E3.02
AC4

IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2007

MSc and EEE PART III/IV: MEng, BEng.and ACGI

Corrected Copy

INSTRUMENTATION

Monday, 30 April 10:00 am

Time allowed: 3:00 hours

There are SIX questions on this paper.

Answer FOUR questions.

All questions carry equal marks

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible First Marker(s) : C. Papavassiliou
Second Marker(s) : S. Lucyszyn

E302 INSTRUMENTATION

1. A medical thermometer is designed using a diode-connected n-channel MOSFET operating in weak inversion. A constant current source of 1nA is applied to the MOSFET and the resulting voltage drop is measured.

The MOSFET has a threshold voltage of $V_T = 0.1V$. In this application it is operating in the weak inversion region so that the I-V relationship of the diode-connected transistor is similar in form to that of a bipolar transistor:

$$I_{DS} = I_0 e^{\frac{e(V_{GS}-V_T)}{kT}} \ , \ I_0 = 500pA \tag{1.1}$$

In the weak inversion regime the FET transit frequency is proportional to the drain bias current:

$$f_T = \frac{g_m}{2\pi C_{GS}} = K \cdot I_{DS} \ , \ K = 10^{12} Hz/A. \tag{1.2}$$

The absolute zero temperature is $-273.15\ °C$. The Boltzman constant is $k = 1.38 \times 10^{-23} J/K$. The electron charge is $e = 1.6 \times 10^{-19} C$.

a) Draw a schematic diagram of the proposed thermometer. Assume the necessary voltmeter is available. Write an expression for the voltage drop on the thermometer as a function of temperature. Calculate this voltage drop at a nominal human body temperature of $T = 37\ °C$. [2]

b) Calculate the small-signal resistance of this thermometer at $T = 37\ °C$. Write an expression for the Johnson noise voltage developed on the thermometer. Calculate the Johnson voltage noise at $T = 37\ °C$. What is the measurement bandwidth if it is determined by the pole formed by the forward resistance and the gate-source capacitance of the FET? [5]

c) Write an expression for the shot noise amplitude of the current source over the measurement bandwidth. Evaluate the shot noise amplitude at $T = 37\ °C$. [3]

d) Write an expression for the signal to noise ratio and evaluate it at the nominal human body temperature. Use the Shannon formula to calculate the maximum data rate in bits/s. [5]

e) Assume the voltmeter used to read this thermometer has a noise figure of $N = 20dB$. What is the resolution of the instrument (in bits) if 1 measurement/second needs to be taken? [5]

[Total: 20]

2. a) The schematic of a generic Wheatstone bridge with arbitrary impedances on its branches is shown of figure 2.1. Write an equation for the voltage developed across terminals N3, N4 of the bridge if an AC voltage $V = V_0$ is applied across terminals N1 N2. [5]

b) Design a capacitance meter using a Wheatstone bridge with a null detector, as follows: Use a fixed parallel resistor-capacitor network on branch Z2. Place the unknown capacitor in parallel with a fixed resistor on a branch of your choice. Use resistors for the remaining branches. Draw a schematic of this instrument, including the driving source and the null detector. Write equations for the balance condition for this instrument assuming a vector voltmeter is used as a null detector. Which of the resistors needs to be variable if we wish the unknown capacitance to be proportional to the resistance of the variable resistor when the bridge is balanced? At what frequency should the bridge be operated? [10]

c) Sketch the null detector voltage dependence on capacitance if an RMS voltmeter is used as a null detector. What is the sensitivity of the bridge in this case? What are the implications of using an RMS null detector in the presence of noise? [5]

[Total: 20]

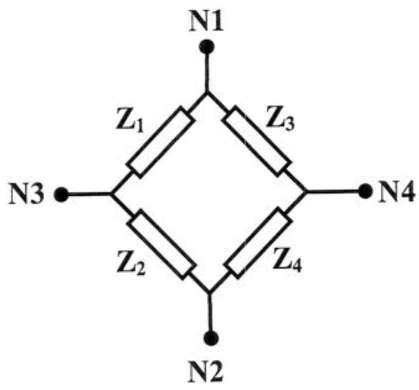


Figure 2.1 A Wheatstone bridge.

3. The following components are available for the design of a frequency multiplier to be operated from a 5V DC power supply.

- XOR gates, TTL counters and op-amps.
- $10pF$ and $100pF$ capacitors
- $10k\Omega$, $47k\Omega$ and $100k\Omega$ resistors
- a VCO whose output frequency is $f = 1kHz + V/2\pi RC$. Where V is the voltage applied at the input of the VCO and R and C are a resistance and a capacitance externally connected to the VCO. Please note that the VCO control voltage needs to be smaller than the power supply.

a) Draw a schematic of a circuit that can multiply the frequency of an input signal by an integer constant. [5]

b) Write the steady-state transfer function of the multiplier in part 3(a) above. What should the loop filter be so that the multiplier can track a linear frequency ramp applied to the input? Derive an expression for the steady state phase error for a frequency ramp input. [8]

c) Design a multiply-by-16 frequency multiplier using only components from the list. The input frequency is $1kHz < f < 10kHz$. The multiplier should be capable of tracking an input linear frequency ramp. Calculate the gains of the circuit blocks of this PLL. Calculate the steady-state phase error of this frequency multiplier under a frequency ramp input of 100Hz/sec. What is the steady-state phase error for this frequency multiplier for a step change in the input frequency? [7]

[Total: 20]

4. a) Draw a diagram for a dual-slope A/D converter and explain its operation. Sketch important waveforms on the converter assuming a constant input. [5]

b) Derive a relationship between the maximum frequency and the maximum amplitude of the input signal to a dual-slope A/D converter. State any assumptions made. Assume the comparator has an internal delay δ. Show that in the limit of the input voltage being much larger than the reference voltage and with a very fast comparator, the product of maximum frequency and maximum amplitude of the input signal needs to be smaller than a certain constant. Calculate the value for this constant. [5]

c) Draw a schematic of a first order $\Sigma\Delta$ A/D converter. Identify all its components. Write expressions for the transfer function of the signal and of the quantisation noise in a $\Sigma\Delta$ A/D converter. What is the transfer function of the signal to quantisation noise ratio? What is the SNQR of the converter? It is given that the SNQR of an N-bit Nyquist converter is $SNQR = 3 \cdot 2^{2N-1}$. [5]

d) Derive an equation relating the effective number of bits of a $\Sigma\Delta$ A/D converter to the signal bandwidth, sampling frequency and order of the loop filter (which is assumed to be an ideal filter with N poles at zero frequency). Calculate the required filter order in order to make a professional audio converter capable of a 24 bit resolution at 96 k samples/sec. The clock frequency is 6.144MHz. Use a 1 bit DAC. [5]

[Total: 20]

5. a) Draw the schematic diagram of a lock-in amplifier and identify all its sub-circuits. Briefly explain the operation of a lock-in amplifier. [5]

b) A radiation meter has an output of $1\mu V$ DC superimposed onto 1mV $50Hz$ RMS AC noise.

i) What is the initial signal to noise ratio of this measurement in dB? [3]

ii) Describe how a lock-in amplifier operating at 10Hz can be used to measure this signal to 3 significant digits. Specify the first order filter required to make this measurement. [6]

iii) How long would this measurement take to settle to 95% of its final value? [3]

iv) What should the order of the filter be for the measurement to be possible in less than 1 hour? [3]

You may assume that any amplifiers you use are noiseless. State any other assumptions you make.

[Total: 20]

6. A collection of many data points $\{x_i, y_i\}$ is available from a measurement. We need to describe this data by a quadratic model $y = \alpha x^2 + \beta \sin x$.

a) Derive an expression that needs to be minimised in order to 'fit' the proposed quadratic model to the data. [10]

b) Derive expressions for the 'best', in the least squares sense, values of the coefficients α, β . [10]

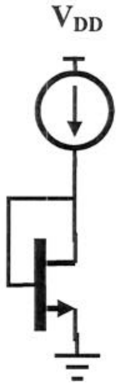
INSTRUMENTATION E3.02

06/07 AC4

ANSWERS

Question 1: (computed example)

a)



$$I_{DS} = I_0 e^{\frac{q}{kT}(V_{gs}-V_T)} \Rightarrow V_{gs} = V_T + \frac{kT}{q}\ln\left(\frac{I_{DS}}{I_0}\right)$$. at T=37C

$, V_{gs} = 0.1 + 26.75mV \times 0.6908 = 0.1185V$

[2]

b) Small signal resistance:

$$R_D = \left(\frac{dI_{ds}}{dV_{gs}}\right)^{-1} \Rightarrow R = \frac{kT}{qI_{DS}} = 26.75M\Omega$$

The noise bandwidth is $B = f_T = 10^{12} \bullet 10^{-9}\,Hz = 10^3\,Hz$

Johnson noise is $V_{J,RMS} = \sqrt{4kTRB} = 21.4\mu V$ [5]

c) Shot noise $I_{RMS} = \sqrt{2eI_{DS}B}$

This results to a voltage noise amplitude of

$V_{S,RMS} = R_D\sqrt{2eI_{DS}B} = 21.4\mu V$

[3]

d) The total noise voltage is: $V_N = \sqrt{V_{J,RMS}^2 + V_{S,RMS}^2} = 22\mu V$

The signal to noise ratio is then (note that the threshold is an offset, so it cannot included in the SNR):

$SNR = \left(\frac{1.848 \bullet 10^{-2}}{22 \bullet 10^{-6}}\right)^2 = 7.1 \bullet 10^5$. The data rate is

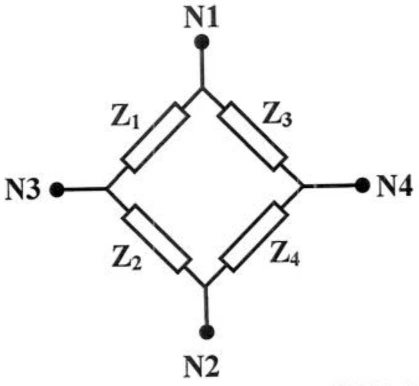
$D = B\log_2(1+SNR) = 1000 * \log_2(7.1 \cdot 10^5) = 1944$ bits/s [5]

If a 20dB noise figure is used to measure this, then SNR at rhe output of the amplifier is $SNR_{OUT} = 7.1 \bullet 10^5 \bullet 0.01 = 7.1 \bullet 10^3$. The number of bits follows from the Shannon capacity formula (or if the student prefers, the quantisation noise formula)

$N = B\log_2(1+SNR) = 19bits$

[5]

Question 2: (bookwork+ application of theory)
a) [bookwork]



N1 is driving, N2 is ground N3-N4 is the null detector voltage.

$$V_{NULL} = v_S \left(\frac{Z_2}{Z_1 + Z_2} - \frac{Z_4}{Z_3 + Z_4} \right) = V_s \left(\frac{Z_2 Z_3 - Z_1 Z_4}{(Z_1 + Z_2)(Z_3 + Z_4)} \right)$$

[5]

b) [application of theory]
Use fixed R//C in branch 2 , and unknown C // known R in branch 4.
The balance condition is:

$$Z_2 Z_3 - Z_1 Z_4 = 0 \Rightarrow R_1 \frac{R_4}{1 + j\omega R_4 C_x} = R_3 \frac{R_2}{1 + j\omega R_2 C_2} \Rightarrow$$

equate real parts: $R_1 R_4 = R_2 R_3$

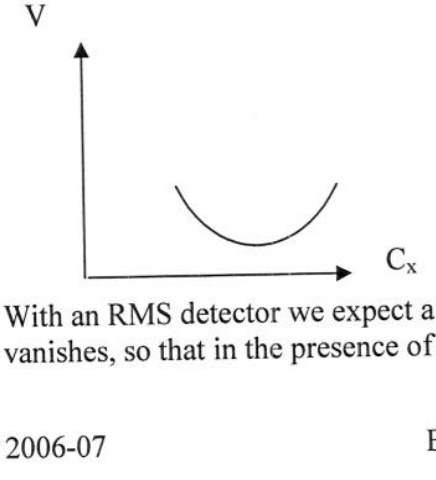
equate imaginary parts: $\omega R_4 C_x R_2 R_3 = \omega R_2 C_3 R_1 R_4 \Rightarrow C_x R_3 = C_3 R_1$

It follows that R_1 needs to be made variable so that $C_x = R_1 / R_3$.

The frequency has cancelled from the balance condition, so it appears not to be critical. The best frequency, though, would be the fixed RC pole frequency.

[10]

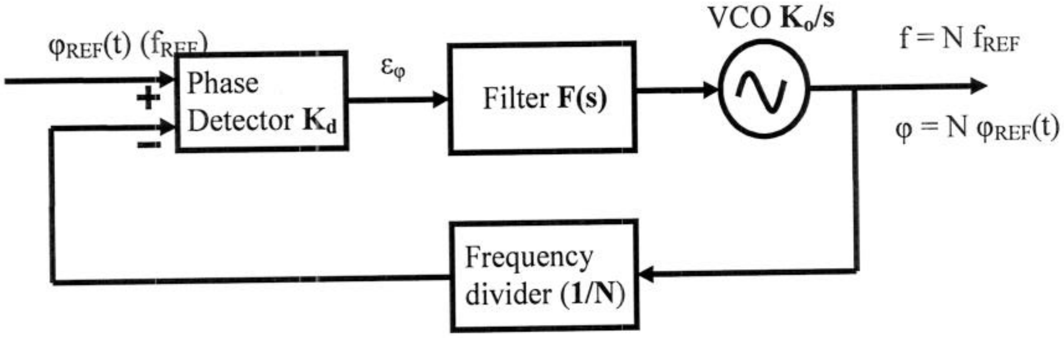
c)



With an RMS detector we expect a parabolic response curve. On balance the sensitivity vanishes, so that in the presence of noise the instrument uncertainty is large. [5]

Question 3)

a) [bookwork]



[5]

b) [bookwork+ application]

$$B(s) = \frac{\varphi_{out}}{\varphi_{in}} = \frac{K_d K_o F(s)}{s + K_d K_o F(s)/N} = \frac{NKF(s)}{s + KF(s)}$$

The steady state phase error is (by the final value theorem:

$$\delta\varphi = \lim_{t\to\infty} \varepsilon_\varphi(t) = \lim_{s\to 0} s\varepsilon_\varphi(s)$$

and

$$\frac{\varepsilon_\varphi(s)}{\varphi_i(s)} = \frac{K_d s}{s + K_d K_o F(s)/N}$$

then,

$$\delta\varphi_{SS} = \lim_{s\to 0} s\varepsilon_\varphi(s) = \lim_{s\to 0} s\theta(s)\frac{K_d s}{s + K_d K_o F(s)/N}$$

since the Laplace transform of the input phase disturbance is: $\mathcal{L}\,\theta(t) = a\mathcal{L}\,t^n = \frac{a}{s^{n+1}}$, n=2

$$\delta\varphi_{SS} = \lim_{s\to 0} s\theta(s)\frac{K_d s}{s + K_d K_o F(s)/N} = \lim_{s\to 0}\frac{as^{-1}}{s + K_d K_o F(s)/N} = \lim_{s\to 0}\frac{a}{sK_d K_o F(s)/N}$$

It follows that $F(s) = a/s$, i.e. the filter must be an ideal integrator.

[8]

c) [computed example]

$K_d = \frac{\pi}{10}$

$K_0 = \frac{1}{RC}$ Since the maximum output frequency needs to be 160kHz=1Mrad/s,

$5K_0 > 10^6 \Rightarrow \frac{5}{RC} > 10^5 \Rightarrow RC < 5\cdot 10^{-5}$. Choose $R = 47k\Omega$, $C = 100pF$. Then,

$$K_0 = \frac{1}{RC} = 2.128 \cdot 10^5 \, rad/V$$

The steady state phase error, if the filter transfer function is $F(s) = \frac{1}{R_{int} C_{int} s}$ (R_{int} , C_{int} are the ideal integrator R and C respectively), is:

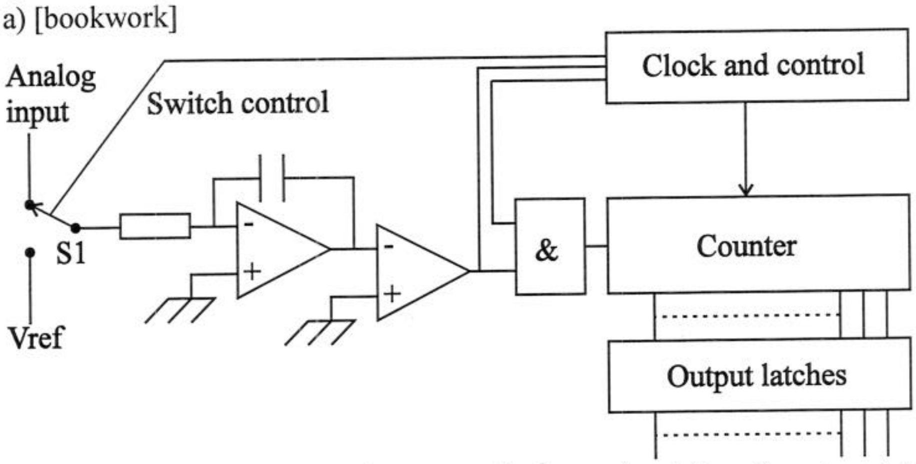
$$\delta\varphi_{SS} = \lim_{s \to 0} \frac{a}{sK_d K_o F_0 / Ns} = \frac{\alpha N}{K_d K_o F_0} = \frac{16 \times 628 \times R_{int} C_{int}}{\pi / 10 \times 2.128 \cdot 10^5} \Rightarrow$$

$\delta\varphi_{SS} \leq 1.5 \times 10^{-6}$ rad

The phase error for a step input change is zero. [7]

Question 4:

a) [bookwork]



A dual slope ramp converter integrates the input signal for a time t_1 and then subtracts from it the integral of a fixed voltage until the output reaches again zero, which takes a time $\mathbf{t_2}$. If an intermediate output $\mathbf{V_{int}}$ is reached after $\mathbf{t_1}$, if $\boldsymbol{\tau}$ is the integrator time constant then:

$$V_{int} = \frac{t_1}{\tau} V_{in} = \frac{t_2}{\tau} V_{ref} \Rightarrow V_{in} = \frac{t_2}{t_1} V_{ref}$$

The logic times $\mathbf{t_2}$.

$$V_{int} = \frac{t_1}{\tau} V_{in} = \frac{t_2}{\tau} V_{ref} \Rightarrow t_2 = t_1 \frac{V_{in}}{V_{ref}}$$ [5]

(b) [theory extension]

The total conversion time is : $T = t_1 + t_2 + \delta$ so that the maximum input frequency is

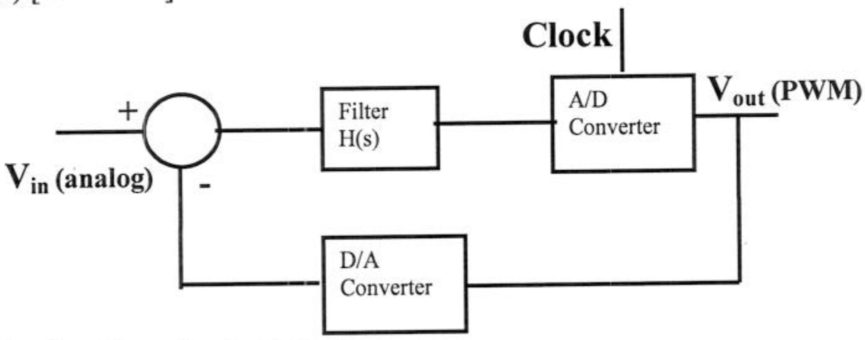
$$f_{max} = \frac{1}{2(t_1 + t_2 + \delta)} = \frac{1}{2t_1 \left(1 + \frac{V_{in}}{V_{ref}} + \frac{\delta}{t_1}\right)}$$

in the limit of input much larger than the reference voltage and negligible delays, this reduces to:

$f_{max} = \frac{V_{ref}}{2t_1 V_{in}} \Rightarrow f_{max} V_{in} = \frac{V_{ref}}{2t_1}$ Which establishes the absolute upper limit for this quantity.

[5]

(c) [bookwork]



the signal transfer function is:

$$G_s(s) = \frac{H(s)}{1+H(s)}$$

while the quantisation noise transfer function is:

$$G_E(s) = \frac{1}{1+H(s)}$$

The SNQR transfer function will then be: $G_{SNQR} = H(s)$.

If we require the signal to quantisation noise ratio at much a lower frequency f_N (since, after all we are sampling at a frequency that is much greater than the Nyquist frequency), the in-band signal to quantisation noise ratio will approximately be:

$$\frac{S}{E}(f_N) = \left(\frac{3 \cdot 2^{2N-1} f_s}{f_N}\right)\frac{H^2(f_N)}{H^2(f_s)}$$ [5]

d) [bookwork+example]

assuming an ideal filter of order M, we can write:

$$|H(s)| = \frac{A}{s^n} \Rightarrow$$

$$\frac{S}{E}(f_N) = \left(\frac{3 \cdot 2^{2N-1} f_s}{f_N}\right)\frac{H^2(f_N)}{H^2(f_s)} = 3 \cdot 2^{2N-1}\left(\frac{f_s}{f_N}\right)^{2n+1}$$

if we define the oversampling ratio: $OSR = f_s / f_N = 2^k$ then we can write

$SNQR = 3 \cdot 2^{2N-1+2k(n+1/2)}$ from which it follows that

$ENOB = N + k(n+1/2)$

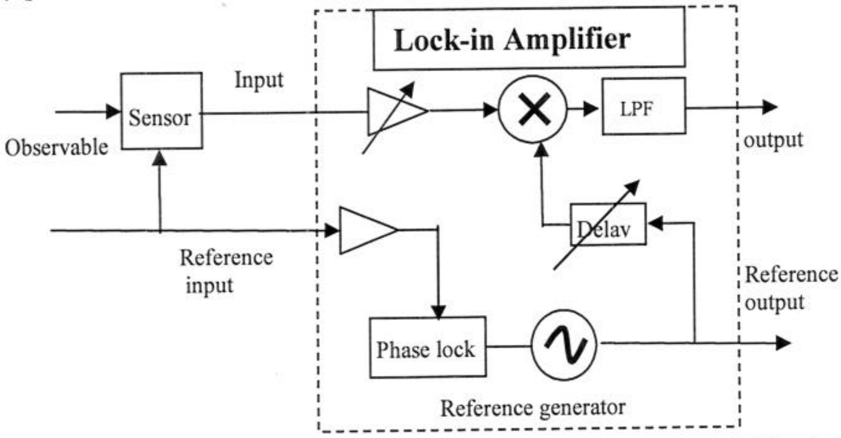
For the numbers given, $OSR = 2^6 \Rightarrow k = 6$, and we require

$ENOB = 24 = 1 + 6(n+1/2) \Rightarrow n = 3.33$. We need a 4th order filter

[5]

Question 5)

a) [bookwork]



The lock-n amplifier is a laboratory version of a chopper stabilized amplifier.
A very low LPF at the output allows arbitrary rejection of interference and/or noise. [5]

(b) [Computed Example]

The initial SNR is: $SNR = \frac{10^{-9}}{10^{-6}} = 20\log\left(10^{-3}\right) = -60dB$

We can use an external 10Hz generator to amplitude modulate the input signal, and then apply this to a lock-in instrument. The low pass filter must be such that at 50 Hz it provides an attenuation of:

$$H(50Hz) = \frac{10^{-12}}{10^{-6}} = 10^{6}$$

For a 1st order filter this implies a settling time of

$$\Delta t = 2.2\frac{10^{6}}{2\pi} = 3.5\cdot 10^{5}\ \text{sec} = 9.7\ \text{hours}$$

To do the measurement within an hour we need a filter of order N:

$$(\omega\tau)^{N} \geq 10^{6}\ ,\ \omega{=}100\pi,\ \tau{<}3600 \Rightarrow$$

$$\ln\omega + N\ln 3600 > \ln 10^{6} \Rightarrow N > 9.8$$

i.e. we would need a 10^{th} order filter

[15]

Question 6) [application]

The model is of the form
$y = \alpha z + bw$, $z = x^2$, $w = \sin(x)$
We want to minimise the objective function:

$$f = \sum_i (y_i - \alpha z_i - bw_i)^2 \Rightarrow$$

$$\left.\begin{array}{l} \dfrac{\partial f}{\partial a} = 0 \Rightarrow \sum_i (y_i - \alpha z_i - bw_i) z_i = 0 \\ \dfrac{\partial f}{\partial \beta} = 0 \Rightarrow \sum_i (y_i - \alpha z_i - bw_i) w_i = 0 \end{array}\right\} \Rightarrow$$

$$\begin{bmatrix} \sum_i z_i^2 & \sum_i z_i w_i \\ \sum_i z_i w_i & \sum_i w_i^2 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \sum_i y_i z_i \\ \sum_i y_i w_i \end{bmatrix} \Rightarrow$$

$$\alpha = \frac{\sum_i y_i z_i \sum_i w_i^2 - \sum_i y_i w_i \sum_i z_i w_i}{\sum_i z_i^2 \sum_i w_i^2 - \left(\sum_i z_i w_i\right)^2}$$

$$\beta = \frac{\sum_i z_i^2 \sum_i y_i w_i - \sum_i y_i z_i \sum_i z_i w_i}{\sum_i z_i^2 \sum_i w_i^2 - \left(\sum_i z_i w_i\right)^2}$$

[20]