UK IMO Next Selection Test 1

Oundle 2006

1. Does there exist a bounded function $f : \mathbb{R} \to \mathbb{R}$ with $f(1) > 0$ satisfying

$$f(x+y)^2 \geq f(x)^2 + f(2xy) + f(y)^2$$

for all $x, y \in \mathbb{R}$.

2. Let ABC be a triangle whose side lengths are all integers, and let D and E be the points at which the incircle of ABC touches BC and AC respectively.

If $|AD^2 - BE^2| \leq 2$, show that $AC = BC$.

3. Find the least value and the greatest value of the expression

$$P = x + y$$

where x, y are real numbers satisfying the condition

$$x - 3\sqrt{x+1} = 3\sqrt{y+2} - y.$$

Time allowed $4\frac{1}{2}$ *hours.*