**BEAM035 / BEFM017**

**UNIVERSITY OF EXETER**

**BUSINESS SCHOOL**

**May / June 2009**

**DERIVATIVES PRICING**
**CREDIT INSTRUMENTS AND DERIVATIVES**

**Module Convenor: Dr Stanley B. Gyoshev**

**Duration: TWO HOURS**

**Candidate Number** ______________________

**Student ID Number** ______________________

**Degree Programme** ______________________

You should answer **ALL FIFTEEN (15)** questions. There are 100 marks available for all questions. The maximum number of marks for this exam is 100.

| | | | | |
|---|---|---|---|---|
| 1 | computation | 20 | marks | 20 |
| 4 | computation | 15 | marks | 60 |
| 10 | True/ False | 2 | marks | 20 |
| | | | | |
| 15 | Total | | marks | 100 |

Always fully explain your answers.

The notation throughout this exam is consistent with that in the lecture notes. If there are new notations they are defined in the exam paper.

Normal distribution tables are attached at the end.

Approved calculators are permitted.

This is a **CLOSED BOOK EXAMINATION.**

**<u>Part A:</u>** Calculations: Question 1-5

**Definition: Cooling Degree Day – CDD**

The number of degrees that a day's average temperature is above $65^0$ Fahrenheit and people start to use air conditioning to cool their buildings. The price of weather derivatives trading in the summer are based on an index made up of monthly cooling degree day (CDD) values. The settlement price for a weather futures contract is calculated by summing a month's CDD values and multiplying by payment rate.

**Question 1.1: (10 marks)**

Suppose that each day during January 2009 the minimum temperature in Rio de Janeiro was $68^{O}$ Fahrenheit and the maximum temperature was $82^{O}$ Fahrenheit. What is the Payoff from a call option on the cumulative CDD during January with a strike price of X = 250 and a payment rate $5,000 per degree day?

**Answer**:

*[10 marks]*

## Question 1.2: (10 marks)

The following table provides the minimum and maximum temperature in Rio de Janeiro for each day during January 2009

| day | minimum | maximum | . | . | . |
|---|---|---|---|---|---|
| 1 | 65.2 $^{O}$ Fahrenheit | 82.5 $^{O}$ Fahrenheit | . | . | . |
| 2 | 67.4 $^{O}$ Fahrenheit | 83.5 $^{O}$ Fahrenheit | . | . | . |
| 3 | 68.8 $^{O}$ Fahrenheit | 83.3 $^{O}$ Fahrenheit | . | . | . |
| 4 | 67.3 $^{O}$ Fahrenheit | 84.1 $^{O}$ Fahrenheit | . | . | . |
| 5 | 69.7 $^{O}$ Fahrenheit | 83.2 $^{O}$ Fahrenheit | . | . | . |
| 6 | 66.2 $^{O}$ Fahrenheit | 82.1 $^{O}$ Fahrenheit | . | . | . |
| 7 | 65.5 $^{O}$ Fahrenheit | 83.3 $^{O}$ Fahrenheit | . | . | . |
| 8 | 69.5 $^{O}$ Fahrenheit | 80.8 $^{O}$ Fahrenheit | . | . | . |
| 9 | 69.1 $^{O}$ Fahrenheit | 82.0 $^{O}$ Fahrenheit | . | . | . |
| 10 | 66.9 $^{O}$ Fahrenheit | 81.4 $^{O}$ Fahrenheit | . | . | . |
| 11 | 65.3 $^{O}$ Fahrenheit | 83.0 $^{O}$ Fahrenheit | . | . | . |
| 12 | 67.0 $^{O}$ Fahrenheit | 82.2 $^{O}$ Fahrenheit | . | . | . |
| 13 | 65.8 $^{O}$ Fahrenheit | 81.6 $^{O}$ Fahrenheit | . | . | . |
| 14 | 66.7 $^{O}$ Fahrenheit | 81.7 $^{O}$ Fahrenheit | . | . | . |
| 15 | 67.1 $^{O}$ Fahrenheit | 84.5 $^{O}$ Fahrenheit | . | . | . |
| 16 | 66.0 $^{O}$ Fahrenheit | 82.8 $^{O}$ Fahrenheit | . | . | . |
| 17 | 68.0 $^{O}$ Fahrenheit | 82.1 $^{O}$ Fahrenheit | . | . | . |
| 18 | 66.6 $^{O}$ Fahrenheit | 81.2 $^{O}$ Fahrenheit | . | . | . |
| 19 | 68.2 $^{O}$ Fahrenheit | 84.7 $^{O}$ Fahrenheit | . | . | . |
| 20 | 66.8 $^{O}$ Fahrenheit | 80.6 $^{O}$ Fahrenheit | . | . | . |
| 21 | 65.3 $^{O}$ Fahrenheit | 80.8 $^{O}$ Fahrenheit | . | . | . |
| 22 | 65.7 $^{O}$ Fahrenheit | 81.1 $^{O}$ Fahrenheit | . | . | . |
| 23 | 68.6 $^{O}$ Fahrenheit | 82.0 $^{O}$ Fahrenheit | . | . | . |
| 24 | 68.9 $^{O}$ Fahrenheit | 81.5 $^{O}$ Fahrenheit | . | . | . |
| 25 | 67.8 $^{O}$ Fahrenheit | 82.8 $^{O}$ Fahrenheit | . | . | . |
| 26 | 67.5 $^{O}$ Fahrenheit | 81.4 $^{O}$ Fahrenheit | . | . | . |
| 27 | 65.2 $^{O}$ Fahrenheit | 84.1 $^{O}$ Fahrenheit | . | . | . |
| 28 | 66.6 $^{O}$ Fahrenheit | 80.4 $^{O}$ Fahrenheit | . | . | . |
| 29 | 69.9 $^{O}$ Fahrenheit | 84.8 $^{O}$ Fahrenheit | . | . | . |
| 30 | 68.9 $^{O}$ Fahrenheit | 81.0 $^{O}$ Fahrenheit | . | . | . |
| 31 | 66.3 $^{O}$ Fahrenheit | 80.3 $^{O}$ Fahrenheit | . | . | . |

What is the Payoff from a call option on the cumulative CDD during January with a strike price of X = 250 and a payment rate $5,000 per degree day

**Answer**:

*[10 marks]*

## Question 2.1

Suppose that the structure for an asset-backed security transaction is as follows: Senior tranche is $230 million. Subordinate tranche 1 is $50 million. Subordinate tranche 2 is $30 million. The value of the collateral for the structure is $320 million. Subordinate tranche 2 is the first loss tranche.

A. How much is the overcollateralization in this structure in millions?

**Answer**:

*[1 marks]*

B. What is the amount of the loss for each tranche if losses due to defaults over the life of the structure total $10 million?

B1. Total Loss (million)

**Answer**:

*[0.5 marks]*

B2. Senior Tranche (million)

**Answer**:

*[0.5 marks]*

B3. Subordinate Tranche 1 (million)

**Answer**:

*[0.5 marks]*

B4. Subordinate Tranche 2 (million)

**Answer**:

*[0.5 marks]*

C. What is the amount of the loss for each tranche if losses due to defaults over the life of the structure total $110 million?

C1. Total Loss (million)

**Answer**:

*[0.5 marks]*

C2. Senior Tranche (million)

**Answer**:

*[0.5 marks]*

C3. Subordinate Tranche 1 (million)

**Answer**:

*[0.5 marks]*

C4. Subordinate Tranche 2 (million)

**Answer**:

*[0.5 marks]*

## Question 2.2

Consider the following CDO transaction: Collateralized debt obligations (CDOs) are a type of asset-backed security or structured finance product.

1. The CDO is a $300 million structure. That is, the assets purchased will be $300 million.
2. The collateral consists of bonds that all mature in 5 years and the coupon rate for every bond is the 5-year US Treasury rate plus 450 basis points.
3. The senior tranche comprises 75% of the structure and pays interest based on the following coupon formula: LIBOR plus 60 basis points.
4. There is only one junior tranche of $50 million with a coupon rate that is fixed. The coupon rate is the 5-year Treasury rate plus 200 basis points.
5. The asset manager enters into an agreement with a counterparty in which it agrees to pay the counterparty a fixed rate each year equal to the 5-year Treasury rate plus 90 basis points and receive LIBOR. The notional amount of the agreement is $225 million.

A. How much is the equity tranche (in million) in this CDO?

**Answer**:

*[1 mark]*

B. Assume that the 5-year Treasury rate at the time the CDO is issued is 5%. Assuming no defaults, what is the interest (in million) from collateral?

**Answer**:

*[2 marks]*

C. Assume that the 5-year Treasury rate at the time the CDO is issued is 5%. Assuming no defaults, what is the interest (in million) to the junior tranche?

**Answer**:

*[2 marks]*

D. Assume that the 5-year Treasury rate at the time the CDO is issued is 5%. Assuming no defaults, what is the interest (in million) to swap counterparty?

**Answer**:

*[2 marks]*

E. Ignoring the asset management fee, what is the amount available each year for the equity tranche?

**Answer**:

*[3 marks]*

## Question 3.1

Using the Public Securities Association Prepayment benchmark, complete the following table:

| Month | PSA | **CPR (**Use **3** digits after the decimal point for CPR**)** | **SMM(**Use **6** digits after the decimal point for SMM**)** |
|---|---|---|---|
| 3 | 100 | | |
| 17 | 80 | | |
| 20 | 175 | | |
| 27 | 50 | | |
| 65 | 200 | | |
| 128 | 75 | | |
| 200 | 225 | | |

**Use 3 digits after the decimal point for CPR and 6 digits after the decimal point for SMM**

A. For PSA100 for month 3 CPR is
**Answer:**

*[1 mark]*

B. For PSA175 for month 20 CPR is
**Answer:**

*[1 mark]*

C. For PSA50 for month 27 CPR is
**Answer:**

*[1 mark]*

D. For PSA100 for month 3 SMM is
**Answer:**

*[1 mark]*

E. For PSA80 for month 17 SMM is
**Answer:**

*[1 mark]*

F. For PSA175for month 20 SMM is
**Answer:**

*[1 mark]*

G. For PSA50 for month 27 SMM is
**Answer:**

*[1 mark]*

H. For PSA200for month 65 SMM is
**Answer:**

*[1 mark]*

I. For PSA75 for month 128 SMM is
**Answer:**

*[1 mark]*

J. For PSA225 for month 200 SMM is

**Answer:**

*[1 mark]*

## <u>Question 3.2</u> *[5 marks]*

Briefly explain the difference between a single monthly mortality rate and an absolute prepayment speed.

**Answer:**

## Question 4.1

Suppose a stock currently trades at a price of $75.00. The stock price can go up 25% or down 12%. Exercise price is $80.00. Risk-free rate is 2.5%.

A. Use a one-period binomial model to calculate the price of a put option.
**Answer:**

*[3 marks]*

B. Suppose the put price is currently $6.50. Show how to execute an arbitrage transaction that will earn more than the risk-free rate using 10,000 put options.
B1. What is the value of the hedged portfolio at expiration if the price goes down?
**Answer:**

*[2 marks]*

B2. What is the value of the hedged portfolio at expiration if the price goes up?
**Answer:**

*[2 marks]*

B3. What is the rate of return of the arbitrage?
**Answer:**

*[2 marks]*

## Question 4.2:

Suppose the put price is currently $105.60 and has a volatility of 0.25. The continuously compounded risk free rate is 1.75%. The option expires in 6 months (T=0.5). The exercise price is $100.

A. Calculate the values of $N(d_1)$ (please provide 4 digits after the decimal point)
**Answer:**

*[1 mark]*

B. Calculate the values of $N(d_2)$ (please provide 4 digits after the decimal point)
**Answer:**

*[1 mark]*

C. Find the Call premium using the Black-Scholes formula.
**Answer:**

*[2 marks]*

D. Find the put premium using the Black-Scholes formula.
**Answer:**

*[2 marks]*

## Question 5

Consider a two-year currency swap with semiannual payments. The domestic currency is the U.S. dollar, and the foreign currency is the U.K. pound. The current exchange rate is $1.15 per pound. ***<u>Please provide 4 digits after the decimal point for all the questions.</u>***

**A.** Calculate the annualised fixed rates for dollars.
The current U.S. term structure is

| Days | LIBOR | |
|---|---|---|
| 180 | Lo (180) | 0.0125 |
| 360 | Lo (360) | 0.0135 |
| 540 | Lo (540) | 0.0145 |
| 720 | Lo (720) | 0.0150 |

**Answer:**

*[1.5 marks]*

**B.** Calculate the annualised fixed rates for pounds.
The U.K. term structure is

| Days | LIBOR | |
|---|---|---|
| 180 | $L_0^{£}$ (180) | 0.0185 |
| 360 | $L_0^{£}$ (360) | 0.0195 |
| 540 | $L_0^{£}$ (540) | 0.0203 |
| 720 | $L_0^{£}$ (720) | 0.0209 |

**Answer:**

*[1.5 marks]*

**C.** Now move forward 120 days. The new exchange rate is $1.10 per pound, and the new U.S. term structure is

| Days | LIBOR | |
|---|---|---|
| 60 | $L_{120}$ (60) | 0.0115 |
| 240 | $L_{120}$ (240) | 0.0125 |
| 420 | $L_{120}$ (420) | 0.0130 |
| 600 | $L_{120}$ (600) | 0.0141 |

The new U.K. term structure is

| Days | LIBOR | |
|---|---|---|
| 60 | $L_{120}^{£}$ (60) | 0.0178 |
| 240 | $L_{120}^{£}$ (240) | 0.0188 |
| 420 | $L_{120}^{£}$ (420) | 0.0198 |
| 600 | $L_{120}^{£}$ (600) | 0.0208 |

Assume that the notional principal is $1 or the corresponding amount in British pounds. Calculate the market values of the following swaps:

**i.** Pay £ fixed and receive $ fixed.
**Answer:**

*[3 marks]*

**ii.** Pay £ floating and receive $ fixed
**Answer:**

*[3 marks]*

**iii.** Pay £ floating and receive $ floating
**Answer:**

*[3 marks]*

**iv.** Pay £ fixed and receive $ floating
**Answer:**

*[3 marks]*

**Part B** True/ False: Question 1-10 *[20 marks]*

**Specify if the following statements are true (T) or false (F).**

_____ 1. An off-market forward contract is established with a nonzero value at the start and there will be a requirement for cash payment at the initiation of the contract.

_____ 2. In marking to market, variation margin is deposited to the account to restore the balance to the maintenance margin.

_____ 3. For all the issues that may be delivered to satisfy a Treasury futures contract, the cheapest-to-deliver issue is the one with the highest implied repo rate.

_____ 4. The role of a clearinghouse is to take an active position in the market to maintain a fair and orderly market by providing liquidity when the normal flow of orders is not adequate.

_____ 5. Prior to expiration, European options prices must always be no less than those of equivalent American options.

_____ 6. To construct a delta-hedged position, a long position in each call is matched with a short position in delta units of the underlying. A delta-hedged position should be adjusted as the delta changes and time passes.

_____ 7. An interest rate collar is created by selling an interest rate cap and buying an interest rate floor.

_____ 8. The lower the strike price relative to the stock's underlying price, the more the call option is worth.

_____ 9. Since an MBS usually comprises several thousand mortgages with various degrees of prepayment risk, an MBS has low interest-rate risk.

_____ 10. The creation of a CMO can eliminate prepayment risk by transferring the various forms of this risk among different classes of bonds called tranches.

# END OF EXAM

## Section 16, Reading 66, Appendix 66A page 216, Cumulative Probabilities for Standard Normal Distribution

| x | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

$$d_1 = \frac{\ln(S_0/ X) + [\, r_c + (\sigma^2/2)\,] * T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma * \sqrt{T}$$

# END OF EXAM