UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 27th APRIL 2006

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

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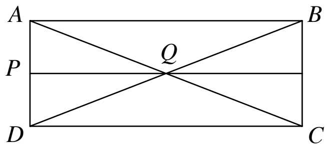
1. **B** $6002 - 2002 = 4000$, so $6002 - 2006 = 4000 - 4 = 3996$.

2. **A** The missing numbers are A: 5, 6, 7; B: 9; C: 12, 1; D: 8, 9; E: 1, 2, 3, 4, 5. So the largest sum of numbers has been eaten on face A.

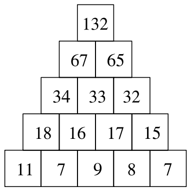
3. **C** There must be at least two boys and two girls in the family. If, for example, there was exactly one boy, then that child would not have a brother. The same argument applies to girls.

4. **C** There are 4 triangles congruent to ΔAQP (including ΔAQP itself), 2 triangles congruent to ΔAQB, 2 triangles congruent to ΔAQD, 4 triangles congruent to ΔACD.



5. **B** Since 9 houses have been knocked down, $123 - 9\,(= 114)$ remain.

6. **D** The completed tower is shown on the right. There is more than one order in which missing numbers may be inserted – and it is not necessary to complete the tower in order to determine n – but one possible order is 8, 17, 32, 33, 16, 7, 18, 11, 67, 132.



7. **A** Let the size of the reflex angle in the quadrilateral be $y°$. Then $x + y = 360$ (angles at a point). Furthermore, $35 + 15 + 25 + y = 360$ (sum of interior angles of a quadrilateral). So $x = 35 + 15 + 25$.

8. **E** Try it for yourself and see!

9. **E** $2 \times \sqrt{64} = 2 \times 8 = 16$; $22 - 2 \times 3 = 22 - 6 = 16$; $2^4 = 16$; $5^2 - 3^2 = 25 - 9 = 16$; $4 + 4 \times 2 = 4 + 8 = 12$.

10. **D** Note that you cannot remove two touching coins; for if you did then any coin making a triangle with those two coins could then slide. So if the middle coin was removed, no other coin could then be removed. So next consider removing first an outer coin. There are two outer coins touching it – which, as argued before, cannot now be removed. We consider two cases. If the next coin to go is that opposite the coin already removed, then each remaining coin is adjacent to a removed coin – and the process stops. Finally, suppose the next coin removed is next but one (around the outer edge) from the first removed coin. Then a third coin, two further around the edge, is the only possibility for removal – and then the process halts again. So the largest number of coins that may be removed is three.

11. **B** The second sign is 143 miles from London and 250 miles from Edinburgh, so it is 393 miles from Edinburgh to London.

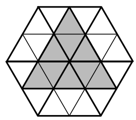
12. **C** The letter ‘e’ already occurs eight times, so both ‘nine’ and ‘eleven’ may be placed in the gap to make the sentence true.

13. **A** When divided by 7, the five options have remainder 6, 1, 3, 5, 0 respectively.

14. **C** The largest and smallest possible fields of play have areas 13000 square yards and 5000 square yards respectively.

15. **D** 6957 is less than 7000 and 31248 is greater than 28000, so $\frac{6957}{31248} < \frac{1}{4}$. It is left as an exercise for the reader to confirm that in the other four fractions the denominator is four times the numerator.
(*Note that each of these fractions uses all the digits from 1 to 9 inclusive exactly once. The question includes the only four fractions of this type which simplify to* $\frac{1}{4}$.)

16. **C** The diagram shows that the regular hexagon may be divided into six congruent equilateral triangles, each of which may be further divided into four smaller congruent equilateral triangles. So the hexagon consists of 24 congruent triangles, 9 of which make up the shaded equilateral triangle.



17. **D** Clearly, there must be a minimum of two ‘on’ switches. Two ‘on’ switches and three ‘off’ switches may be set in only one way: ‘off’, ‘on’, ‘off’, ‘on’, ‘off’. Three ‘on’ switches and two ‘off’ switches may be set in six different ways. Four ‘on’ switches and one ‘off’ switch may be set in five different ways. Finally, five ‘on’ switches can be set in just one way. So there are 13 ways in all.
(*Note: it is left as an exercise for the reader to show that if that there are n switches then the number of different ways in which they may be set so that no two adjacent switches are in the ‘off’ position is the (n + 2)th term of the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, …*)

18. **A** The total of the numbers in the magic square is 7 + 8 + 9 + …15, that is 99. So each row, column and main diagonal has sum 33 and we deduce that the number in the top right hand corner of the square is 12. The remaining blank squares may now be completed in terms of n, as shown. Considering the middle row: $31 - 2n + 19 - n + 7 = 33$, so $n = 8$.

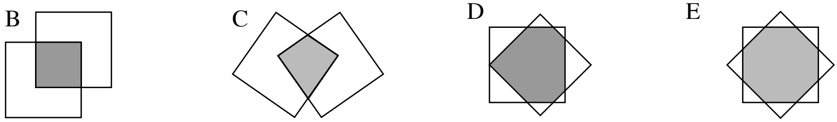
n	$21 - n$	12
$31 - 2n$	$19 - n$	7
$n + 2$	$17 - n$	14

Alternate solution: it is known that the number in the centre of a magic square is always equal to the mean of the nine numbers making up the square, in this case 11. So $n = 33 - 14 - 11 = 8$.

19. **D** After Pinocchio has told 9 lies, the length of his nose will be $2^9 \times 5\,\text{cm}$ $= 512 \times 5\,\text{cm} = 25.6\,\text{m}$. This is close to the length of a tennis court, which is 23.8m.

20. **E** As 2 is the only even prime, it must be one of the three primes which total 40 since any three odd numbers have an odd total. So the other two primes sum to 38. Different odd numbers which total 38 are (1, 37), (3, 35), (5, 33), (7, 31), (9, 29), (11, 27), (13, 25), (15, 23), (17, 21).
Of these pairs, only (7, 31) consists of two primes, so the required difference is 24.

21. **A** If the overlapping region is a triangle, two of its sides must be adjacent sides of one of the squares. So one of its angles will be a right angle and therefore an equilateral triangle could not appear as the overlapping region. The diagrams show how the other options could appear as the overlapping region.



22. **B** The number is 1 less than a multiple of 3, 1 less than a multiple of 4 and also 1 less than a multiple of 5. So, since 3, 4 and 5 have no factors in common, it is 1 less than a multiple of the product of 3, 4 and 5, that is 1 less than a multiple of 60. As it is less than 100, the only possibility is 59, which leaves remainder 3 when divided by 7.
(*Note that the Chinese Remainder Theorem provides a general method for solving problems like this*.)

23. **B** Let the number of girls at the camp be x. Then there are $\frac{3x}{4}$ boys and $\frac{7x}{5}$ adults. So the ratio of children to adults is $\frac{7x}{4} : \frac{7x}{5}$, that is 5:4.

24. **C** The total of the four triples is three times the total of the original four numbers. So these numbers have sum $(115 + 153 + 169 + 181) \div 3$, that is 206. Hence the largest of Amrita's numbers is $206 - 115$, that is 91.

25. **B** If $\frac{1}{2}n$ is a three-digit whole number, then n is an even number between 200 and 1998 inclusive. If $2n$ is a three-digit whole number, then n is a whole number between 50 and 499 inclusive. So for both conditions to hold, n is an even number between 200 and 499 inclusive and there are 150 such numbers.