UK JUNIOR MATHEMATICAL CHALLENGE

April 25th 2013

EXTENDED SOLUTIONS

These solutions augment the printed solutions that we send to schools. For convenience, the solutions sent to schools are confined to two sides of A4 paper and therefore in many cases are rather short. The solutions given here have been extended (with the summary solutions shown at the end of the document). In some cases we give alternative solutions, and we have included some ***Extension Problems*** for further investigations.

The Junior Mathematical Challenge (JMC) is a multiple choice contest, in which you are presented with five alternative answers, of which just one is correct. Sometimes you can find the correct answers by working backwards from the given alternatives, or by showing that four of them are not correct. Also, no reasons for the answers need to be given. However, providing good and clear explanations is the heart of doing Mathematics. So here we have aimed to give full solutions with all steps explained. We therefore hope that these solutions can be used as a model for the type of written solution that is expected in the Junior Mathematical Olympiad and similar competitions.

We welcome comments on these solutions, and, especially, corrections or suggestions for improving them. Please send your comments,

either by e-mail to enquiry@ukmt.co.uk, or by post to JMC Solutions, UKMT Maths Challenges Office, School of Mathematics, University of Leeds, Leeds LS2 9JT.

Quick Marking Guide

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
E	E	C	C	D	A	D	A	B	E	C	E	D	B	A	B	E	A	D	B	C	E	D	B	C

1. Which of the following has the largest value?

A $1 - 0.1$ B $1 - 0.01$ C $1 - 0.001$ D $1 - 0.0001$ E $1 - 0.00001$

Solution: **E**

In each case a number is subtracted from 1. The smaller the number we subtract, the larger the answer will be. So the largest value is obtained when the smallest number is subtracted.

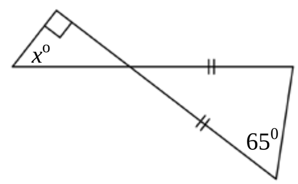
2. Heidi is 2.1 m tall, while Lola is only 1.4 m tall. What is their average height?

A 1.525 m B 1.6 m C 1.7 m D 1.725 m E 1.75 m

Solution: **E**

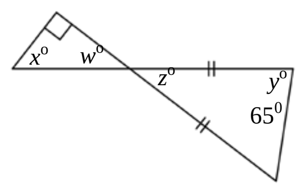
Their average height, in metres, is $\frac{2.1+1.4}{2} = \frac{3.5}{2} = 1.75$.

3. What is the value of x?



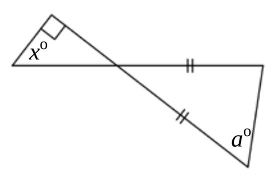
A 25 B 35 C 40 D 65 E 155

Solution: **C**



Let the other angles be as marked in the diagram. Then $y = 65$ as the angles marked 65º and yº are the base angles of an isosceles triangle. Because the angles in a triangle sum to 180º, $z = 180 - 65 - 65 = 50$. Now, $w = z$, as the angles marked wº and zº are vertically opposite. Therefore $w = 50$. So, the angle marked xº is the third angle of a triangle in which the other two angles are 50º and 90º. Therefore $x = 180 - 50 - 90 = 40$.

Extension Problem



3.1 In the diagram alongside, the angle that was previously labelled 65º is now labelled aº. Find a formula for x in terms of a. Check that your formula gives the same answer in the case that $a = 65$ as you gave to Question 3.

3.2 Is there a maximum value that a can have?

4. Gill went for a five-hour walk. Her average speed was between 3 km/h and 4 km/h. Which of the following could be the distance she walked?

A 12 km B 14 km C 19 km D 24 km E 35 km

Solution: **C**

Walking at 3km/h for five hours, Gill would walk $5 \times 3\,\text{km} = 15\,\text{km}$, and at 4km/h she would walk $5 \times 4\,\text{km} = 20\,\text{km}$. As her average speed is between 3 km/h and 4 km/h, she walks between 15 km and 20km. Of the options given, only 19 km is in this range.

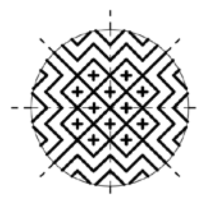
5. The diagram shows a weaver's design for a *rihlèlò*, a winnowing tray from Mozambique.

How many lines of symmetry does the design have?

A 0 B 1 C 2 D 4 E 8

Solution: **D**

The four lines of symmetry are shown in the diagram.



6. What is the value of $((1-1)-1)-((1-(1-1))$?

A -2 B -1 C 0 D 1 E 2

Solution: **A**

We have $((1-1)-1)-((1-(1-1)) = (0-1)-(1-0) = (-1)-(1) = -2$.

Extension Problem

6.1 The answer to Question 6 shows that by putting brackets in the appropriate places in

$$1 - 1 - 1 - 1 - 1 - 1 - 1 - 1$$

we obtain an expression whose value is –2. How many different values can be obtained by inserting brackets in different ways?

7. After tennis training, Andy collects twice as many balls as Roger and five more than Maria. They collect 35 balls in total. How many balls does Andy collect?

A 20 B 19 C 18 D 16 E 8

Solution: **D**

Suppose that Andy collects x balls. Since Andy collects twice as many balls as Roger, Roger collects $\frac{1}{2}x$ balls. Andy collects five more balls than Maria. So Maria collects $x-5$ balls. So between them, Andy, Roger and Maria collect $x+\frac{1}{2}x+(x-5)$ balls. They collect 35 balls in total. Therefore

$$x+\tfrac{1}{2}x+(x-5)=35.$$

Multiplying this equation by 2 gives

$$2x+x+2x-10=70.$$

This is equivalent to

$$5x=80.$$

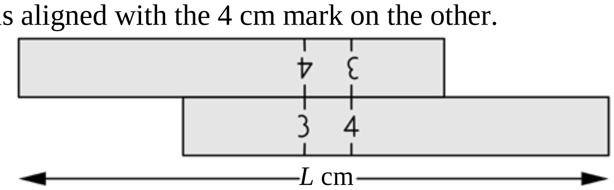
It follows that $x=16$.

Note: As we are asked for the number of balls that Andy collects, it is natural to begin the problem by letting x be this number. However, we see then see that this leads to an equation which includes a fraction. If you look ahead, you might prefer to let the number of balls that Roger collects be x. Then Andy collects $2x$ balls and Maria collects $2x-5$ balls. We then obtain the equation

$$2x+x+(2x-5)=35$$

with no fractions in it. This equation is equivalent to $5x=40$, from which we deduce that $x=8$. This calculation is easier that the one we gave above, but, if you use this method, you need to remember that the answer we are asked for is $2x$ and not just x.

8. Two identical rulers are placed together, as shown (not to scale).
Each ruler is exactly 10 cm long and is marked in centimetres from 0 to 10. The 3 cm mark on each ruler is aligned with the 4 cm mark on the other.



The overall length is L cm. What is the value of L?

A 13 B 14 C 15 D 16 E 17

Solution: **A**

The distance between the 4 cm mark and the 10 cm mark at the end of the ruler is $(10-4)\text{cm}=6\,\text{cm}$. The overall length is made up of the length on each ruler from the 4cm mark to the 10 cm mark, plus the length from the 3 cm mark to the 4 cm mark. This latter distance is 1 cm. So, $L=6+6+1=13$.

9. Peter has three times as many sisters as brothers. His sister Louise has twice as many sisters as brothers. How many children are there in the family?

A 15 B 13 C 11 D 9 E 5

Solution: **B**

Suppose Peter has b brothers and hence $3b$ sisters. So, including Peter, there are $b+1$ boys and $3b$ girls in the family. So Louise has $b+1$ brothers and $3b-1$ sisters. Since Louise has twice as many sisters as brothers, $3b-1=2(b+1)$. This equation is equivalent to $3b-1=2b+2$. So $b=3$ and there are 4 boys and 9 girls in the family, making 13 children altogether.

10. On standard dice the total number of pips on each pair of opposite faces is 7. Two standard dice are placed in a stack, as shown, so that the total number of pips on the two touching faces is 5.

What is the total number of pips on the top and bottom faces of the stack?

A 5 B 6 C 7 D 8 E 9

Solution: **E**

The total number of pips on the top and bottom faces of the two dice is $7+7=14$. As there is a total number of 5 pips on the touching faces, there are $14-5=9$ pips altogether on the top and bottom faces of the stack.

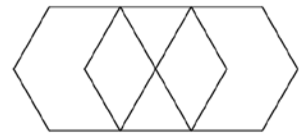
11. Usain runs twice as fast as his mum. His mum runs five times as fast as his pet tortoise, Turbo. They all set off together for a run down the same straight path. When Usain has run 100 m, how far apart are his mum and Turbo the tortoise?

A 5 m B 10 m C 40 m D 50 m E 55 m

Solution: **C**

When Usain has run 100m his mum has run half this distance, that is, 50 m and Turbo has run one-fifth of his mum's distance, that is, 10 m. So the distance between his mum and Turbo is $(50-10)\text{ m} = 40\text{ m}$.

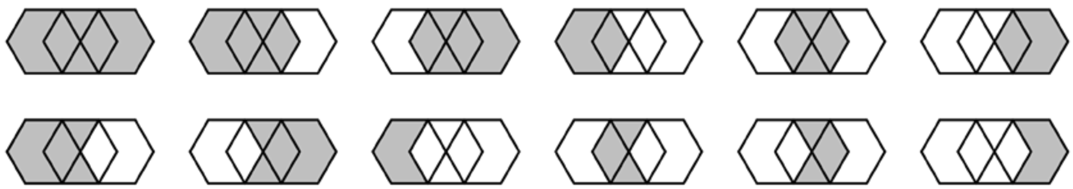
12. How many hexagons are there in the diagram?



A 4 B 6 C 8 D 10 E 12

Solution: **E**

The twelve different hexagons are shown shaded in the diagram below.



Extension Problems

12.1 Are there any triangles in the diagram of Question 12?

12.2 How many quadrilaterals are there in this diagram?

12.3 How many pentagons are there in this diagram?

12.4 For which values of n can you find at least one polygon with n sides in this diagram?

13. When painting the lounge, I used half of a 3 litre can to complete the first coat of paint. I then used two thirds of what was left to complete the second coat. How much paint was left after both coats were complete?

A 150 ml B 200 ml C 250 ml D 500 ml E 600 ml

Solution: **D**

The first coat uses half the paint, so half remains. Two thirds of this is then used so one third of one half, that is one sixth remains. So the volume remaining is $\frac{1}{6}\times 3$ litres. So there remains 0.5 litres, that is, 500 ml of paint.

14. Each side of an isosceles triangle is a whole number of centimetres. Its perimeter has length 20 cm. How many possibilities are there for the lengths of its sides?

A 3 B 4 C 5 D 6 E 7

Solution: **B**

Let the length of the two equal sides of the isosceles triangle be *a* cm. Since the triangle has perimeter 20 cm, the third side will have length $20 - 2a$ cm. Since this must be a positive length, $20 - 2a > 0$ and so $a < 10$. In a triangle the length of one side must be less than the sum of the lengths of the other two sides. So $20 - 2a < 2a$. This gives $20 < 4a$ and hence $5 < a$. So, we have $5 < a < 10$. Therefore, as *a* is a whole number, there are just four possible values for *a*, namely 6, 7, 8 and 9. So there are four possibilities for the side lengths of the triangle,

6, 6, 8; 7, 7, 6; 8, 8, 4 and 9, 9, 2.

Extension Problems

14.1 How many different possibilities are there for the side lengths of an isosceles triangle each of whose sides is a whole number of centimetres and whose perimeter is 24 cm?

14.2 Can you find a formula for the number of different isosceles triangles each of whose sides is a whole number of centimeters and whose perimeter has length *n* centimeters, where *n* is a positive integer with $n \geq 3$?

We now consider the analogous problem for triangles which are not necessarily isosceles.

14.3 For each whole number *n*, with $3 \leq n \leq 10$, find the number of different triangles whose side lengths are all a whole number of centimetres and whose perimeter is *n* cm.

14.2 Can you find a formula for the number of different triangles each of whose sides is a whole number of centimeters and whose perimeter has length *n* centimeters, where *n* is a positive integer with $n \geq 3$?

15. The Grand Old Duke of York had 10 000 men. He lost 10% of them on the way to the top of the hill, and he lost 15% of the rest as he marched them back down the hill. What percentage of the 10 000 men were still there when they reached the bottom of the hill?

A $76\frac{1}{2}\%$ B 75% C $73\frac{1}{2}\%$ D $66\frac{2}{3}\%$ E 25%

Solution: **A**

After losing 10% of the men, the Grand Old Duke of York was left with 90% of them. After losing 15% of these, he was left with 85% of the remaining 90% men. So he is left with $\frac{85}{100} \times \frac{90}{100}$ $= \frac{7650}{10000}$ $= \frac{76.5}{100}$ of the men he started, that is, $76\frac{1}{2}\%$ of the original number of men.

16.	Ulysses, Kim, Mei and Tanika have their 12th, 14th, 15th and 15th birthdays today. In what year will their ages first total 100?

A 2023 B 2024 C 2025 D 2057 E 2113

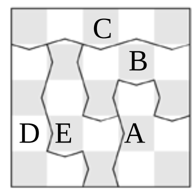
Solution: **B**

Today the total of their ages is $12+14+15+15=56$. Each year they are all grow 1 year older, and so the total of their ages increases by 4. This needs to increase by $100-56=44$ years in order to reach 100 years. This will take $\frac{44}{4}=11$ years. In 11 years time it will be $2013+11=2024$.

17. A 5 cm × 5 cm square is cut into five pieces, as shown.

Each cut is a sequence of identical copies of the same shape but pointing up, down, left or right.

Which piece has the longest perimeter?



A B C D E

Solution: **E**

We let the length of the shape be x cm. Since this shape is longer than the side length of one of the squares, $1<x$. The perimeters of the five pieces are, in centimetres, A $4+6x$, B $2+10x$, C $7+5x$, D $6+6x$ and E $1+11x$, respectively. As $1<x$, E has the longest perimeter.

18. Weighing the baby at the clinic was a problem. The baby would not keep still and caused the scales to wobble. So I held the baby and stood on the scales while the nurse read off 78 kg. Then the nurse held the baby while I read off 69 kg. Finally I held the nurse while the baby read off 137 kg. What was the combined weight of all three ?

A 142 kg B 147 kg C 206 kg D 215 kg E 284 kg

(*This problem appeared in the first Schools' Mathematical Challenge in 1988 – 25 years ago.*)

Solution: **A**

We let the weights of the baby, the nurse and myself be x kg, y kg and z kg, respectively. The information we are given implies that $x+z=78$, $x+y=69$ and $y+z=137$. Adding these three equations gives $(x+z)+(x+y)+(y+z)=78+69+137$, that is, $2x+2y+2z=284$. It follows that $x+y+z=142$. So the combined weight of all three was 142 kg.

Extension Problem

18.1 What were the individual weights of the baby, the nurse and me?

19. A swimming club has three categories of members: junior, senior, veteran. The ratio of junior to senior members is 3:2 and the ratio of senior members to veterans is 5:2 .

Which of the following could be the total number of members in the swimming club?

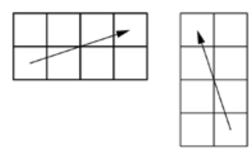
A 30 B 35 C 48 D 58 E 60

Solution: **D**

Suppose that there are x junior members. Then, as the ratio of junior to senior members is 3:2, there are $\frac{2}{3}x$ senior members. Hence, as the ratio of senior members to veterans is 5:2, there are $\frac{2}{5}(\frac{2}{3}x) = \frac{4}{15}x$ veterans. Therefore, the total number of members is $x + \frac{2}{3}x + \frac{4}{15}x = \frac{29}{15}x$. For this to be an integer, x must be a multiple of 15. It follows that $\frac{29}{15}x$ is a multiple of 29. Of the given options, only 58 is a multiple of 29. [This corresponds to taking $x = 30$. In this case there are 30 juniors, 20 seniors and 8 veterans in the club.]

Note: This is another problem where fractions can be avoided by thinking ahead. You may notice that the number of senior members must be a multiple of 2 and of 5. So this number must be a multiple of 10. So let 10*x* be the number of senior members. In then follows that there are 15*x* junior members and 4*x* veterans. So there are 29*x* members altogether. It follows that the number of members is a multiple of 29.

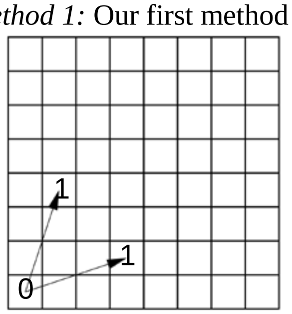
20. A 'long knight' moves on a square grid. A single move, as shown, consists of moving three squares in one direction (horizontally or vertically) and then one square at right angles to the first direction. What is the smallest number of moves a long knight requires to go from one corner of an 8 × 8 square board to the diagonally opposite corner?



A 4 B 5 C 6 D 7 E 8

Solution: **B**

Method 1: Our first method is systematic but long winded.



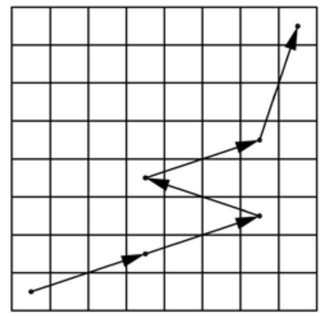
2		2					
		2		2			
	1						
2				2		2	
			1				
0		2				2	

	3		3		3		5
2		2		4		4	
	3		3		3		3
4		2		2		4	
	1		3		3		3
2		4		2		2	
	3		1		3		3
0		2		4		2	

In the left hand diagram we have put 0 in the square where the long knight starts and 1s in the squares which the long knight can reach in 1 move. In the middle diagram we have put a 2 in all the squares, other than that already marked 0, that the long knight can reach in one move from the squares marked 1. The nine squares marked with a 2 in this diagram are all the squares that the long knight can reach in two moves.

Next we put a 3 in all the squares, other than those already marked, which the long knight can reach in one move from the squares marked with a 2, and so on. The completed diagram is shown on the right. The number in a square indicates the smallest number of moves it takes the long knight to reach that square. We see from this that it takes the long knight five moves to reach the corner which is diagonally opposite the square where it started.

Method 2: We note that to get from the bottom left-hand corner to the top right-hand corner the long knight must end up 7 squares up and 7 squares right from its original position, a total of 14 squares. In any one move, it goes at most 3 squares up and 1 squares to the right, or at most 1 square up and 3 squares to the right, in each case a total of 4 squares.

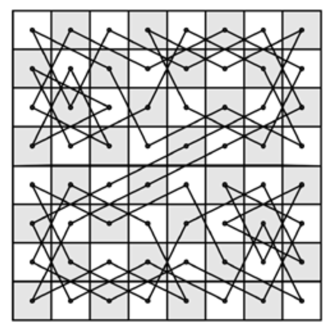


So after 3 or fewer moves it cannot end up in the top right-hand square. In 4 moves the number of squares it will have moved upwards must be even, as it will be the sum of four odd numbers, and so cannot be the required 7 squares to reach the top right-hand corner. Therefore at least 5 moves are needed. The diagram shows one way in which the task may be achieved in 5 moves.

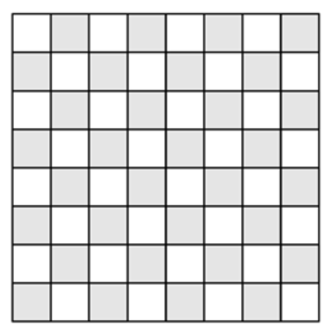
Extension Problems

20.1 In how many different ways can the long knight get from the bottom left-hand corner to the top right-hand corner in 5 moves?

20.2 A *knight's tour on a chessboard* is a sequence of 64 moves by a chess knight on an 8×8 grid in which it ends up on the square where it begins, and visits each of the other squares exactly once. An example of a knight's tour is shown on the right. This knight's tour was found by the great Swiss mathematician Leonhard Euler (1707-1783).



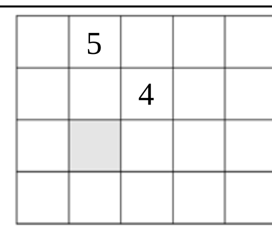
It is not possible to find a complete long knight's tour of a chessboard because, as we have seen in the solution to Question 20, a long knight starting on a particular square cannot reach half of the squares on the chessboard. If it starts on a white square it can only reach the other white squares, and if it starts on a black square it can only reach the other black squares.



Can you find a long knight's tour of the black squares, that is, a sequence of 32 moves by a long knight, starting on a black square, ending up on the square where it begins, and visiting each of the other black squares exactly once?

21. The 5×4 grid is divided into blocks. Each block is a square or a rectangle and contains the number of cells indicated by the integer within it.

Which integer will be in the same block as the shaded cell?

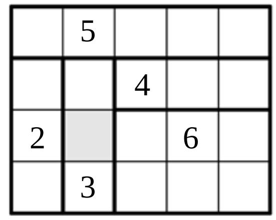


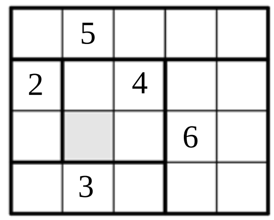
A 2 B 3 C 4 D 5 E 6

2 6

3

Solution: **C**





The 5 must be in a 5×1 block, and this can only be made up of the top row of the grid. There is not room in the grid for a 6×1 block, so the 6 must be in a 3×2 block. There are only two possibilities for a 3×2 block which does not include any of the other numbers and which does not overlap the 5×1 block. These are shown in the two diagrams on the right.

If the 3×2 block is arranged as in the top diagram, the 3 must be in the 3×1 block shown. But then there is not space to put the 4 in a block of four squares. Therefore the 3×2 block must be as in the lower diagram Then the 4 must be in a 2×2 block and the diagram may be completed as shown.

We therefore see that the shaded square must be in the same block as the 4.

22. Two numbers in the 4 × 4 grid can be swapped to create a Magic Square (in which all rows, all columns and both main diagonals add to the same total).

What is the sum of these two numbers?

A 12 B 15 C 22 D 26 E 28

9	6	3	16
4	13	10	5
14	1	8	11
7	12	15	2

Solution: **E**

We have included, in italics, the row and column totals. The circled numbers are the totals of the numbers in the two main diagonals.

The sum of all the numbers in the square is $1+2+...+15+16$ .This is the 16th triangular number, $\frac{1}{2}(16\times17)=136$. So the total of each row, column and main diagonal should be $\frac{1}{4}(136)=34$.

	9	6	3	16	*34*
	4	13	10	5	*32*
	14	1	8	11	*34*
	7	12	15	2	*36*
34	*34*	*32*	*36*	*34*	*32*

We thus see that we need to increase the totals of row 2 and column 2 by 2, and decrease the totals of row 4 and column 3 by 2. Clearly, the only way to achieve this by swapping just two numbers is to swap 13, in row 2 and column 2, with 15 in row 4 and column 3. When we swap 13 and 15 the total on the main diagonal from top left to bottom right is increased by 2 to 34, while the total on the other main diagonal is unchanged. So by swapping 13 and 15 we create a magic square. $13+15=28$. The discussion above makes it clear that this is the only swap that makes the total of each row and column, and the main diagonals, equal to 34.

23.	In our school netball league a team gains a certain whole number of points if it wins a game, a lower whole number of points if it draws a game and no points if it loses a game. After 10 games my team has won 7 games, drawn 3 and gained 44 points. My sister's team has won 5 games, drawn 2 and lost 3. How many points has her team gained?

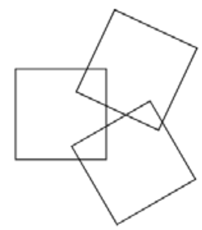
A 28 B 29 C 30 D 31 E 32

Solution: **D**

Suppose that there are w points for a win, and d points for a draw. Since my team gains 44 points from 7 wins and 3 draws, $7w+3d=44$. Here w and d are positive integers with $w>d$. Since $7w<44$, we have $1\le w\le 6$. Since $3d=44-7w$, $44-7w$ must be a multiple of 3. The only whole numbers in the range $1\le w\le 6$, for which $44-7w$ is a multiple of 3, are $w=2$ and $w=5$. When $w=2$, $3d=44-7w=30$, giving $d=10$, contradicting $w>d$. When $w=5$, $3d=44-7w=9$, giving $d=3$. In this case $w>d$. So $w=5$ and $d=3$. Therefore, my sister's team with 5 wins and 2 draws has gained $5\times5+2\times3=31$ points.

24. Three congruent squares overlap as shown. The area of the three overlapping sections are 2 cm^2, 5 cm^2 and 8 cm^2 respectively. The total area of the non-overlapping parts of the square is 117 cm^2.

What is the side-length of each square?

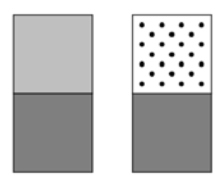


A 6 cm B 7 cm E 8cm D 9cm E 10 cm

Solution: **B**

The total area of the three squares is the sum of area of the non-overlapping parts and twice the areas of the overlapping sections as each of these forms part of two squares. So the total area of the squares is, in cm^2, $117+2\times(2+5+8)=117+2\times15=117+30=147$. Hence the area of one of the squares is one third of this, that is, 49 cm^2. Therefore the side-length of each square is 7 cm.

25. For Beatrix's latest art installation, she has fixed a 2 × 2 square sheet of steel to a wall. She has two 1 × 2 magnetic tiles, both of which she attaches to the steel sheet, in any orientation, so that none of the sheet is visible and the line separating the two tiles cannot be seen. As shown alongside, one tile has one black cell and one grey cell; the other tile has one black cell and one spotted cell.



How many different looking 2 × 2 installations can Beatrix obtain?

A 4 B 8 C 12 D 14 E 24

Solution: **C**

In the 2×2 square sheet, there are 4 positions in which Beatrix could place the spotted cell. For each position of the spotted cell, there remain 3 positions where she could place the grey cell. Once she has placed these, she has no choice for the two black cells. Hence there are $4\times3=12$ possible installations. It can be checked that Beatrix can make each of these installations by positioning the two tiles appropriately. [She can create each of the installations in which the black squares occupy the diagonal positions in two ways, and all the other installations in just one way.] So Beatrix can create 12 differently looking installations.

UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 25th APRIL 2013

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

Institute and Faculty of Actuaries

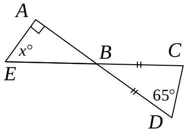
SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **E** All of the alternatives involve subtracting a number from 1. The largest result, therefore, will correspond to the smallest number to be subtracted, i.e. 0.00001.

2. **E** Their average height is $\frac{2.1 + 1.4}{2}$ m $= 1.75$ m.

3. **C** Triangle BCD is isosceles, so $\angle BCD = \angle BDC = 65°$.
The sum of the interior angles of a triangle is 180° so $\angle CBD = (180 - 2 \times 65)° = 50°$.
Therefore $\angle ABE = 50°$ (vertically opposite angles). So $\angle AEB = (180 - 90 - 50)° = 40°$.



4. **C** Distance travelled = average speed × time of travel, so Gill travelled between 15 km and 20 km. Of the alternatives given, only 19 km lies in this interval.

5. **D** The diagram shows the four lines of symmetry.

6. **A** $((1 - 1) - 1) - (1 - (1 - 1)) = (0 - 1) - (1 - 0) = -1 - 1 = -2$.

7. **D** Let the number of balls collected by Roger be x. Then Andy collects $2x$ balls and Maria collects $(2x - 5)$ balls. So $x + 2x + 2x - 5 = 35$, i.e. $5x = 40$, i.e. $x = 8$. So Andy collected 16 balls.

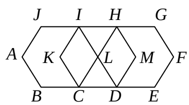
8. **A** The number 3 on the top ruler (which is 7cm from the left-hand end) aligns with the 4 on the bottom one (which is 6cm from the right-hand end). Thus $L = 7 + 6 = 13$.

9. **B** Let there be b boys and g girls in the family. Then Peter has g sisters and $(b - 1)$ brothers. So $g = 3(b - 1)$. Louise has $(g - 1)$ sisters and b brothers. So $g - 1 = 2b$. Therefore $2b + 1 = 3b - 3$, i.e. $b = 4$. So $g = 9$.
Therefore there are 4 boys and 9 girls in the family, i.e. 13 children in total.

10. **E** The top and bottom faces of the stack and the two touching faces form two pairs of opposite faces.
So the total number of pips on these four faces is $2 \times 7 = 14$. Therefore the total number of pips on the top and bottom faces of the stack is $14 - 5 = 9$.

11. **C** After Usain has run 100 m, his mum has run 50 m and Turbo has 'run' 10 m. So the distance between Usain's mum and Turbo is 40 m.

12. **E** Figure $ABEFGJ$ itself is a hexagon. There are three hexagons congruent to $ABCLIJ$; two hexagons congruent to $ABDMHJ$; four hexagons congruent to $ABCKIJ$; two hexagons congruent to $ABDLHJ$. So in total there are twelve hexagons.



13. **D** After the first coat, half of the paint is left. So after the second coat, the volume of paint remaining is one third of half of the capacity of the tin, i.e. one sixth of three litres = 500 ml.

14. **B** Let the two equal sides of the isosceles triangle have length a and the other side have length b. Then $2a + b = 20$. Since the sum of the lengths of any two sides of a triangle is greater than the length of the third, $2a > b$. Hence $4a > 2a + b$. So $4a > 20$, i.e. $a > 5$. Also $a < 10$ since $2a + b = 20$. So the possibilities are $a = 6, b = 8$; $a = 7, b = 6$; $a = 8, b = 4$; and $a = 9, b = 2$.

15. **A** When he starts to come down the hill, the Grand Old Duke of York has 90% of his men left. He loses 15% of these, so at the bottom of the hill he has 85% of 90% of the original number left. As $\frac{85}{100} \times 90 = 76\frac{1}{2}$, this means that $76\frac{1}{2}$% of his men were still there when they reached the bottom of the hill.

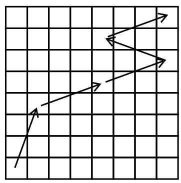
16. **B** The sum of the ages of the four children is $12 + 14 + 15 + 15 = 56$. Each year on their birthday, this sum increases by 4. So the number of years before the sum reaches 100 is $(100 - 56) \div 4 = 11$. Therefore their ages will first total 100 in 2024.

17. **E** Let x cm be the length of the ﹀ shape. Although x is not given, it is clear that $x > 1$. The lengths, in cm, of the perimeters of pieces A, B, C, D, E are $4 + 6x$, $2 + 10x$, $7 + 5x$, $6 + 6x$, $1 + 11x$ respectively. As $4 + 6x < 6 + 6x$, the piece with the longest perimeter is B, C, D or E. As $x > 1$, it may be deduced that $7 + 5x < 6 + 6x < 2 + 10x < 1 + 11x$, so E has the longest perimeter.

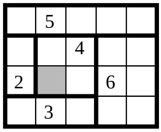
18. **A** Let the weights, in kg, of baby, nurse and me be x, y, z respectively. Then $x + z = 78$; $x + y = 69$; $y + z = 137$. Adding all three equations gives $2x + 2y + 2z = 284$, so $x + y + z = 284 \div 2 = 142$.
(*To find the combined weight, it is not necessary to find the individual weights, but baby weighs 5kg, nurse weighs 64 kg and I weigh 73 kg.*)

19. **D** For every 2 senior members in the swimming club there are 3 junior members. For every 5 senior members there are 2 veteran members. The lowest common multiple of 2 and 5 is 10, so it may be deduced that the number of senior members is a multiple of 10. For every 10 senior members in the swimming club there are 15 junior members and 4 veteran members. So the total number of members is a multiple of 29. Of the alternatives given, the only multiple of 29 is 58.

20. **B** The ‘long knight’ needs to move exactly seven squares to the right and exactly seven squares upwards. Although it is possible to move seven squares to the right in three moves (1, 3 and 3), in doing so it could move upwards by a maximum of five squares (3, 1 and 1). Similarly, it could move seven squares upwards in three moves, but could then move a maximum of five squares to the right. In four moves, the number of squares moved to the right must be even, since it is the sum of four odd numbers. So at least five moves are required and the diagram shows one way in which the task may be achieved in five moves.



21. **C** As 5 is a prime number, it must lie in a 5×1 rectangle. So the only possibility is the rectangle which covers the top row of the grid. Now consider 6: there is insufficient room for a 6×1 rectangle so it must lie in a 3×2 rectangle. There are only two such rectangles which include 6 but do not include either 4 or 3. If 6 comes in the middle of the top row of a 2×3 rectangle then there is space for a 3×1 rectangle including 3. But then there is not enough space for a rectangle including 4. So 6 must be placed in the rectangle shown. There is now insufficient room to place 4 in a 4×1 rectangle so it must lie in the 2×2 square shown, which includes the shaded square. This leaves the grid to be completed as shown.



22. **E** The diagram shows the totals of the rows and columns. The circled numbers are the total of the numbers in the two main diagonals. Note, by considering the average values of the rows and columns, that each should total 34. Row 2 and column 2 are both 2 short. So their common entry, 13, needs to increase by 2. So 13 must be interchanged with 15. (This change also reduces row 4 and column 3 by 2 and increases the main diagonal by 2, thus making all the sums equal 34 as desired.) So the sum of the numbers to be swapped is 28.

	9	6	3	16	34
	4	13	10	5	32
	14	1	8	11	34
	7	12	15	2	36
34	34	32	36	34	32

23. **D** Let the points awarded for a win and a draw be w and d respectively. Then $7w + 3d = 44$. The only positive integer solutions of this equation are $w = 2$, $d = 10$ and $w = 5$, $d = 3$. However, more points are awarded for a win than for a draw so we deduce that 5 points are awarded for a win and 3 points for a draw. So the number of points gained by my sister's team is $5 \times 5 + 2 \times 3 = 31$.

24. **B** Each of the overlapping areas contributes to the area of exactly two squares. So the total area of the three squares is equal to the area of the non-overlapping parts of the squares plus twice the total of the three overlapping areas i.e. $(117 + 2(2 + 5 + 8))\text{ cm}^2 = (117 + 30)\text{ cm}^2 = 147\text{ cm}^2$.
So the area of each square is $(147 \div 3)\text{ cm}^2 = 49\text{ cm}^2$. Therefore the length of the side of each square is 7 cm.

25. **C** By arranging the tiles in suitable positions it is possible to place the 1×1 spotted square in any one of four corners of the steel sheet and then to place the grey square in any one of the other three corners. The other two corners will then be occupied by black squares. So, in total, there are $4 \times 3 = 12$ different looking installations.)