Next Selection Test: Paper 2

Oundle School

30th May 2011

1. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that:

   (a) $f(x + f(x + 2y)) = f(2x) + f(2y)$ for all integers x, y;
   (b) $f(0) = 2$.

2. Let n be a positive integer, and let $\mathcal{L}$ be a collection of n lines in the plane in general position (which means that no three are concurrent and no two are parallel). Consider the interiors of the incircles of the triangles formed by each set of three lines in $\mathcal{L}$.

   For each n, what is the largest and smallest possible number of these circle interiors which are intersected by no lines from $\mathcal{L}$?

3. Let ABC be a triangle with a right angle at C. Let CN be an altitude. A circle Γ is tangent to the line segments BN, CN, and the circumcircle of ABC. If D is where Γ kisses BN, prove that CD bisects BCN.

Each question is worth seven marks.
Time: 4 hours, 30 minutes.