promote a band-to-band transition, where h is Planck's constant, ν is the optical frequency, e is the electronic charge and E_g is the energy gap between the bands. This condition may be written as $hc/\lambda \geq eE_g$, where c is the velocity of light and λ is the wavelength. Thus, absorption will only occur if $\lambda \leq hc/eE_g$, or up to a maximum wavelength of $\lambda_{max} = 1.24/E_g$, where λ_{max} is measured in μm and E_g in eV. **3**

In a p-n junction photodiode, the carrier pairs are swept apart by the strong electric field in the depletion layer, and then diffuse to the contacts. Motion of an electron to one contact and a hole to the other is equivalent to one electron passing between the two contacts. When a beam of power P strikes the detector, the number of photons delivered per second is $P/h\nu$, or $P\lambda/hc$. The number of carrier pairs generated per sec is $\eta P\lambda/hc$, where η is the quantum efficiency, so the photocurrent is $I_p = \eta Pe\lambda/hc$. **3**

b) The responsivity is $R = I_p/P = \eta e\lambda/hc$, or $0.8056\eta\lambda$, where λ is measured in μm. When $\eta = 1$, the responsivity should increase linearly with wavelength until $\lambda = \lambda_{max}$. The maximum responsivity is then $R_{max} = 0.8056\lambda_{max} = 1/E_g$. The variation of responsivity with wavelength is then as below.



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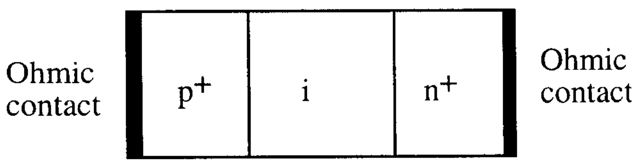
Material	E_g (eV)	λ_{max} (μm)	R_{max} (A/W)
Ge	0.66	1.878	1.515
$In_{0.53}Ga_{0.47}As$	0.74	1.676	1.351
Si	1.14	1.088	0.877
InP	1.35	0.918	0.741
GaAs	1.42	0.873	0.704

5

c) The quantum efficiency of a surface-entry p-n photodiode is variable, and peaks at $\approx$ 80%. At short wavelengths, η is reduced, because the attenuation is so strong that photons are absorbed before reaching the depletion layer. At long wavelengths, it falls because photons are transmitted right through. The latter limitation is overcome in the p-i-n structure. Here, a region of intrinsic or lightly-doped material is introduced between two heavily-doped p- and n-layers. Because the doping is so low here, the depletion layer can extend right through it under a modest reverse bias. The effective depletion layer width may therefore be fixed at a value far greater than the 'natural' one, roughly the intrinsic layer width, making a better absorber for long wavelengths.



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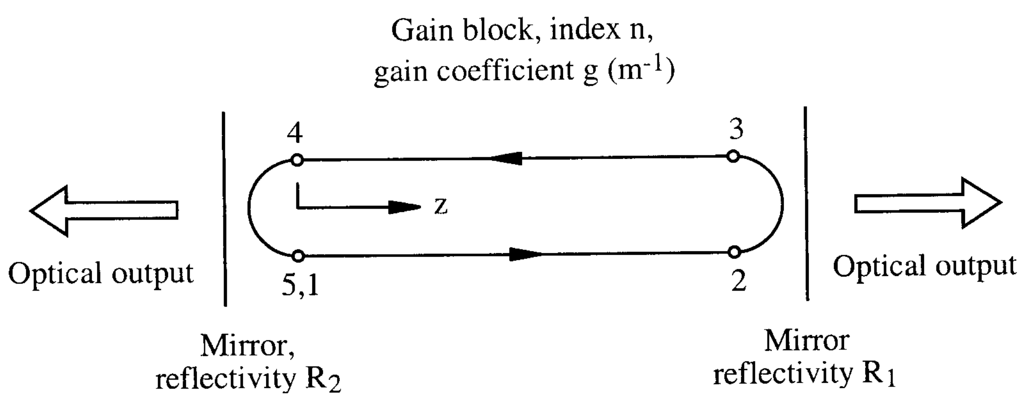
The two materials that are suitable for a substrate-entry heterostructure p-i-n photodiode are InP and $In_{0.53}Ga_{0.47}As$. The two are lattice-matched, so that the latter may be grown epitaxially on the former. An n^+ InP substrate will be transparent to the oncoming radiation, while an intrinsic $In_{0.53}Ga_{0.47}As$ layer acts as the absorber. The p^+ layer may be obtained by doping the InGaAs.

2

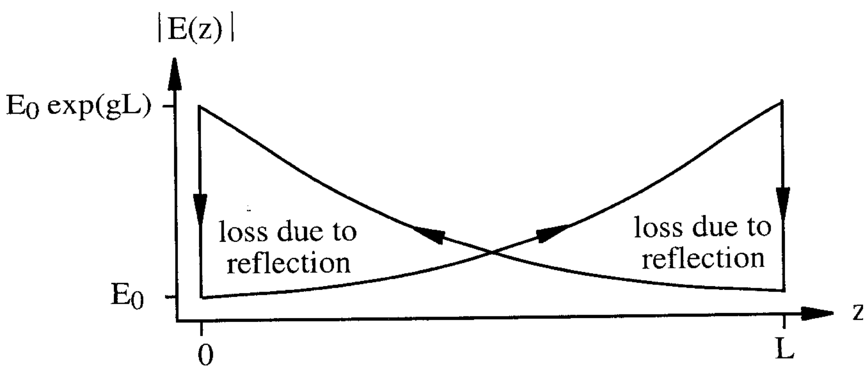
5. a) Consider the Fabry-Perot laser model shown below. The longitudinal resonance condition may be established by considering the amplitude of a wave as it propagates through one round trip of the cavity. Assuming an effective index and a gain coefficient of n and g, a travelling wave will propagate in the gain block as $E = E_0 \exp\{-jk_0nz\} \exp\{gz\}$, where $k_0 = 2\pi/\lambda$ and λ is the wavelength. If the mirror reflectivities are R_1 and R_2, the fields at points 1 ... 5 may be written:



$E_1 = E_0$
$E_2 = E_0 \exp\{-jk_0nL\} \exp\{gL\}$
$E_3 = E_0 R_1 \exp\{-jk_0nL\} \exp\{gL\}$
$E_4 = E_0 R_1 \exp\{-j2k_0nL\} \exp\{2gL\}$
$E_5 = E_0 R_1 R_2 \exp\{-j2k_0nL\} \exp\{2gL\}$

For longitudinal resonance, we require $E_1 = E_5$, so that:
$R_1 R_2 \exp\{2gL\} = 1$ (gain condition) **2**
$2k_0nL = 2\nu\pi$ or $2nL = \nu\lambda$ (phase condition) **2**

The variation of field amplitude as the wave travels up and down the cavity is then as below.



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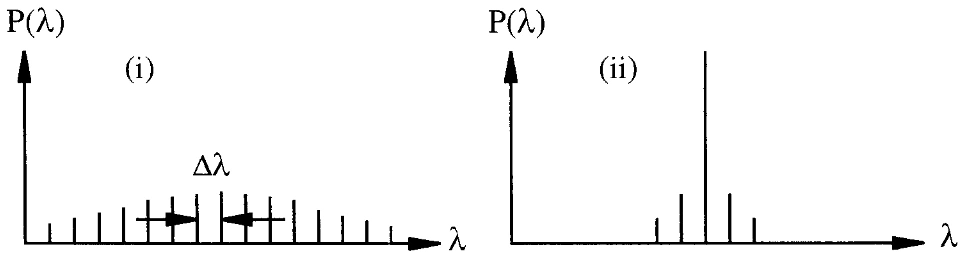
b) For two adjacent longitudinal modes, we may write:
$2nL = \nu\lambda_\nu$ and $2nL = (\nu+1)\lambda_{\nu+1}$ so $\lambda_\nu - \lambda_{\nu+1} = 2nL\{1/\nu - 1/(\nu+1)\} \approx 2nL/\nu^2$
The longitudinal mode separation is then $\Delta\lambda \approx \lambda_\nu^2/2nL$ **3**

i) Below threshold, the laser essentially acts as an LED. There are many modes, all weak, and the overall mode spectrum is limited by the laser gain bandwidth. **1**

ii) Above threshold, lasing takes place, and the nonlinear characteristics of the laser narrow the mode spectrum considerably. There are considerably fewer modes, with a much larger difference in power between the mode nearest to the peak in the gain bandwidth and adjacent modes. **1**

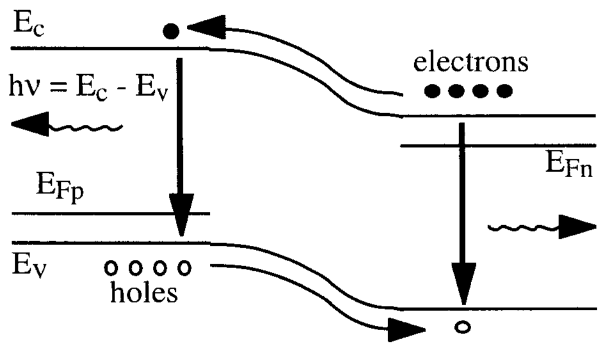


2+2

c) Re-arranging the expression for the longitudinal mode separation, the cavity length can be found as $L \approx \lambda_v^2/2n\Delta\lambda$. For a nominal optical wavelength of 1.5 μm and an effective index of 3.5, we then obtain a length of $L \approx (1.5 \times 10^{-6})^2 / (2 \times 3.5 \times 1.28 \times 10^{-9})$ m $\approx$ 250 μm. **2**

Re-arranging the gain condition, the gain coefficient for lasing is $g = (1/2L) \log_e\{1/R_1R_2\}$
For cleaved end mirrors, $R_1 = R_2 = \{n - 1\} / \{n + 1\} = \{3.5 - 1\} / \{3.5 + 1\} = 0.5555$
The required gain coefficient is then $g = (1/500) \log_e\{1/0.555^2\}$ $\mu m^{-1} = 2.35 \times 10^{-3}$ μm^{-1}. **2**

6. a) Recombination occurs in a semiconductor when electrons in the conduction band combine with holes in the valence band to create photons (light, useful) and phonons (heat). The photons have energy $h\nu = E_c - E_v$, where E_c and E_v are the energies of the two bands. Electroluminescence is a form of electrically-promoted radiative recombination, and can take place in a forward-biased p-n junction. The figure below illustrates the process; electrons from the n-side are injected into the p-side, where they combine with the large number of holes already present. At the same time, holes are injected into the n-side, where they combine with electrons; however, the junction is normally highly asymmetric, so that light generation takes place mainly on one side. **3**



2

The internal efficiency is limited by non-radiative recombination, which wastes a fraction of the injected electrons. The external efficiency is limited by total internal reflection of the vast majority of the light, due to the high refractive index of the semiconductor. The modulation bandwidth is limited by the recombination lifetimes. **3**

b) The rate equations for an LED are $dn/dt = I/ev - n/\tau_e$ and $d\phi/dt = n/\tau_{rr} - \phi/\tau_p$
In the steady state, $dn/dt = 0$ so that $n = I\tau_e/ev$; similarly, $d\phi/dt = 0$,
so the rate of loss of photons per unit volume is $\phi/\tau_p = n/\tau_{rr} = (I/ev)\ (\tau_e/\tau_{rr})$

The net optical flux is then $\Phi = \phi v/\tau_p = nv/\tau_{rr} = (I/e)\ (\tau_e/\tau_{rr})$
The number of photons generated per electron (i.e. the DC efficiency) is then $\eta' = \tau_e/\tau_{rr}$

When the LED is modulated, the drive current must contain a DC term together with the AC term, so assume that $I = I' + I'' \exp(j\omega t)$, $n = n' + n'' \exp(j\omega t)$ and $\Phi = \Phi' + \Phi'' \exp(j\omega t)$

When $dn/dt \neq 0$, substitution into the electron rate equation gives:
$n' = I'\tau_e/ev$ and $n'' = (I''/ev)\ \{\tau_e / (1 + j\omega\tau_e)\}$

Since $1/\tau_p$ is generally so big, we can still neglect $d\phi/dt$, so that $\Phi = nv/\tau_{rr}$ as before.
Equating the DC and AC components of Φ and nv/τ_{rr} separately, we obtain:
Thus, $\Phi' = (I'/e)\ \eta'$ and $\Phi'' = (I''/e)\ \eta''$ where $\eta'' = \eta' / (1 + j\omega\eta'\tau_{rr})$.
This is a first-order response, with a break point when $\omega\eta'\tau_{rr} = 1$.
The AC efficiency is $|\eta''| = \eta' / \sqrt{(1 + \omega^2\eta'^2\tau_{rr}^2)}$ **6**

c) Rearranging the expression for $|\eta''|$, we get $\tau_{rr} = \sqrt{\{(\eta' / |\eta''|)^2 - 1\}}\ (1/\omega\eta')$
From the data table, $\eta' = 0.5$ and $|\eta''| = 0.25$ at 10^8 Hz
(Data at 10^4 Hz must be well below the breakpoint) **1**

Hence $\tau_{rr} = \sqrt{\{(0.5/0.25)^2 - 1\}}\ .\ 1/(2\pi \times 10^8 \times 0.5)$ sec $= \sqrt{3} / (\pi \times 10^8)$ sec $= 5.51$ nsec **3**
Since $\eta' = \tau_e/\tau_{rr}$, $\tau_e = 0.5 \times 5.51$ nsec $= 2.75$ nsec **2**