UK JUNIOR MATHEMATICAL CHALLENGE

TUESDAY 29th APRIL 2003

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

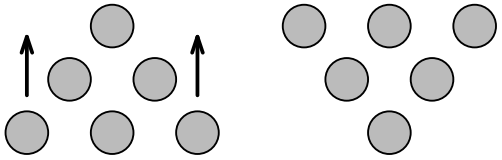
This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **E** ½ of 199 = ½(200 − 1) = 100 − ½ = 99½.

2. **D** There are 100 teeth and 99 gaps so the length of the comb = 199mm = 19.9 cm.

3. **A** He will save 9 × 7 minutes.

4. **B** The task cannot be done by moving one plate, since removing any one plate from the formation of plates does not leave a formation which is part of the desired final formation. However, the formation may be turned upside down, by moving two of the plates in the bottom row, as shown.



5. **C** The next time the six digits are all different, the meter will read 098671.

6. **B** Orange and yellow are B and D, though not necessarily in that order. Blue is E and, as orange is not next to blue, it cannot be D. So orange is B.

7. **C** Buying a bottle of juice, drinking the juice and returning the bottle involves a net cost of $1. Therefore, after doing this seven times, you will have $3 left i.e. just sufficient to buy an eighth bottle. After drinking the juice and returning this bottle, you will have $2 left, but this is not enough to buy a ninth bottle.

8. **E** When these numbers are divided by 9, the remainders are 5, 5, 5, 5 and 6 respectively.
(*The digital root of a number is the sum of the digits of that number. If this is greater than 9, then the sum of its digits is found. If necessary, the procedure is repeated until a single digit is obtained. Sometimes called 'casting out nines', when a whole number is divided by 9, the remainder is the digital root of that number unless the digital root is 9, in which case the number is a multiple of 9 so the remainder is zero.*)

9. **B** The total age of the original four members is 4 × 19 = 76. When the fifth member joins, their total age is 76 + 24 = 100. So the mean age is now 100 ÷ 5 = 20.

10. **D** As 18 lies between 16 and 25, $\sqrt{18}$ lies between 4 and 5. Of the options available, only 4.2 is in this range. To confirm that 4.2 is correct (to one decimal place), we note that $4.15^2 = 17.2225$ and $4.25^2 = 18.0625$ so $\sqrt{18}$ lies between 4.15 and 4.25.

11. **B** Let the cards intended for Carol, Holly and Ivy be C, H and I respectively. Nicolas can send Carol either H or I. In both cases, the recipients of the other two cards will then be determined as shown below.

Carol gets H, so Ivy gets C (as she cannot then get I) and Holly gets I.
or Carol gets I , so Holly gets C (as she cannot then get H) and Ivy gets H.

12. **A** Angles on a straight line add up to 180°. The base angles of the triangle are 67° and 71°. The angles of a triangle add to 180°. Therefore $x = 180° - (67° + 71°) = 42°$.

13. **D** As 2005 has units digit 5, 2005 is a multiple of 5. The sum of the digits of 2007 is 9, so 2007 is a multiple of 9; the sum of the digits of 2013 is 6, which means 2013 is a multiple of 3. This leaves 2009 and 2011. Now $2009 \div 7 = 287$ so four of the five options have been eliminated. To confirm that 2011 is prime, it is necessary to show that it is not a multiple of any prime less than $\sqrt{2011}$ and this is indeed the case.

14. **C** As each number after the second is the sum of all the previous numbers in the sequence, each number after the third is double the previous number. So the first 10 terms of the sequence are 2, 3, 5, 10, 20, 40, 80, 160, 320, 640.

15. **C** The information given suggests that each human swallows a spider while sleeping approximately once every nine years. Therefore the number of spiders consumed in this way in the UK in one year is approximately one ninth of 60 million, or approximately 7 million.

16. **D** For C to give a units digit of 8 when multiplied by 7, its value must be 4. There is a carry of 2, so B must be 1 to give 9 in the tens column. There is no carry so 7×6 gives 2 in the hundreds column, i.e. E is 2, with a carry of 4. With this carry, A must be 5 to give 9 in the thousands column and there is a carry of 3 into the ten thousands column, so it is D that is 3.

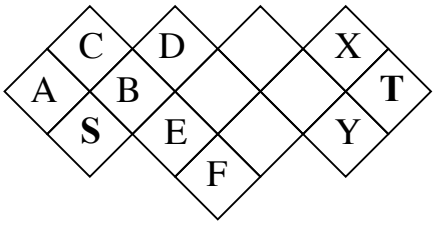
17. **E** The new rectangle will measure $(2a + 6)$ cm by 4 cm, so its area will be $(8a + 24)$ cm^2.

18. **D** The single strip of paper forms a Möebius band which has only one surface, so all three 'UKMT's are written on the same side of the paper!

19. **D** Let the number in the second cell be x. Then the sum of the numbers in the first and third cells is $2x$, so the number in the third cell is $2x - 8$. This means that $2x - 8$ is the mean of x and 20 so $x + 20 = 4x - 16$. Therefore $x = 12$. We can now deduce that the numbers are 8, 12, 16, 20 and 24.
(*Note that the number in each cell is 4 more than that in the adjoining cell i.e. the numbers are in 'arithmetic progression'. One of the properties of such a sequence is that each term is the mean of the two terms on either side of it.*)

20. **C** The grid may be considered to consist of four strips of wire of length a and five strips of length $\frac{3a}{4}$. So the total length required is $4a + 5 \times \frac{3a}{4} = \frac{31a}{4}$.

21. **C** From the starting square **S** the counter must first be moved to square A or square B. On the second move it must either go back to S, which is clearly not advisable, or to one of the squares C, D or E. Now to end at square T, the penultimate move must take the counter to either square X or square Y. As it is not possible to reach square X or Y in one move from any of squares C, D or E, the task must require at least five moves. One way of doing this is S → B → E → F → X → T, although this is not the only possible five-move sequence.



22. **E** Consider what happens when both builders sell 60 bricks (60 is the LCM of 10 and 12). Bob will receive £36, while Geri will receive £35, so for every 60 bricks which each builder sells, Bob gains £1 more than Geri.

23. **A** As triangle ABC is isosceles, $\angle ABC = \angle ACB = w°$. Also, $\angle ABC = x°$ (vertically opposite angles) so $w = x$. Now triangle ACD is also isosceles, so $\angle DAC = \angle DCA = w° = x°$. We now see that the second statement is true as the three angles on the straight line at A have sizes $y°$, $x°$ and $z°$. In a triangle, an exterior angle is equal to the sum of the two interior opposite angles, so in triangle ABC: $\angle ABC + \angle ACB = z°$. Therefore $z = x + w = 2x$ (since $w = x$). Thus all three of the statements are true.

24. **B** The sum of three digits is at most 27, so the first digit of the house number is 4. Hence the second digit is less than 3. If this second digit were even, then the house number and the sum of its digits would have the same parity (i.e. both would be even or both would be odd), so their sum would be even. However, their sum is odd so the second digit is 1. Now if the last digit is d then $410 + d + (4 + 1 + d) = 429$, so $d = 7$ and the product of the digits is 28.

25. **A** The resulting shape will consist of six squares. Of these, two will have three 'cut' sides (sides which are formed by the cuts made along the edges of the cube and which as a result are not also sides of adjoining squares); two will have two 'cut' sides which are perpendicular to each other and two will have two 'cut' sides which are parallel to each other. Only shapes A and E satisfy these conditions. Furthermore, we note that the squares with three 'cut' sides both share a side with a square which has two 'cut' sides parallel to each other. Shape A satisfies this also, whereas shape E does not.