UK IMO Next Selection Test 2

Oundle 2006

1. Find all pairs (x, y) of positive integers satisfying the equation

$$(x + y)^x = x^y.$$

2. On a horizontal line, 2005 points are marked, each of which is either white or black. For every point, one finds the sum of the number of white points on the right of it and the number of black points on the left of it. Among the 2005 sums, exactly one number occurs an odd number of times. Find all possible values of this number.

3. Triangle ABC has incentre I. The line AI meets the circumcircle of ABC again at D. The feet of the perpendiculars from I to BD and CD are denoted E and F respectively. Given that $IE + IF = \frac{1}{2}AD$, determine the possible values of $\angle BAC$.

Time allowed $4\frac{1}{2}$ hours.