TEAM SELECTION TEST 1

MONDAY 28 MAY 2001

08.30-13.00

1. Let O be the circumcentre and H the orthocentre of the acute-angled triangle ABC. Show that the minimum value of $OP + HP$, as P varies over the perimeter of the triangle, is exactly the circumradius of ABC.

2. Show that $\left\{n\sqrt{3}\right\} > \frac{1}{n\sqrt{3}}$ for every positive integer n. Show also that, for every real $c > 1$, there exists a positive integer n such that $\left\{n\sqrt{3}\right\} < \frac{c}{n\sqrt{3}}$.
[For any positive real x, the symbol $\{x\}$ denotes the fractional part of x, in other words the part of x to the right of the decimal point – for example, we have $\{3 \cdot 1415\} = 0 \cdot 1415$.]

3. The function F is defined on the non-negative integers and takes non-negative integer values. For every non-negative integer n it satisfies
(i) $F(4n) = F(2n) + F(n)$
(ii) $F(4n + 2) = F(4n) + 1$
(iii) $F(2n + 1) = F(2n) + 1$.
Show that, for any positive integer m, the number of integers n with $0 \leq n < 2^m$ satisfying $F(4n) = F(3n)$ is precisely $F(2^{m+1})$.