IMPERIAL COLLEGE LONDON

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
EXAMINATIONS 2005

EEE/ISE PART II: MEng, BEng and ACGI

COMMUNICATIONS 2

Corrected Copy

Monday, 6 June 2:00 pm

Time allowed: 2:00 hours

There are FOUR questions on this paper.

Q1 is compulsory.
Answer Q1 and any two of questions 2-4.
Q1 carries 40% of the marks. Questions 2 to 4 carry equal marks (30% each).

Any special instructions for invigilators and information for candidates are on page 1.

Examiners responsible	First Marker(s) :	P.A. Naylor, P.A. Naylor
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SPECIAL INFORMATION FOR CANDIDATES

Some useful relationships:

$$\log_2(x) = 3.32\log_{10} x$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\cos(A)\cos(B) = \tfrac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\cos^2(A) = \tfrac{1}{2}\left[1 + \cos(2A)\right]$$

$$\sin^2(A) = \tfrac{1}{2}\left[1 - \cos(2A)\right]$$

$$\cos(A)\sin(A) = \tfrac{1}{2}\sin(2A)$$

1 (a) Justify the representation:

$$n(t) = \sum_k a_k \cos(2\pi f_k t + \theta_k)$$

for band-limited white noise of which a representative frequency is f_k, and values θ_k are random phases which are independent and uniformly distributed over 0 to 2π.

[6]

(b) Show that this bandpass noise can be written as

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

and explain what band of frequencies are present in each of $n_c(t)$ and $n_s(t)$.

[6]

(c) Consider the signal $x(t) = Ae^{-j(\omega t+\theta)}$, where A and ω are constants, and θ is a random variable having a probability density function that is uniformly distributed in the range 0 to π.

Draw the probability density function of θ and evaluate

(i) the mean value of $x(t)$

(ii) the mean square value of $x(t)$.

[6]

(d) Consider pulse-code modulation (PCM) of an analog signal. State what is meant by quantization noise and derive an expression for the mean-square quantization error in terms of the quantization step size. Assume a uniform quantizer.

[8]

(e) The input to a uniform n-bit quantizer is the sine wave $A_m \sin(2\pi f_m t)$. Derive an expression for the signal-to-noise ratio (in decibels) at the output of the quantizer. Assume that the dynamic range of the quantizer is $-A_m$ to A_m.

[8]

(f) Define channel capacity of a noisy channel of bandwidth B. Find the channel capacity when the SNR at the receiver is 11.8 dB.

[6]

2. (a) Consider an FM receiver consisting of an ideal band-pass filter followed by an FM demodulator. If the carrier power is much greater than the noise power at the output of the bandpass filter, then the signal-to-noise ratio at the output of the receiver is given by:

$$SNR_o = 3\beta^2 \frac{P}{|\max m(t)|^2} SNR_{base}$$

where P is the average power of the message signal $m(t)$, and we assume that the noise is zero-mean Gaussian with a flat power spectral density.

Explain why SNR_o cannot be increased arbitrarily simply by increasing β.

HINT: The transmission bandwidth of FM is given by Carson's rule as: $B_T = 2(\beta+1)W$

[6]

(b) Explain pre-emphasis and de-emphasis and indicate why they are used in FM systems.

[6]

(c) Consider an AM receiver using a square-law detector whose output is proportional to the square of the receiver input $x(t)$, as indicated in Figure 2.1 below:

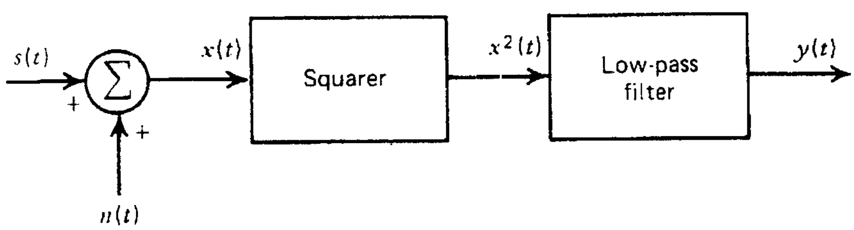


Figure 2.1

The AM waveform is :

$$s(t) = A[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

where μ is the modulation index. Assume that the additive noise at the receiver input is white Gaussian bandpass noise with zero mean. Show that the output signal-to-noise ratio of the receiver is given by:

$$SNR_{\text{out}} = \frac{2\mu^2\rho^2}{1+\rho(2+\mu^2)}$$

where ρ is the carrier-to-noise ratio at the input to the receiver. Assume that a capacitor is included at the output of the receiver to block DC.

[18]

3. (a) Demonstrate that a long string of N symbols from a source alphabet S, whose entropy is $H(S)$, can be represented by $NH(S)$ binary digits.

[12]

(b) State the source coding theorem, and define the efficiency of a variable length code.

[6]

(c) Explain what is meant by a prefix code, and construct such a code for the 5-symbol alphabet {A,B,C,D,E} whose symbols occur independently with respective probabilities {0.05, 0.12, 0.22, 0.08, 0.53}. Comment on the efficiency of this code.

[12]

4. Consider sending a file of F=1 Mbit from node A to B. There are Q=3 nodes between A and B, and the links are uncongested (no queueing delays). Each of the links has length D=100 m, rate R=10 Mbits/s and propagation speed C=2.8x10^8 m/s. Assume that any processing delay is insignificant.

Link-layer packets (whenever they are needed) can be up to L=10 Kbits long of which H=100 bits corresponds to a header. Assume that higher layers add no additional overhead to the packet. A connection establishment takes exactly S=10 ms from A to B.

Provide an expression and/or numerical solution for the delay in sending the file from A to B for the following types of networks:

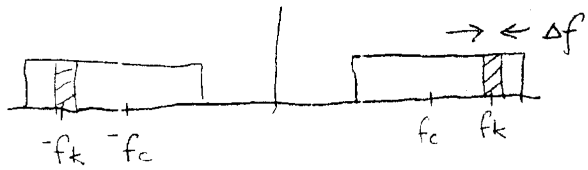
(a) Packet-switched datagram network with connectionless service [10]

(b) TDM Circuit switching. Assume an ideal partitioning of N=25 channels per link [10]

(c) Virtual circuit switching [10]

1 a) $$n(t) = \sum_k a_k \cos(2\pi f_k t + \theta_k)$$

- white noise has a flat power spectral density:



- for Δf small, the shaded component can be represented by a randomly-phased sinusoid of frequency f_k, and random phase θ_k, and amplitude a_k.
- summing these random sinusoids over the entire band gives the representation required.

b)

- let $f_k = (f_k - f_c) + f_c$

$$\therefore n_k(t) = a_k \cos\left[2\pi(f_k - f_c)t + \theta_k + 2\pi f_c t\right]$$

but $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\therefore n_k(t) = a_k \cos(2\pi(f_k - f_c)t + \theta_k)\cos(2\pi f_c t) - a_k \sin(2\pi(f_k - f_c)t + \theta_k)\sin(2\pi f_c t)$$

$$\therefore n(t) = \sum_k n_k(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

where $n_c(t) = \sum_k a_k \cos(2\pi(f_k - f_c)t + \theta_k)$

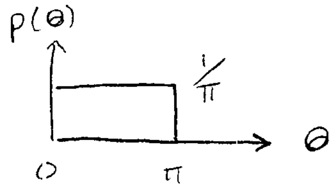
$n_s(t) = \sum_k a_k \sin(2\pi(f_k - f_c)t + \theta_k)$

Each of $n_c(t)$ & $n_s(t)$ contain frequencies $(f_k - f_c)$

Since f_k are centred around f_c, hence frequencies $(f_k - f_c)$ present in $n_c(t)$ & $n_s(t)$ are centred around 0, ie they are baseband

PAN RL

(i)



(ii) $$E\{x(t)\} = \int_{-\infty}^{\infty} A e^{-j(\omega t+\theta)}\, p(\theta)\, d\theta$$

$$= \frac{2}{\pi} A e^{-j\omega t} \int_0^{\pi} e^{-j\theta}\, d\theta$$

$$= \frac{2}{\pi} A e^{-j\omega t} \frac{-1}{j}(-1-1)$$

(iii) $$E\{x^2(t)\} = \int_{-\infty}^{\infty} A^2 e^{-2j(\omega t+\theta)}\, p(\theta)\, d\theta$$

$$= \frac{2}{\pi} A^2 e^{-j2\omega t} \int_0^{\pi} e^{-j2\theta}\, d\theta$$

$$= \frac{2}{\pi} A^2 e^{-j2\omega t} \frac{-1}{2j}[1-1]$$

$$= 0.$$

d:

PCM consists of:

1. sampling at or above Nyquist rate
2. quantizing each sample into discrete levels
3. encoding into a digital stream.

Quantization noise is introduced in step 2. It is caused by the fact that errors are introduced when amplitude is rounded to the nearest quantization level.

For a uniform quantizer with separation of Δ volts between levels, quantization error is a random variable bounded by $-\Delta/2 \le q \le \Delta/2$. It is approximately uniformly distributed with pdf:

$$p(q) = \begin{cases} \frac{1}{\Delta}, & -\Delta/2 \le q \le \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$

Mean square error is thus

$$E\{e^2\} = \int_{-\infty}^{\infty} q^2 \, p(q) \, dq$$

$$= \int_{-\Delta/2}^{\Delta/2} q^2 \frac{1}{\Delta} \, dq$$

$$= \frac{\Delta^2}{12}$$

(e)

For a sine wave $A_m \sin(2\pi f_m t)$, the average power is:

$$P_S = \frac{A_m^2}{2}$$

Average noise power (from (b)) is:

$$P_N = \frac{\Delta^2}{12}$$

The range of the quantizer is $2A_m = L\Delta$ where L is the no. of levels, which for a n-bit quantizer is $L = 2^n - 1 \approx 2^n$

$$\therefore \Delta = \frac{2A_m}{2^n} \quad \& \quad \Delta^2 = \frac{4A_m^2}{2^{2n}}$$

$$SNR = \frac{P_S}{P_N} = \frac{A_m^2}{2} \times \frac{12 \times 2^{2n}}{4A_m^2}$$

$$= 3/2 \times 2^{2n}$$

In decibels,

$$SNR_{dB} = 10 \times 2n \log_{10} 2 + 10 \log_{10} 3/2$$
$$= 6.02n + 1.8 \text{ dB.}$$

(f)

Define the *channel capacity*, C, as the maximum rate of *reliable* information transmission over a *noisy* channel. In other words, it is the maximum rate of information transfer with an arbitrarily small probability of error. Shannon proved the following fundamental theory of communications regarding channel capacity.

Channel Capacity C = B log2 (1 +S/N). For SNR of 11.8 dB, this gives S/N = 15. Hence capacity is given as C = B log2 (16) = 4B bits/s.

2.a For FM, transmission bandwidth is given by

$$B_T = 2(\beta+1)W,$$

and the receiver will have an input BPF tuned to this frequency band. As β increases, so the bandwidth of this BPF increases, thereby letting in more noise. But this will increase noise power relative to carrier power, & condition that carrier power >> noise power will no longer hold. So β cannot be increased arbitrarily.

PSD of message is typically : 

whereas PSD of noise at FM output is : 

b. Pre-emphasis is used before transmission to artificially boost HF components of message, thereby improving SNR. After detection, HF components are de-emphasised so that message is undistorted.

Received signal is:

$$x(t) = (A(1+\mu \cos\omega_m t) + n_c)\cos\omega_c t - n_s \sin\omega_c t$$

Squared signal is:

$$y(t) = x^2(t)$$
$$= [A(1+\mu\cos\omega_m t) + n_c]^2 \cos^2\omega_c t - n_s^2 \sin^2\omega_c t$$
$$- 2[A \ldots]\cos\omega_c t \sin\omega_c t$$

Using trig. ids this becomes; after LPF:

$$y_L(t) = \frac{1}{2}(A(1+\mu\cos\omega_m t) + n_c)^2 - \frac{1}{2}n_s^2$$

$$= \frac{1}{2}\Big\{A^2 + 2A^2\mu\cos\omega_m t + \frac{A^2\mu^2}{2} + 2An_c$$
$$+ 2A\mu n_c \cos\omega_m t + n_c^2 + n_s^2\Big\}$$

After removing DC terms this is:

$$y_D(t) = A^2\mu\cos\omega_m t + An_c(t) + A\mu n_c(t)\cos\omega_m t$$
$$+ \frac{1}{2}n_c^2 + \frac{1}{2}n_s^2$$

Signal term is: $A^2\mu\cos\omega_m t$

$$\therefore P_S = \frac{A^4\mu^2}{2}$$

cont. . .

Noise terms are:

$$A n_c(t) + A\mu n_c(t) \cos\omega_m t + \tfrac{1}{2} n_c^2(t) + \tfrac{1}{2} n_s^2(t)$$

$$\therefore P_N = A^2 \sigma^2 + \frac{A^2\mu^2}{2} \sigma^2 + \sigma^4$$

where $\sigma^2 = E\{n_c^2\} = E\{n_s^2\}$

$$\therefore P_N = \tfrac{1}{2} A^2 (2+\mu^2) \sigma^2 + \sigma^4$$

$$= \sigma^4 \left[\rho(2+\mu^2) + 1\right]$$

where $\rho = \frac{A^2}{2} / \sigma^2$ is carrier to noise ratio

Output SNR is:

$$\frac{P_S}{P_N} = \frac{A^4\mu^2}{2\sigma^2} \frac{1}{1+\rho(2+\mu^2)}$$

$$= \frac{2\rho^2\mu^2}{1+\rho(2+\mu^2)}$$

Now consider an alphabet $S = \{s_1, \ldots, s_K\}$ with respective probabilities p_k, $k = 1, \ldots, K$. During a long period of transmission in which N symbols have been generated (where N is very large), there will be Np_1 occurrences of s_1, Np_2 occurrences of symbol s_2, etc. If these symbols are produced by a discrete memoryless source (so that all symbols are independent), the probability of occurrence of a typical sequence S_N, will be

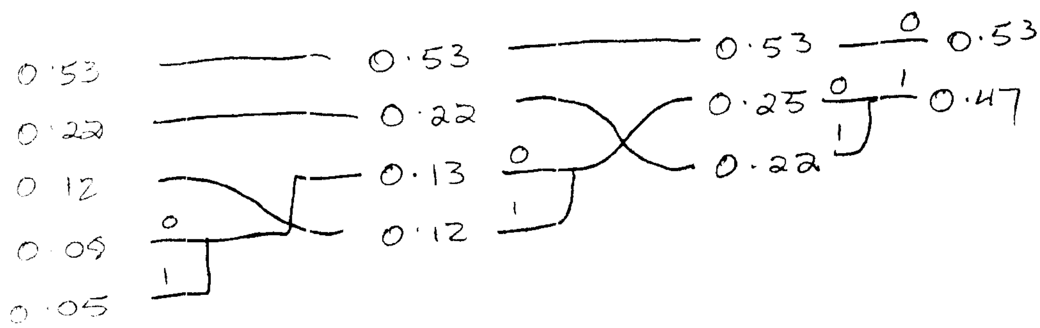
$$p(S_N) = {p_1}^{Np_1} \times {p_2}^{Np_2} \times \ldots \times {p_K}^{Np_K}$$

Since any particular sequence of N symbols is equally likely, the number of bits required to represent a typical sequence S_N is

$$\begin{aligned} L_N &= \log_2 \frac{1}{p(S_N)} = -\log_2({p_1}^{Np_1} \times \ldots \times {p_K}^{Np_K}) \\ &= -Np_1 \log_2 p_1 - Np_2 \log_2 p_2 - \ldots - Np_K \log_2 p_K \\ &= -N \sum_{k=1}^{K} p_k \log_2 p_k = NH(S). \end{aligned}$$

Theorem 5.1 (Source Coding Theorem)
Given a discrete memoryless source of entropy $H(S)$, the average codeword length $\bar{L}$ for any source coding scheme is bounded as

$$\bar{L} \geq H(S).$$



0.53 :	0
0.22 :	1 1
0.12 :	1 0
0.08 :	1 0 0 0
0.05 :	1 0 0 1

Avg codeword length is

$$\bar{L} = \sum l_k p_k$$

$$= 1.85 \text{ bits/symbol}$$

Entropy is : $H = -\sum p_k \log_2 p_k$

$$= 1.8407$$

∴ Efficiency is : $\dfrac{1.8407}{1.85} = 99.5\%$

4

a.

The total delay is the sum of the propagation delay and the transmission delay of the datagrams:

Total delay = Tp + Tt

First we need to determine the number of datagrams needed to transport the file. Each datagram can carry up to L - H = 10000 - 100 = 9900 bits of data. That means that $\lceil F/(L-H) \rceil$ packets are needed to send the entire file of which:

$\lfloor 1 \times 10^6 / 9900 \rfloor$= 101 = *n* datagram are of size L
1 packet is of size Ls = H + F - n (L-H) = 100 + 1 x 10^6 - 101 * 9900 = 200 bits

There are Q nodes between A and B. This means that there are Q+1 = 4 hops between A and B

Propagation delay (Tp) is therefore:

Tp = (Q+1) D / C = 4 (100) / 2.8x10^8 = 1.4286 μs

Transmission delay is:

Td = n L / R + Ls / R = 101 (10000 / 10 x 10^6) + 200 / 10 x 10^6 = 0.10102 sec

Therefore, the total delay is (Q+1) D / S + n L/R + Ls / R

0.10102 s + 1.4286 μs = 0.10102 s

b

The nominal rate of each channel is Rc = R / N =10 x 10^6 / 25 = 0.4 Mbps.

Now there are no packets, but there is a call establishment time of S seconds that adds time to the total delay. Propagation delay is again Tp as in part a.

Total delay = S + F / Rc + Tp = S + F N / R + Tp ~= 10 x 10^{-3} + (1 x 10^6) / 0.4 x 10^6 ~= 2.501 sec

c

The delay now is because of a call establishment time plus the total delay of the packets (as found in part a):

Total delay = S + (Q+1) D / S + n L/R + Ls / R = 10 x 10^{-3} + 0.10102 = 0.11102 sec