Next Selection Test: Exam 4

IMO camp, Oundle School

28-v-2008

after correction · *for K in places 5, true for K=5, false for K=5, real, K=5.*

Problem 10 Consider an infinite sequence of distinct positive integers (a_n) such that $a_n \leq 5n$ for all positive integers n. Show that the sequence contains infinitely many terms which have the property that their decimal digit sum is not a multiple of 5.

Problem 11 Let $m \geq 2$ be an integer. A positive integer n is called *m-good* if every positive integer a which is coprime to n has the property that n divides $a^m - 1$.

Show that any m-good number is at most $4m(2^m - 1)$.

Problem 12 A point X lies in the plane of triangle ABC. A circle Γ passing through X meets XA, XB and XC again at P, Q and R respectively. Let Γ meet the circles BXC, AXC and AXB again at K, L and M respectively.

Prove that PK, QL and RM are concurrent.

Time allowed: 4 hours 30 minutes