UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Wednesday 6 June 2001 1.30 to 3.30

PAPER 18

LOGIC AND COMBINATORICS

*Answer any **THREE** questions. The questions carry equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

An **incline** is a structure with two associative and commutative binary operations $+$ and $\cdot$ satisfying:

(i) $(\forall xyz)\big(x \cdot (y+z) = (x \cdot y) + (x \cdot z)\big)$

(ii) $(\forall x)(x + x = x)$

(iii) $(\forall xy)\big(x + (x \cdot y) = x\big)$

We define a relation $\leq$ by setting $x + y \leq x$ for all x, y.

Prove that $\leq$ is a quasi-order.

Let $\big(I, +, \cdot\big)$ be a finitely generated incline. Show that $\big(I, \leq\big)$ is a WQO.

2

Write an essay on BQO theory.

3

Prove Kruskal's theorem and deduce Friedman's Finite Form of it.

4

Explain why Friedman's Finite Form of Kruskal's theorem proves the consistency of Peano arithmetic.

5

Write an essay on ordinal arithmetic and the fast-growing hierarchy.