UK JUNIOR MATHEMATICAL CHALLENGE

TUESDAY 27th APRIL 2004

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **B** Of the letters in question, only S does not have at least one line of symmetry.

2. **E** $112 \div 7 = 16$. As none of the other options differs from 112 by a multiple of 7, 112 is the only one of these numbers which is exactly divisible by 7.

3. **A** The required condition will next be met in 2014.

4. **D** Every allowable route must pass through the centre point of the bow tie. There are two routes from P to the centre point, and for each of these there are two routes from the centre point to Q. So the total number of different routes $= 2 \times 2 = 4$.

5. **C** Eighteen has 8 letters and 18 is not a multiple of 8. Of the other options, 6 is a multiple of 3, 12 is a multiple of 6, 70 is a multiple of 7 and 90 is a multiple of 6.

6. **E** The differences between the given fractions and 1 are, respectively, $\frac{11}{23}, \frac{11}{34}, \frac{11}{45}, \frac{11}{56}$ and $\frac{11}{67}$. The smallest of these is $\frac{11}{67}$, so $\frac{56}{67}$ is nearest to 1.

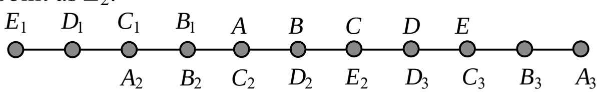
7. **C** There are 8 demisemiquavers in a crotchet and 4 crotchets in a semibreve, so there are 32 demisemiquavers in a semibreve.

8. **E** The original pyramid had 8 edges. Cutting off the top corner adds 4 edges, whilst cutting off the other 4 corners adds 3 extra edges in each case. So the total number of edges is $8 + 4 + 4 \times 3 = 24$.

9. **C** Pa Bean does not eat more than half the beans, so he eats at most 11 beans. Ma Bean eats the same number of beans as both children together, so she eats an even number of beans which is at least one quarter of the total number of beans eaten. Therefore she eats at least 6 beans. If she does eat 6 beans, then Pa Bean eats 11 beans, which is consistent with the information given. However, if Ma Bean eats 8 or more beans, then Pa Bean eats at most 7 beans and this is impossible as we are told that Pa Bean eats more beans than Ma Bean. So Pa Bean eats 11 beans.

10. **B** The coins consisting of one of each type must add up to £1.25. The only way that this total may be made with coins of different denominations is by using a £1 coin, a 20p coin and a 5p coin.

11. **D** The diagram shows the positions of the points after the rotations. Note that A_1, which is not marked, is the same point as A. Similarly, E_3, which is not marked, is the same point as E_2.



12. **B** On Tuesday the White Rabbit will be 8 minutes late, on Wednesday 4 minutes late, on Thursday 2 minutes late and on Friday 1 minute late. So on the following Monday the White Rabbit will be 30 seconds late and on the day after that he will be 15 seconds late.

13. **A** As $\angle QPR = 40°$, $\angle PQR + \angle PRQ = 180° - 40° = 140°$.
So $\angle SQR + \angle SRQ = 140° \div 2 = 70°$. Therefore $\angle QSR = 180° - 70° = 110°$.

14. **C** The 8 individuals will each wrestle 6 others. This suggests that the number of bouts is $8 \times 6 = 48$. However, each bout has been counted twice in this calculation, so the number of bouts is $48 \div 2 = 24$.

15. **D** After Thursday, two thirds of Granny's pension is left. So after she has spent one quarter of this amount on Friday, the fraction of the original amount which remains is three quarters of two thirds, i.e. one half.

16. **C** The total angle turned through after each of the first 4 moves is 10°, 30°, 60° and 100°. So the robot does not face due East at the end of a move in its first complete revolution. The total angle it has turned through after each of the next 5 moves is 150°, 210°, 280°, 360° and 450°, so at the end of the 9th move the robot *does* face due East. As the robot moves 5m in each move, the distance it travels is 45m.
Note that this solution assumes that the robot is not starting the process close to the North Pole!

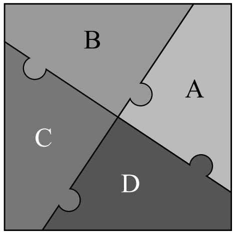
17. **D** Statements such as those in this question may sometimes be shown to be false by considering the units digit of the expressions on each side. The units digit of $44^2 + 77^2$ is 5; the units digit of $55^2 + 66^2$ and hence also of $66^2 + 55^2$ is 1; the units digit of $88^2 + 33^2$ is 3 and that of $99^2 + 22^2$ is 5. So four of the statements are definitely false. It remains to check that $88^2 + 33^2 = 7744 + 1089 = 8833$.

18. **B** Each of the 2004 squares, apart from those at the top and bottom of the shape, contributes 2 cm to the perimeter of the figure.
The other two squares contribute 3 cm each, so the perimeter is $(2002 \times 2 + 2 \times 3)$ cm $= 4010$ cm.

19. **B** From the three equations we see that $(abc)^2 = 2 \times 24 \times 3 = 144$ and so, since abc is positive, $abc = 12$. Then the third equation tells us that $a = \frac{1}{2}$, the second that $b = 4$ and the first that $c = 6$. Therefore $a + b + c = 10\frac{1}{2}$.

20. **A** As $PQRST$ is a regular pentagon, each of its internal angles is 108°. The internal angles of the quadrilateral $PRST$ add up to 360° and so by symmetry $\angle PRS = \angle RPT = \frac{1}{2}(360° - 2 \times 108°) = 72°$. Each interior angle of a regular hexagon is 120°, so $\angle PRU = 120°$.
Therefore $\angle SRU = \angle PRU - \angle PRS = 120° - 72° = 48°$.

21. **E** In total, the five pieces have four "holes" and six "tabs", so we can deduce that either B or E, both of which have two "tabs", will not be used.
Shape A, therefore, is used and, after a clockwise rotation of 90°, will fit together with shape B. This suggests that it is shape E which is not used and shapes C and D will indeed complete the jigsaw after rotations of 90° and 180° anticlockwise respectively.



22. **D** In the four options other than D, the digits may be rearranged as follows:

A $21 \times 3 = 63$ B $18 \times 4 = 72$ C $38 \times 2 = 76$ E $29 \times 3 = 87$.

23. **E** Consider the thousands column. The letters represent different digits so, as S is 3, M is 2 and there is a carry of 1 from the hundreds column. Therefore A is 9, U is 0 and there is also a carry of 1 from the tens column. In the units column, $O + Y$ produces a units digit of 3, so $O + Y = 3$ or $O + Y = 13$. However, $O + Y = 3$ requires one of O, Y to equal zero (impossible as $U = 0$) or 2 (also impossible as $M = 2$). So $O + Y = 13$. We can also deduce that N is 8, since, in the tens column, $1 + 3 + N = 12$. The pairs of digits which produce a sum of 13 are 4 and 9, 5 and 8, 6 and 7. As A is 9 and N is 8, the only possible values for O and Y are 6 and 7. These are interchangeable, but in both cases $Y \times O = 42$.

24. **A** Let the length and breadth of each of the rectangles be a cm and b cm respectively. Then $2a = 3b$ and $a + b = 15$. So $2a + 2b = 30$.
Therefore $3b + 2b = 30$, that is $b = 6$.
So the total area covered by the five rectangles is that of a rectangle measuring 18 cm by 15 cm, i.e. 270 cm^2.

25. **D** Let the first two terms of the sequence be a and b respectively. Then the next three terms are $a + b$, $a + 2b$, $2a + 3b$. So $2a + 3b = 2004$. For a to be as large as possible, we need b to be as small as possible, consistent with their both being positive integers. If $b = 1$ then $2a = 2001$, but a is an integer, so $b \neq 1$.
However, if $b = 2$ then $2a = 1998$, so the maximum possible value of a is 999.