Paper Number(s): **E3.02**
AM4

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE
UNIVERSITY OF LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2001

MSc and EEE PART III/IV: M.Eng., B.Eng. and ACGI

INSTRUMENTATION

Monday, 30 April 10:00 am

There are SIX questions on this paper.

Answer FOUR questions.

Corrected Copy

Qu 1(e)

Time allowed: 3:00 hours

Examiners: Papavassiliou,C. and Burdett,A.J.

Special instructions for invigilators: None

Information for candidates: None

1. This question is about noise and signal recovery.

   a) Define the noise factor and noise figure of a signal processing device. Argue whether the output *S/N* ratio of a device can be larger, smaller than or equal to that at the input. What is the *S/N* ratio of a voltage source V_s with internal resistance R_s ? What is, therefore, the *S/N* ratio of a 1.5 volt battery of internal impedance 0.1Ω over a bandwidth of 100 Hz?

   [5 marks]

   b) Write an expression for the noise factor of a cascade of three amplifiers of gains and noise factors G_1 F_1, G_2 F_2 and G_3 F_3 respectively. Derive an expression for the noise figure of the cascade of an infinite number of identical amplifiers of gain *G* and noise factors *F*. (Hint: notice that the cascade forms a geometric series) What is the limit of this noise factor as *G* tends to zero? as *G* tends to infinity?

   [5 marks]

   c) Derive the signal to noise ratio of a quantised sinusoidal signal, in terms of the quantisation step size.

   [5 marks]

   d) With the aid of a block diagram describe the ΣΔ A/D converter. Explain how it achieves multibit resolution despite its using a 1 bit converter internally. If a final sampling bandwidth B is required, what is the maximum resolution of the converter in terms of the oversampling ratio?

   [5 marks]

   e) Explain how the *S/N* ratio of a measurement can be enhanced by averaging a number *N* of repetitions of the measurement. How many measurements must be averaged if the *S/N* ratio is to be enhanced by a factor of 10? How many measurements are needed to enhance the Voltage signal to noise ratio by a factor of 10?

   [5 marks]

13:40pm. ↑ peak

2. In this question we study a binary weighted D/A converter.

a) Design a 4 bit binary weighted D/A converter. Draw and fully label schematics for this converter, including component values. You should choose component values so the error arising from the 10 μA input bias and offset current of the OPAMP you use is less than ½ LSB. Write an expression for the voltage output of this converter involving the 4 binary inputs.

[5 marks]

b) Define when a converter is monotonic. Assume the resistors you use to construct a binary weighted D/A converter are supplied with a tolerance of 2%, i.e. a resistor is guaranteed to have an actual value within 2% of its nominal value. Write an expression for the converter output involving the individual resistor tolerances. What is the maximum number of bits for which a binary weighted converter is guaranteed to be monotonic? Is your 4 bit converter of part (a) above guaranteed to be monotonic?

[10 marks]

c) Define the sensitivity of an instrument with respect to a component value. What are the sensitivities of the binary weighted D/A converter of part (a) with respect to each of the resistors you use?

[10 marks]

3. In this question we will study a transimpedance amplifier

a) Draw and fully label the schematic for a simple OPAMP based transimpedance amplifier.

[2 marks]

b) You are given the following specifications for the OPAMP:

- It is a dominant pole OPAMP with an open loop gain G_0 and a pole at ω_0.
- The OPAMP has zero input admittance and zero output impedance.
- The OPAMP has zero voltage offset and current offset and bias.

Write an expression for the transimpedance gain as a function of frequency for a feedback resistance R_f. Write an expression for the input impedance of the transimpedance amplifier as a function of frequency.

[6 marks]

c) Write expressions for the sensitivities as a function of frequency of the transimpedance gain to

- the value of the feedback resistor
- the value of the open loop gain of the OPAMP
- the value of the dominant pole frequency

What happens to these sensitivities in the extreme cases that the open loop gain is large or unity?

[9 marks]

d) Describe and write expressions for the errors in the current measurement that will arise if the OPAMP deviates from ideality in each of the following ways:

- Has a non-zero offset current.
- Has a non-zero bias current (Hint: this is equivalent to a non-zero input admittance)

[8 marks]

4. General measurements

a) Define the resolution and sensitivity of a measurement in general.

[5 marks]

b) List and briefly discuss 2 methods for measuring each of the following quantities. Qualitatively comment on the sensitivity and resolution of each of the measurements you describe. Use appropriate diagrams to clarify your descriptions.

- Voltage
- Distance
- Frequency
- Magnetic field
- Velocity

[2 marks each method]

5. Wheatstone bridges.

a) Draw a diagram for a wheatstone bridge. Describe its operation. Derive its balance condition.

[5 marks]

b) Calculate the bridge voltage output sensitivity to the impedance value of one of its branches.

[5 marks]

c) Devise an AC bridge that can be used to measure capacitance. You may use variable resistors, capacitors or inductors. Solve your bridge for the balance condition, and derive its sensitivity to the change in the unknown capacitance. How can we minimise the dependence of the sensitivity on the magnitude of the unknown capacitance?

[5 marks]

d) Suggest a way to extend the range of this instrument, and discuss advantages and drawbacks.

[5 marks]

e) Design, draw a circuit diagram and discuss an amplifier suitable for amplifying the error signal of the bridge. As the error lies on a large common mode signal your amplifier must have a high *CMRR*. Write an expression for the differential gain of this amplifier

[5 marks]

6. Oscilloscopes.

a) Define the risetime of an oscilloscope. Write an expression for the risetime if the vertical amplifier has a 3dB bandwidth B.

[5 marks]

b) The electron gun in the CRT has a current I_G. If 1 photon is generated for each electron hitting the screen derive an expression for the trace intensity (i.e. number of photons per unit trace length) when a sine wave of frequency f is displayed at a timebase setting such that the total time displayed horizontally is T.

[6 marks]

c) Describe and draw a qualitative schematic of the trigger circuit. Explain how events preceeding the nominal trigger time can be displayed.

[6 marks]

d) Describe how the electronically intensified sweep method works. Draw an appropriate diagram to assist your description. Explain how this circuit can be used to augment the trace intensity.

[8 marks]

Question 1

a) Noise factor: $F = \dfrac{S/N|_{in}}{S/N|_{out}}$ (S, N powers) [1]

Noise figure: $N = 10\log_{10} F = (\text{dB})\, F$ [1]

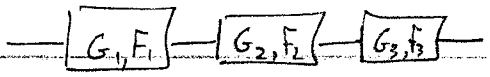
Always $F > 1$ or $S/N|_{IN} \geq S/N_{out}$ [1]

A voltage source $\{V_s, R_s\}$ has Johnson noise:

$V_N^2 = 4kTRB$. then, the signal to noise ratio

is: $S/N = \dfrac{V_s^2}{4kTRB}$ [1]

Substitute $V_s = 1.5$, $R = 0.1$ and $B = 100\,Hz$ [1]

b) 

$$F = F_1 + \frac{F_2 - 1}{G} + \frac{F_3 - 1}{G_1 G_2} + \dots$$ [2 marks]

If all amplifiers identical,

$$F_\infty = F + \frac{F-1}{G} + \frac{F-1}{G^2} + \dots \frac{F-1}{G^n} = \dots = \frac{FG-1}{G-1}$$ [2 marks]

if $G \to \infty$ $F_\infty \to F$

if $G \to 0$ $F_\infty \to \infty$ [1]

c) Q is the quantisation step.

for a converter output N, for input V,

$V = NQ + V_n$ and $|V_n| \leq \frac{1}{2} Q$.

Noise power: $P_N = V_n^2$ then the average noise is:

$$\overline{P_n} = \frac{2}{Q}\int_0^{Q/2} V_n^2 \, dV_n = \frac{1}{12} Q^2 .$$ [2]

Assuming the converter length n bits,

a sinusoidal can have amplitude

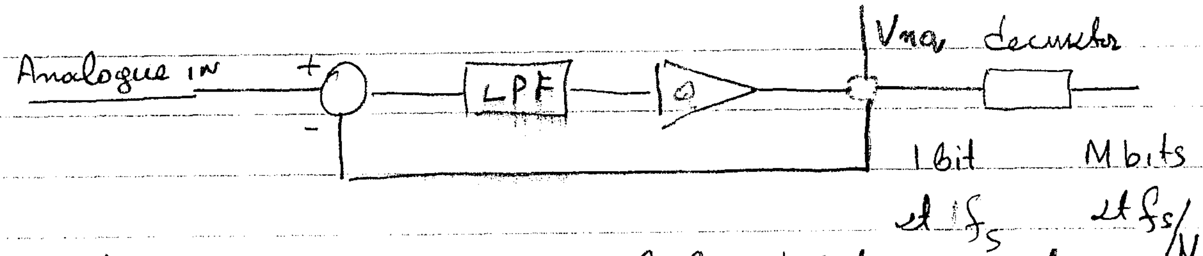
$V_0 = (2^{n-1}) Q$ and a corresponding power:

$$\overline{S} = 2^{2n-3} Q^2$$ [2]

the signal-to-noise ratio is:

$$S/N = \frac{2^{2n-3} Q^2}{\frac{1}{12} Q^2} = 3 \cdot 2^{2n-1}$$ [1]

(d)



Operation: the oversampled 1 bit quantiser output is feed back and subtracted from the signal. This results into the signal transfer function being low pass while that of the quantisation noise high pass.

For a simple low pass loop filter the signal to noise ratio scales as a function of Bandwidth:

$$S/N(B) \sim \frac{1}{B^2}$$

[5]

(e) If M measurements are averaged,

Signal adds algebraically. =>

$$\overline{V_S} = \frac{\sum V_S}{M} = V_{S\phi}$$

while noise vectorially:

$$\overline{V_N} = \frac{1}{N}\sqrt{\sum V_N^2} = \frac{1}{M}\sqrt{M}\,V_{N\phi} = \frac{1}{\sqrt{M}} V_{N\phi}$$

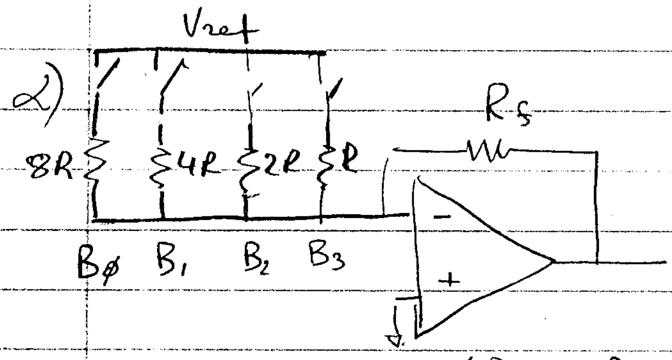
$$\text{then } S/N_v = \frac{V_S^2}{V_N^2} = M \frac{V_{S\phi}^2}{V_{N\phi}^2}$$

If S/N is to be enhanced by a factor of 10 then we need to average 10 measurement. Note that the VOLTAGE S/N scales as $\sqrt{M}$.

[5]

Question 2

a)



[1]

$$V_{out} = -V_{ref}\left(\frac{B_0}{8R} + \frac{B_1}{4R} + \frac{B_2}{2R} + \frac{B_3}{R}\right)R_F$$

We want $\frac{I_{LSB}}{2} \geq i_{in} \Rightarrow \frac{10V}{8R} \geq 20\mu A \Rightarrow R \leq 6.25K\Omega$

[2]

Full scale should be −10V, then:

$15 \cdot I_{LSB} \cdot R_F = 10 \Rightarrow 15 \cdot \frac{10V}{8R} \cdot R_F = 10 \Rightarrow$

$R_F = \frac{8R}{15}$

[2]

b) A converter is monotonic if for two input codes $C_1 > C_2$ $V_{out}(C_1) > V_{out}(C_2)$

for the present converter which is current mode, this translates to

$C_1 > C_2 \Rightarrow i(C_1) > i(C_2)$

Converter output with resistors that may vary:

$$V_{out} = -V_{ref} \cdot R_F(1+\varepsilon_F) \cdot \left(\frac{B_0}{8R(1+\varepsilon_0)} + \frac{B_1}{4R(1+\varepsilon_1)} + \frac{B_2}{4R(1+\varepsilon_2)} + \right.$$

$$+ \frac{B_3}{R(1+\varepsilon_3)}\Bigg) \simeq$$

$$= -V_{Ref} \cdot R_f(1+\varepsilon_f)\left[\left(\frac{B_0}{8R} + \frac{B_1}{4R} + \frac{B_2}{2R} + \frac{B_3}{R}\right) - \left(\frac{B_0\varepsilon_0}{8R} + \frac{B_1\varepsilon_0}{4R} +\right.\right.$$

$$\left.\left. + \frac{B_2\varepsilon_0}{2R} + \frac{B_3\varepsilon_0}{R}\right)\right]$$

the biggest possibility for non-monotonicity is the transition $0111 \to 1000$.

in this case, the output step is: ...

$$\delta V_{out\,7\to 8} = -V_{ref} R_f(1+\varepsilon_f)\left[\frac{1}{8R} - \frac{\varepsilon_0}{R} + \frac{\varepsilon_1}{2R} + \frac{\varepsilon_2}{4R} + \frac{3\varepsilon_3}{8R}\right]$$

In terms of the various ε, worst case is $\varepsilon_0 = -\varepsilon_2 = -\varepsilon_1 = -\varepsilon_0$. then for non monotonicity,

$$\frac{1}{8R} - \frac{15\varepsilon}{8R} = \phi \Rightarrow \varepsilon = \frac{1}{15} = 6\%$$

the 4 bit converter is monotonic.

the general expression comes to:

$$(2^{n}-1)\varepsilon = 1 \qquad [10]$$

c) Sensitivity:

$$S = \frac{d \ln A}{d \ln B}$$

'A is the instrument reading', B the component value.

the converter equation is, again:

$$V_{out} = -V_{ref} R_f \left(\frac{B_0}{R_0} + \frac{B_1}{R_1} + \frac{B_2}{R_2} + \frac{B_3}{R_3} \right)$$

$$S_{RF} = \frac{\partial \ln V_{out}}{\partial \ln R_f} = \frac{\partial V_{out}}{\partial R_f} \frac{R_F}{V_{out}} = \frac{V_{out}}{V_{out}} = 1.$$

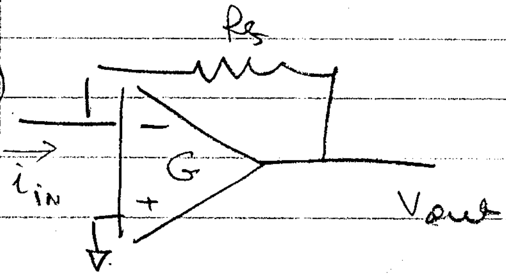
$$S_{R_0} = \frac{R_0}{V_{out}} \frac{\partial V_{out}}{\partial R_0} = \frac{R_0}{V_{out}} \quad V_{ref} R_f \frac{B_0}{R_0^2} = \frac{V_{ref} R_f B_0}{V_{out} R_0}.$$

the sensitivity depends on the code, as well as on the components!

☐⊙]

Question 3.

a)



[2]

b) $G = \dfrac{G_0}{1+j\omega/\omega_0}$ [1]

transimpedance gain:

$$Z_T = \frac{\partial V_{out}}{\partial i_{IN}} = -\frac{G R_f}{1+G} = -\frac{\frac{G_0}{1+j\omega\tau}}{1+\frac{G_0}{1+j\omega\tau}} R_f =$$

$$= -\frac{G_0}{(1+G_0)+j\omega\tau} R_f = -\frac{G_0}{1+G_0} R_f \frac{1}{1+j\omega\frac{\tau}{1+G_0}}$$

[6]

(c) $S_{R_f} = 1$

$S_{G_0} = \dfrac{G_0}{Z_T} \dfrac{\partial Z_T}{\partial G_0} = \ldots$

$S_{\omega_0} = \dfrac{\omega_0}{Z_T} \dfrac{\partial Z_T}{\partial \omega_0} = \ldots$ [8]

(d) An offset current offsets the output:

$V_{out} = i Z_T + i_{off} Z_T$ [4]

A bias current with a non zero input admittance forms an admittance divider:

$i_{effective} = i_{IN} \cdot \dfrac{Y_T}{Y_T + Y_{in}}$, $Y_{in} = \dfrac{1+G}{R_f}$ [4]

Question 4

(a) Resolution: Smallest detectable input change [2].

Sensitivity: Ratio of electrical output change to input change [3].

(b) Voltage: Voltmeter
V/f converter
A/D
Oscilloscope

Distance: Meter/vernier
Reflectometry: light, sound.
Interferometry

Frequency: Counters
Clocks

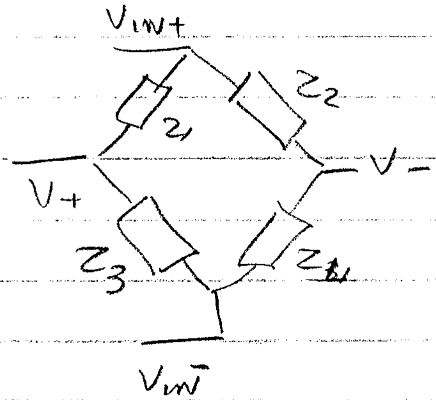
Magnetic field: Hall Bars
Rot. Magnetometers
Quantum Hall effect.
SQUID

Velocity: timing.
Interferometry.
Doppler

... [2 ea]
10 answers total

Question 5

a)



Balance $V_+ - V_- = (V_{in+} - V_{in-}) \left(\frac{Z_3}{Z_1+Z_3} - \frac{Z_4}{Z_2+Z_4} \right) =$

$$= V_{in} \frac{Z_3 Z_2 - Z_1 Z_4}{(Z_1+Z_3)(Z_2+Z_4)}$$

Balance when $V_{out} = 0 \Rightarrow Z_3 Z_2 = Z_1 Z_4$

[5]

b) Sensitivity:

$S = \frac{\partial V_{out}}{\partial Z_i}$: for Z_2 or Z_3

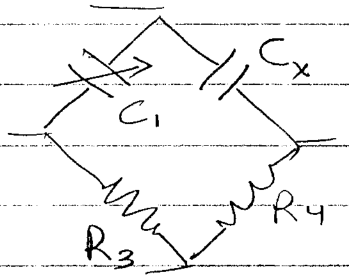
$$S_2 = \frac{(Z_1+Z_3)(Z_2+Z_4) Z_3 - (Z_3 Z_2 - Z_1 Z_4)(Z_1+Z_3)}{(Z_1+Z_3)^2 (Z_2+Z_4)^2} V_{in}$$

$$= \frac{2 Z_1 Z_3 Z_4 + (Z_1^2 + Z_3^2) Z_4}{(Z_1+Z_3)^2 (Z_2+Z_4)^2} V_{in}$$

Similarly the other sensitivities, $S_1, S_4 < 0$.

[5]

c) A possibility



then $C_x = C_1 \frac{R_4}{R_3}$

$$S = \frac{2\frac{R_3 R_4}{j\omega C_1} + \left(-\frac{1}{\omega^2 C_1^2} + R_3^2\right)}{\left(R_3 + \frac{1}{j\omega C_1}\right)^2 \left(\frac{1}{j\omega C_x} + R_4\right)}$$

to make the bridge linear, make

$\frac{1}{\omega C_1} << R_3$ and $\frac{1}{\omega C_x} << R_4$

[5]

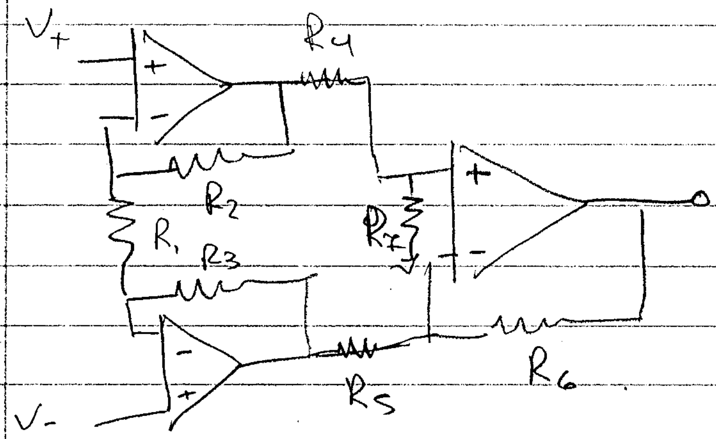
d) the range is presumably limited by the range of the variable C_1.

Since $C_x = C_1 \frac{R_4}{R_3}$ we can alter the ratio $\rho = \frac{R_3}{R_4}$. the sensitivity depends on this ratio:

$$S = \frac{\frac{2\rho}{j\omega C_1} + \left(\rho^2 - \frac{1}{\omega^2 C_1^2}\right)}{\left(\rho + \frac{1}{j\omega C_1 R_4}\right)\left(1 + \frac{1}{j\omega C_x R_4}\right)}$$

[5]

e) For the output we can use an instrumentation amplifier:



$R_2 = R_3$, $R_4 = R_5$, $R_6 = R_7$.

if $V_{in} = V_+ - V_-$ the differential gain is

$$G_d = \left(1 + 2\frac{R_2}{R_1}\right)\left(\frac{R_6}{R_5}\right)$$

and common mode gain is approx. 1.

[5]

Question 6

a) If an ideal step of amplitude V_0 is applied to the input of the scope, the rise time is the time the trace needs to go from 10% to 90% of the final value

Assuming input amplifier is a low pass filter with a pole at τ,

$V(t) = V_0(1 - e^{-t/\tau})$ then the rise time is defined between t_1: $V(t_1) = 0.1V_0$

$\Rightarrow t_1 = \tau \ln(0.9)$ and

t_2: $V(t_2) = 0.9V_0 \Rightarrow t_2 = \tau \ln(0.1) \Rightarrow$

$t_{RISE} = t_2 - t_1 = \tau \ln(9) = 2.2\tau$. [2]

b) The minimum intensity is at the maximum trace velocity, ie. near $\sin(\omega t) = 0$.

If the timebase is $T/10\,\text{cm}$ the max velocity is $\sqrt{2} \times \frac{10\,\text{cm}}{T}$ and the linear intensity is just: $I_{min} = I_{gun}/V_{max} = \frac{I_{gun} T}{14.2\,\text{cm}}$

For a gun current of $1\mu A$ and a timebase of 10 nsec, the number of photons per linear cm is:

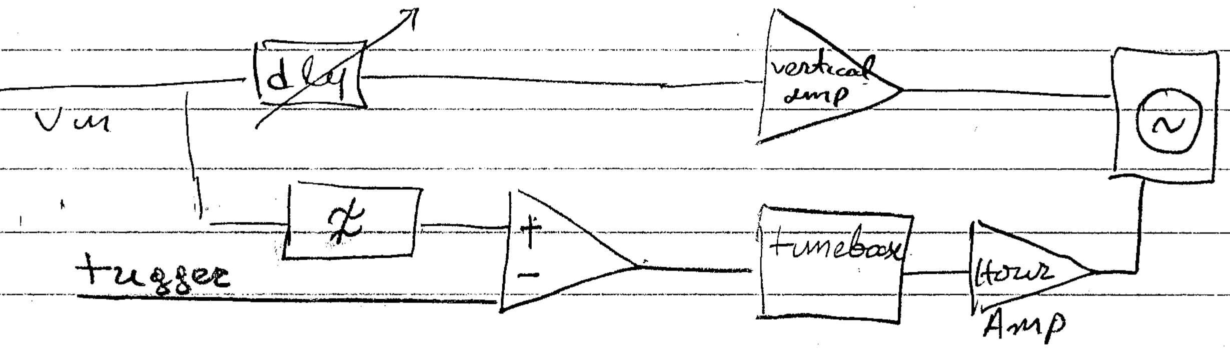
$$N = \frac{0.1 \cdot 10^{-6}}{1.6 \times 10^{-19}} \times \frac{10^{-8}\ \text{sec}}{14.1\ \text{cm}} = 4.4 \times 10^{4}$$

per pixel:

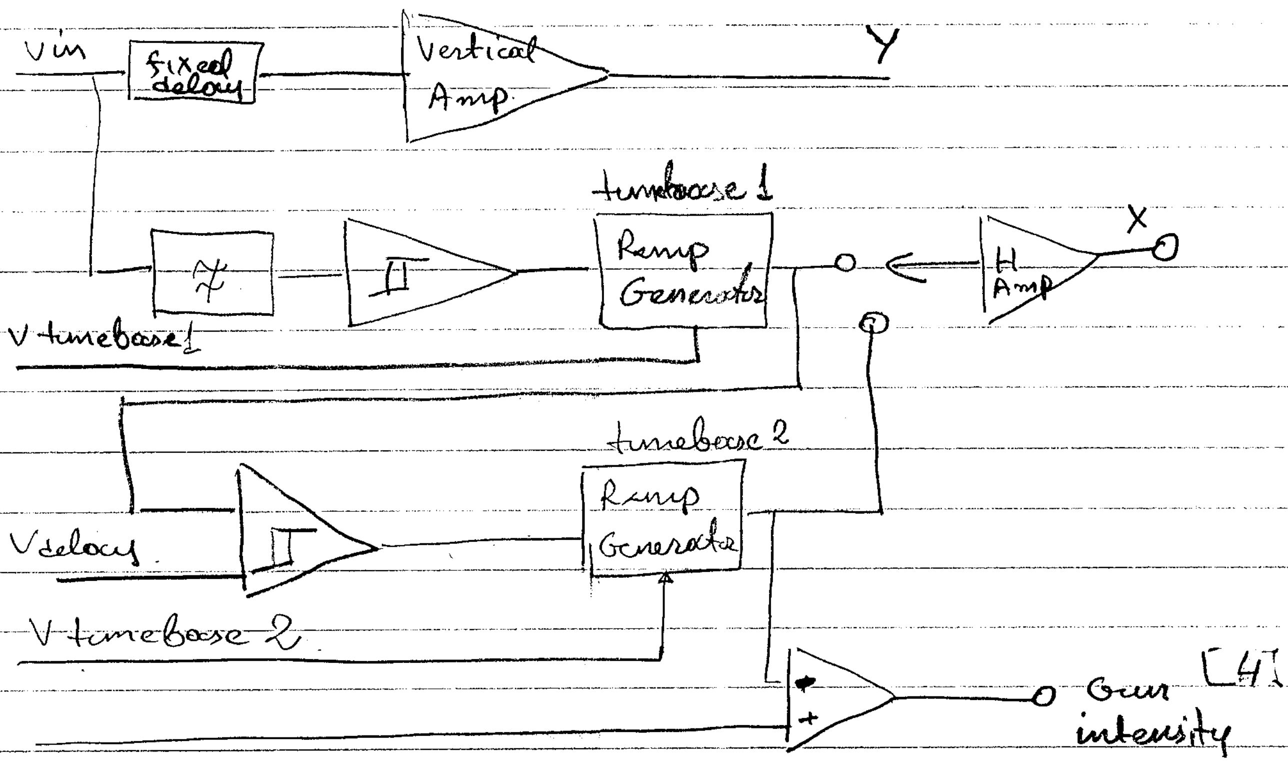
$$n = N/100 = 440$$

This is too small to be visible [5]

c) The trigger circuit consists of a smitt trigger comparing the trigger setting with a DC value. The comparator output fires a to synchronize the horizontal sweep generator. A delay generator in the vertical amp circuit allows the signal to be delayed relative to the trigger.



d)



[4]

And verbal explanation of the above.

- the horizontal delay [2]
- second timebase [2]
- gun control. [2]

e) A sampling oscilloscope uses interleaved sampling gates to effect a higher effective sampling rates. [2]

The interleaving can also be random [1].