Geometry & Vectors

Exercises 1

1) Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be lines and $\mathcal{P}$ a plane. $\mathcal{L}_1$ does not lie in $\mathcal{P}$, that is $\mathcal{L}_1 \notin \mathcal{P}$, but intersects the plane in a point P, that is P$\in \mathcal{L}_1$ and P$\in \mathcal{P}$. Furthermore, $\mathcal{L}_2$ lies in the plane $\mathcal{P}$ but does not go through the point P, i.e. P$\notin \mathcal{L}_2$ and $\mathcal{L}_2 \in \mathcal{P}$.

i) Do the lines $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect?

ii) State the argument to support your answer using the axioms provided in the lecture.

2) Three distinct points A,B,C lie in two different planes $\mathcal{P}_1$ and $\mathcal{P}_2$, i.e. A,B,C $\in \mathcal{P}_1$ and A,B,C $\in \mathcal{P}_2$.

Are the two planes necessarily identical?

3) Can a three legged table wobble?

4) Two lines $\mathcal{M}$ and $\mathcal{N}$ cross in some point X. A line $\mathcal{L}_1$ crosses $\mathcal{N}$ in the point A and $\mathcal{M}$ in the point B. A further line $\mathcal{L}_2$ is parallel to $\mathcal{L}_1$ and crosses $\mathcal{N}$ in the point C and $\mathcal{M}$ in the point D. The point X is on the line segment BD and AC.

Draw the relevant sketch and use the similarity axiom to show that

$$\text{XA} : \text{AC} = \text{XB} : \text{BD}.$$

5) ABCD constitutes a parallelogram. The point O is on the line $\overleftrightarrow{AB}$. The line $\overleftrightarrow{OC}$ crosses the line $\overleftrightarrow{AD}$ in the point P.

Draw the relevant sketch and show that

$$\text{AP} : \text{PD} = \text{OA} : \text{AB}.$$

6) ABCD constitutes a parallelogram. The point W is the midpoint of the line segment BC. The lines $\overleftrightarrow{AW}$ and $\overleftrightarrow{BD}$ intersect in the point X.

Draw the relevant sketch and show that

$$\text{DX} : \text{XB} = 2 : 1.$$

See handout for solutions.

Exercises 1

1) i) No.

ii) – suppose L_1 and L_2 intersect in some point $P' \neq P$

– then L_1 has two points in the plane

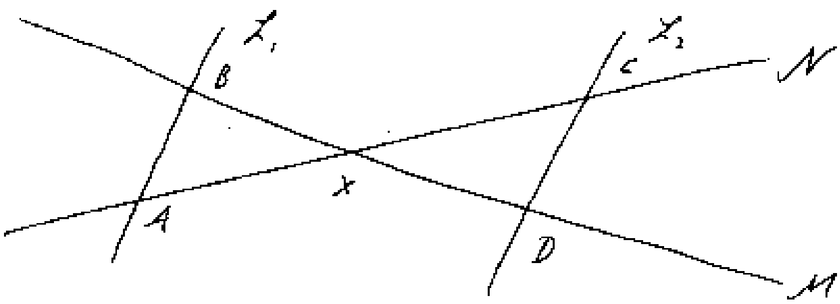
– by axiom 4 follows that $L_1 \in \mathcal{P}$ which contradicts the assumption

2) – if A, B, C are noncollinear it follows from axiom 2 that $\mathcal{P}_1 = \mathcal{P}_2$

– if A, B, C are collinear then it could be that $\mathcal{P}_1 \neq \mathcal{P}_2$ and the line $\mathcal{L} \ni A, B, C$ could be the intersection of the two planes (axiom 6)

3) No by axiom 3.

4)



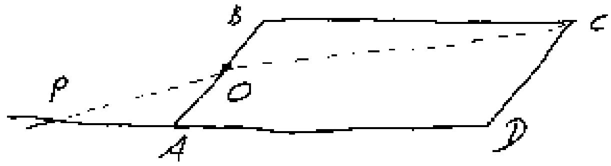
axiom 9:

$$XA : XC = XB : XD$$

$$\Rightarrow \frac{XC}{XA} = \frac{XD}{XB} \Rightarrow 1 + \frac{XC}{XA} = 1 + \frac{XD}{XB} \Rightarrow \frac{XA + XC}{XA} = \frac{XB + XD}{XB}$$

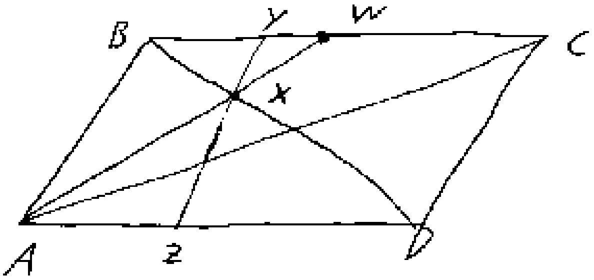
$$\Rightarrow \frac{AC}{XA} = \frac{BD}{XB} \Rightarrow XA : AC = XB : BD$$

5)



$$\left.\begin{array}{l} PA : PD = PO : PC \\ OA : AB = OP : PC \end{array}\right\} \Rightarrow AP : PD = OA : AB$$

6)



Draw the line $\overleftrightarrow{ZY}$ through X parallel to the lines $\overleftrightarrow{AB}$, $\overleftrightarrow{DC}$

$$\left.\begin{array}{l} BY : YC = BX : XD \\ WX : XA = WY : YB \\ BX : XD = XW : AX \\ (BY + YW) : (BY + YC) = BW : BC = 1 : 2 \end{array}\right\} \Rightarrow DX : XB = 2 : 1$$