Oundle Test 4

30 May 2007

1. Consider triangle ABC. B_1 is on the line AC and the line BB_1 passes through the incentre I. The point C_1 is similarly defined. The line B_1C_1 meets the circumcircle of triangle ABC at M and N. Prove that the circumradius of triangle MIN is twice the circumradius of triangle ABC.

2. Find all pairs of functions $f, g : \mathbb{R} \to \mathbb{R}$ which satisfy:

   (a) $f(x \cdot g(y+1)) + y = x \cdot f(y) + f(x + g(y))$, for all $x, y \in \mathbb{R}$;

   (b) $f(0) + g(0) = 0$.

3. A positive integer N is said to be an "*n-good number*" if it satisfies the following two properties:

   (Property 1) N is divisible by at least n distinct primes

   (Property 2) There exist distinct positive divisors $1, x_2, \ldots, x_n$ of N such that
   $$1 + x_2 + \cdots + x_n = N.$$

   Show that there exists an "*n-good number*" for each $n \geq 6$.