Oundle Test 1

27 May 2007

1. Triangle ABC has circumcentre O and centroid M. The lines OM and AM are perpendicular. Let AM meet the circumcircle of ABC again at A'. Lines BA' and AC intersect at D. Lines CA' and AB intersect at E. Prove that the circumcentre of triangle ADE lies on the circumcircle of ABC.

2. A pawn is placed on a square of an $n \times n$ board. There are two types of legal moves:

   (a) A pawn can move to a neighbouring square which shares a common edge with its current square.

   (b) A pawn can move to a neighbouring square which shares a common vertex, but not a common edge, with the current square.

   Consecutive moves must be of different types. Find all integers $n \geq 2$ for which there is a starting square and a sequence of moves such that the pawn visits each square exactly once. It is not required that the pawn returns to its starting square.

3. Prove that there do not exist different positive integers x and y which satisfy the equation

$$x^{2007} + y! = y^{2007} + x!.$$