Next Selection Test: Exam 3

IMO camp, Oundle School

27-v-2008

Problem 7 Find all injective functions $f : \mathbb{N} \longrightarrow \mathbb{N}$ such that, for each n,

$$f(f(n)) \leq \frac{n + f(n)}{2}.$$

Note that a function $g : A \longrightarrow B$ is injective iff whenever $f(x) = f(y)$, then $x = y$.

Problem 8 Let $ABCD$ be a parallelogram but not a rhombus. Let E be the foot of the perpendicular from B to AC. The line through E which is perpendicular to BD intersects BC at F and AB at G.

Show that $EF = EG$ if, and only if, $ABCD$ is a rectangle.

Problem 9 Find all positive integers n for which there are distinct integers $a_1, a_2, \ldots, a_n$ such that

$$\frac{1}{a_1} + \frac{2}{a_2} + \cdots + \frac{n}{a_n} = \frac{a_1 + a_2 + \cdots + a_n}{2}.$$

note correction: positive

Time allowed: 4 hours 30 minutes